where a is the nuclear scattering length and $\vec{P}$ is the polarization vector of the neutron beam. On substituting the above expression for the electronic matrix element, we obtain

$$\frac{d\sigma}{d\Omega'} = |\sum_{\vec{n}} e^{i\vec{K}\cdot\vec{n}}|^2 \left\{ a^2 + 2a\frac{\gamma e^2}{mc^2} Sf(\vec{K})\vec{P}\cdot\vec{q}(\vec{K}) + \left(\frac{\gamma e^2}{mc^2}\right)^2 S^2 |f(\vec{K})|^2 \vec{q}^2(\vec{K}) \right\}. \quad (10)$$

The polarization-dependent term here affords a method of determining the direction of $\vec{q}(\vec{K})$, for the cross section depends on the angle between $\vec{P}$ and $\vec{q}(\vec{K})$. If the cross section is measured for two different reflections as a function of the direction of $\vec{P}$, the variation in the direction of $\vec{q}(\vec{K})$ due to a noncollinear density can be detected. It is also possible to use the analysis of the direction of polarization of the final beam to detect this effect. The derivation of this is similar to the above and will not be given here.

In addition to being of inherent experimental and theoretical interest, noncollinear spin densities can greatly complicate the interpretation of neutron form-factor measurements and spin-structure determinations. An examination of Eq. (8) shows the sort of difficulties which may arise. Ordinarily one interprets the change in intensity of the scattered beam as a function of $\vec{K}$ to be due to the variation in the form factor. If $\hat{\eta}(\vec{K})$ changes direction, however, as a function of $\vec{K}$, the factor $\vec{q}^2(\vec{K})$ will also contribute to the variation in intensity. This will, accordingly, lead to difficulty in accurate determination of form factors, for the separation of these two variations is not possible unless the change of direction of $\vec{q}(\vec{K})$ has been measured in a polarized beam experiment. Similar difficulties occur in spin-structure experiments. Here the absence of certain reflections otherwise expected to occur is taken to indicate $\vec{q}^2 = 0$. According to our discussion, however, the condition $\vec{q}^2 = 0$ determines the direction of $\hat{\eta}(\vec{K})$, which is not necessarily that of the direction of magnetization $\hat{\eta}(0)$. One may be led by this to incorrect conclusions about the direction of magnetization if the presence of noncollinear densities is unsuspected.

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DOUBLE EXCITATION OF HELIUM BY ELECTRONIC IMPACT

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We report a preliminary calculation of the cross section for electron impact excitation of helium to the doubly excited $(2p)^2\ ^3P_g$ state, for which the scattered beam has a maximum of intensity at right angles to the incident beam. Similar calculations for several other states have been performed and will be reported elsewhere. The case considered here is of special interest because it involves the lowest lying, doubly excited state which is stable to autoionization.[1,2] Thus, while most doubly excited (or "anomalous") states decay by electron ejection within 10^{-13}-10^{-14} second, helium in the $(2p)^2\ ^3P_g$ state and certain other states can survive for as long as 10^{-9}-10^{-10} second before it ultimately decays by dipole radiation. It is therefore possible for these highly excited atoms to live long enough to participate in chemical reactions analogous to the inverse dissociative-recombination process.

The $(2p)^2\ ^3P_g$ state is the lowest with this symmetry. By representing the angular portions of the wave functions as products of spherical harmonics for the individual electrons, trial functions can be constructed which are orthogonal to all lower states, even the infinite number of $1snp\ ^3P_u$ states. Thus the variation theorem can be applied directly without reference to lower lying states. Therefore energy integrals calculated

with these trial functions provide rigorous upper bounds to the energy eigenvalue for $(2p)^2\ ^3P_g$. Using an antisymmetrized product of identical hydrogenic orbitals,

$$\chi_t = 2^{-1/2}[\Psi_{21m}(1)\Psi_{21n}(2) - \Psi_{21m}(2)\Psi_{21n}(1)],$$
$$m \neq n = 1, 0, -1,$$

the lowest energy is $E' = -1.39758$ rydbergs with a screening constant 1.67188. A better estimate of the energy has been provided by Holøien[3] ($E' = -1.42021$ rydbergs, which is 59.68 eV above the exact ground state), and calculations using this wave function are in progress. The exact value for E' has not been confirmed experimentally, although there has been some speculation.[4]

Since the direct scattering of the Born approximation contributes nothing to the cross section for a transition from a singlet to a triplet state, we employ the Born-Oppenheimer approximation, which allows for electron exchange. The scattering amplitude then is given by[5]

$$f = (3^{1/2}/4\pi)\iiint d\tau_1 d\tau_2 d\tau_3 \exp(i\vec{k}_0\cdot\vec{r}_1 - i\vec{k}_t\cdot\vec{r}_3)\chi_t^*(1,2)(H-E)\chi_0(2,3), \quad (1)$$

where H denotes the Hamiltonian for the three electrons in the field of the nucleus, and $\vec{k}_0$ and $\vec{k}_t$ the propagation vectors for the incident and ejected electrons, Nos. 1 and 3, respectively. For χ_0, the wave function of the ground state $(1s)^2\ ^1S_g$, we used an Eckart[6] type function with two screening constants 2.18316 and 1.18854 and energy $E'' = -5.75132$ rydbergs. The total energy of the system is $E = E'' + k_0^2 = E' + k_t^2$. The energy difference between the two states, $E' - E''$, which is needed in the calculation of k_t for a given k_0, was taken to be that value given by the approximate wave functions, 59.19 eV, although the correct value is probably somewhat larger.

All the terms in $(H - E)$ except $(2/r_{12} + 2/r_{23})$ vanish due to the orthogonality of $\Psi_{21m}(2)$ in χ_t and $\Psi_{100}(2)$ in χ_0. For the remaining integrations it is convenient to choose $\vec{k}_0$ as the polar axis. The term with r_{12} vanishes by symmetry considerations. Furthermore, all the terms with $m = \pm 1$ for electron No. 1, e.g., $\Psi_{211}(1) = u(r_1)P_1^1(\theta_1)\times\exp(i\phi_1)$, vanish when the integration is performed over ϕ_1. There remains only a term of the form

$$\int d\tau_1 \exp(i\vec{k}_0\cdot\vec{r}_1)\Psi_{210}^*(1)\iint d\tau_2 d\tau_3 \times\exp(-i\vec{k}_t\cdot\vec{r}_3)\Psi_{21m}^*(2)(1/r_{23})\Psi_{100}(3)\Psi_{100}(2), \quad (2)$$

with the two different states denoted by $m = +1$ and $m = -1$ making identical contributions to the cross section. It will be noted that the state with both p orbitals at right angles to $\vec{k}_0$, i.e., $\Psi_{21-1}(1)\times\Psi_{211}(2) - \Psi_{21-1}(2)\Psi_{211}(1)$, makes no contribution. The double threefold integral in (2) may easily be evaluated when the polar axis is changed to $\vec{k}_t$.

The striking result of this calculation is that the differential cross section $I(\theta,\phi) = (k_t/k_0)|f|^2$ is proportional to $\sin^2\theta$, where θ and ϕ are the polar

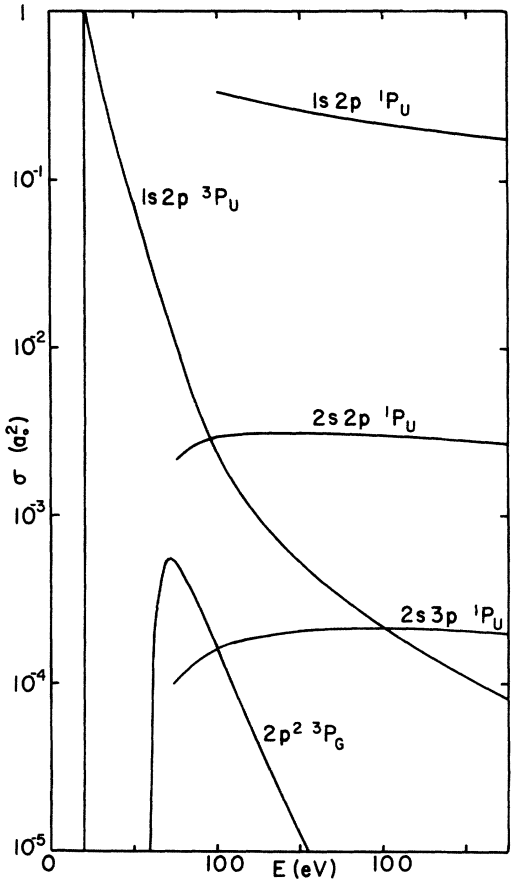

FIG. 1. Cross section σ (in units of a_0^2, where a_0 is the Bohr radius) as a function of incident electron energy E for the $(2p)^2\ ^3P_g$ excitation, with other transitions shown for comparison. The $2s2p\ ^1P_u$ and the $2s3p\ ^1P_u$ transitions were calculated without electron exchange (the Born approximation) by H. S. W. Massey and C. B. O. Mohr [Proc. Cambridge Phil. Soc. 31, 604 (1935)]. Threshold for these excitations occurs near 60 eV, as well as for the $(2p)^2\ ^3P_g$ case. The $2s2p\ ^{1,3}P_u$ transitions were calculated with electron exchange (the Born-Oppenheimer approximation) by Massey and Moiseiwitsch (see reference 5) below 50 eV, and by H. S. W. Massey and E. H. S. Burhop [Electronic and Ionic Impact Phenomena (Clarendon Press, Oxford, 1952), p. 172], above 100 eV. Threshold for these excitations is near 20 eV.

angles for the scattered beam. In other words, this theory predicts that the scattered beam has a maximum of intensity at right angles to the incident beam. As an aid to the interpretation of this result, one can strongly contend that the incident electron must be captured into the p orbital Ψ_{210} which is oriented parallel to the initial direction of approach $\vec{k}_0$. In the final $(2p)^2\ ^3P_g$ state, the other bound electron must therefore lie in one of the p orbitals Ψ_{21-1} or Ψ_{211} oriented perpendicular to $\vec{k}_0$. Consequently, due to the preferential ejection of the third electron (induced by the perturbation $1/r_{23}$), it can be argued that the intermediate complex, a helium negative ion, is composed of a doubly occupied Ψ_{211} or Ψ_{21-1} orbital and a singly occupied Ψ_{210} orbital.

The total cross section $\sigma = 2\iint I(\theta, \phi) d\phi d\theta \sin\theta$ shows the fairly sharp peak and rapid decay with increased incident electron energy characteristic of exchange reactions. In Fig. 1 the cross section for the transition is compared with other processes.

Even though the Born-Oppenheimer approximation is not very accurate, and the helium wave functions are not very good, the results suggest that the experiments be run, perhaps after the manner of Swift and Whiddington,[7] looking for a transition with the properties described. In order that a better prediction may be made, further calculations are in progress using better atomic wave functions and the more accurate distorted-wave and strong-coupling scattering theories.

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ELECTROMAGNETIC STRUCTURE OF THE GIANT DIPOLE RESONANCE*

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Inelastic electron scattering yields valuable information about the spatial nature of the transition charge, current, and magnetization densities in nuclei. Since the electron part of the process is completely calculable, one can directly compare the experimental inelastic form factors (functions of the momentum transferred to the nucleus) with the predictions of nuclear theory. Different nuclear models can give quite different behavior for these form factors, and thus electron scattering provides a unique tool for elucidating nuclear structure. We wish to present some experimental and theoretical results on the nature of the giant dipole resonance in C^{12} illustrating these points.

The differential cross section at 180° for electron excitation of the giant dipole resonance is given in first Born approximation ($Z/137 \ll 1$) by[1]

$$\frac{d\sigma}{d\Omega} = \frac{16\pi\alpha^2 K_2^2}{\Delta^4} \left| \langle J = 1^- || T_1^{\mathrm{el}}(q) || J = 0^+ \rangle \right|^2, \qquad (1)$$

where K_2 is the final electron wave number, $\hbar\vec{q}$ and $\hbar\Delta$ are the three- and four-momenta transferred to the nucleus, and we have neglected the electron mass and nuclear recoil. One needs the reduced transition matrix element (the "inelastic form factor") of the transverse electric dipole