frequencies $\lesssim 30$ Mc/sec. Observations, however, become more difficult at low frequencies, and in any case this spectral change may reflect a change in emission or absorption rates and not in the actual electron density spectrum. For the present we therefore choose to extrapolate from the well-known spectrum at frequencies $\gtrsim 100$ Mc/sec.

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EVIDENCE FOR A TWO-PION DECAY MODE OF THE ω MESON

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A two-pion decay mode of the ω meson is inferred from the effective-mass distribution of the two pions produced in the reaction

$$\pi^- + p \to n + \pi^- + \pi^+ \qquad (1)$$

at an incident pion momentum of 1.7 BeV/c. 2137 events of Reaction (1) show, above background, approximately 900 events in the ρ-resonance band. When the events of the reaction are analyzed as a function of momentum transfer to the nucleon, it is found that a statistically significant number of events fall in a narrow band peaked at a mass close to the mass of the ω meson. Thus both ρ and ω intermediate states for Reaction (1) are implied:

$$\pi^- + p \to n + \rho^0, \quad \rho^0 \to \pi^- + \pi^+, \qquad (2)$$

$$\pi^- + p \to n + \omega, \quad \omega \to \pi^- + \pi^+. \qquad (3)$$

The accepted properties of the resonances, obtained from the decays $\rho \to 2\pi$ and $\omega \to 3\pi$, are summarized in Table I. The ω resonance has the same spin and parity as the ρ, but different isotopic spin; its mass is about 4% greater and its width much less than the mass and width of the ρ.

The ω can decay into two pions only through electromagnetic interaction. Glashow[1] suggested that, because of the small ω - ρ mass difference, electromagnetic interference between the two states may greatly enhance the G-forbidden decay $\omega \to 2\pi$. Calculations by Bernstein and Feinberg[2] showed that this forbidden decay might be observed in $\pi^+\pi^-$ effective-mass distributions as a spike near the ω mass superimposed on the broad ρ peak. Some indications of this structure have appeared in several experiments.[3]

Table I. Properties of ρ and ω mesons.

Particle	Mass (MeV)	Γ (MeV)	I	J	P	G
ρ	~750	~120	1	1	-	+
ω	~780	$\leq$ 20	0	1	-	-

To study this effect further, the BNL 20-inch hydrogen bubble chamber was exposed to negative pions from the AGS separated beam; the nominal beam momentum was 1.7 BeV/c and the spread approximately ±0.5%. 60 000 pictures were taken with 20-25 tracks per picture. Bubble density was kept low so that the chamber's excellent ionization discrimination could be employed to separate positive pions from protons.

Events were selected in a small fiducial volume corresponding to about one fourth of the total chamber. The volume was placed well upstream to insure long secondary tracks for accurate momentum measurements. All two-prong stars within the fiducial volume were measured and kinematically fitted with the exception of those in which, at the scanning stage, the positive secondary was identified unambiguously as a proton by its ionization. 5535 events were measured; in addition to the 2137 identified as Reaction (1), 1196 were from the reaction

$$\pi^- + p \to \pi^- + \pi^+ + n + (1 \text{ or more } \pi^0). \qquad (4)$$

The remainder consisted of events with secondary protons not eliminated in the scanning.

A Chew-Low plot of all events from Reaction (1) is shown in Fig. 1. The large concentration of events at low four-momentum transfer (Δ) and in the region of the ρ mass [0.56 (BeV/c)^2] is obvious. Figure 2(a) shows the $\pi^+\pi^-$ effective-mass distribution. The broad band, 650-850 MeV, typical of the ρ, is apparent, but the distribution is asymmetric about 750 MeV, with 393 events between 750 and 800 MeV and 298 events between

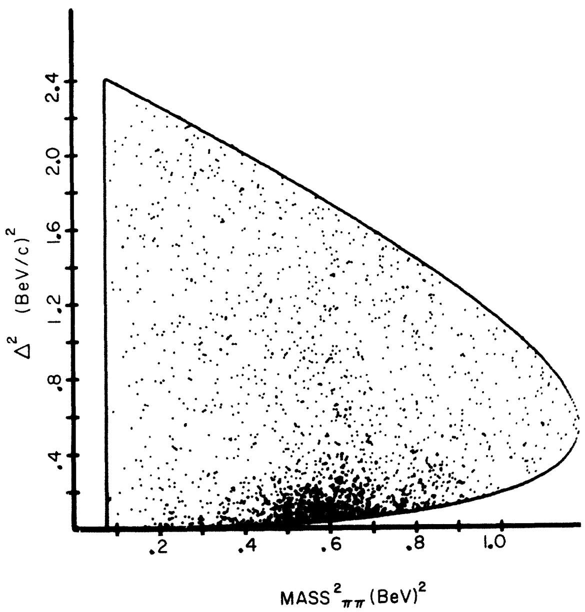

FIG. 1. Chew-Low plot of 2137 events of the type $\pi^- + p \to \pi^- + \pi^+ + n$. The square of the two-pion effective mass is plotted on the abscissa, and the square of the four-momentum transfer to the nucleon is plotted on the ordinate. There is a strong concentration of events around the ρ mass at low Δ^2.

750 and 700 MeV. If it is assumed that the neutral ρ has the same mass and symmetrical distribution as the charged ρ, the asymmetry suggests an accumulation of events in the region of the ω mass, 780 MeV.

The production of ρ in peripheral collisions (low Δ^2) is consistent with one-pion exchange (OPE). G-parity conversation forbids production of ω by OPE, so that ω production could be peaked at higher Δ^2. Accordingly, the data were divided into momentum-transfer intervals.

Figures 2(b) and 2(c) show $\pi^+\pi^-$ mass distributions for the two regions $\Delta^2 < 0.15$ (BeV/c)^2 and $0.25 \leq \Delta^2 < 0.70$ (BeV/c)^2, regions selected to give maximum separation of the two states with significant statistics. The low momentum-transfer selection shows a typical ρ peak.

The mass distribution for the higher momentum-transfer selection, Fig. 2(c), shows a narrow peak at 790 MeV. While no peak stands out at the value expected for a ρ, 750 MeV, the distribution below that value is consistent with some ρ production. The high-mass range, above 850 MeV, shows considerable background from other processes. Interpolation between 750 MeV (ρ peak) and the high-mass regions yields a background of 9-10 events per 10-MeV interval around 790 MeV. In the two intervals 780-800 MeV around the 790 peak, 46 events are observed, so that there are about 27 events above background. Thus the $\pi^+\pi^-$ mass distribution shows a peak at a mass close to the ω mass, about 4 standard deviations above background. The peaks beyond 850 MeV may be statistical fluctuations.

The peak at 790 MeV in Fig. 2(c) has a full width at half-maximum of 20-30 MeV, much narrower than the ρ peak. Our resolution in mass in this region, as calculated from the GUTS error matrix, is about ± 8 MeV, so that the distribution is consistent with a width less than ~15 MeV.

At high momentum transfers, there is considerable (3, 3) isobar production. This forms part of the background for the events of Fig. 2(c), and is strong in events with $\Delta^2 \geq 0.70$ (BeV/c)^2 in Fig. 2(d). In these latter events there is no sign of the ρ and little of the ω peak.

Three-pion ω decays, produced in the reaction

$$\pi^- + p \to n + \omega, \quad \omega \to \pi^- + \pi^+ + \pi^0, \tag{5}$$

cannot be identified, since the final state includes two neutral particles. Hence, the relative production $\sigma_\omega/\sigma_\rho$ cannot be given. However, an upper limit for this ratio was obtained by counting as ω's all those events of Reaction (4) kinematically satisfying Reaction (5). 908 events satisfied this criterion, giving

$$(\sigma_\omega/\sigma_\rho)_{max} \simeq 1. \tag{6}$$

It is estimated that 100-150 events, $\omega \to 2\pi$, are produced in this experiment, but because this production is clear only for a limited range of momentum transfer, the estimate is only approximate.

The branching ratio $R = \Gamma(\omega \to 2\pi)/\Gamma(\omega \to 3\pi)$ for ω decay into 2 and 3 pions may, in principle, be found by adjusting the mixing parameters of the Bernstein-Feinberg equation so as to fit the observed $\pi\pi$ mass distribution, but the requisite relative phases of the ρ and ω states and the ratio $\sigma_\omega/\sigma_\rho$ are not known. However, a minimum value for R is obtained by using (6) and by adjusting phases so as to maximize the ω peak. This procedure yields $R \geq 0.05$ if the width of the pure ω is assumed to be 10 MeV, and $R \geq 0.07$ if that width is assumed to be 1 MeV.[4,5]

We are happy to acknowledge our indebtedness to the AGS operating staff, to the separated beam and 20-inch bubble chamber groups, and to our

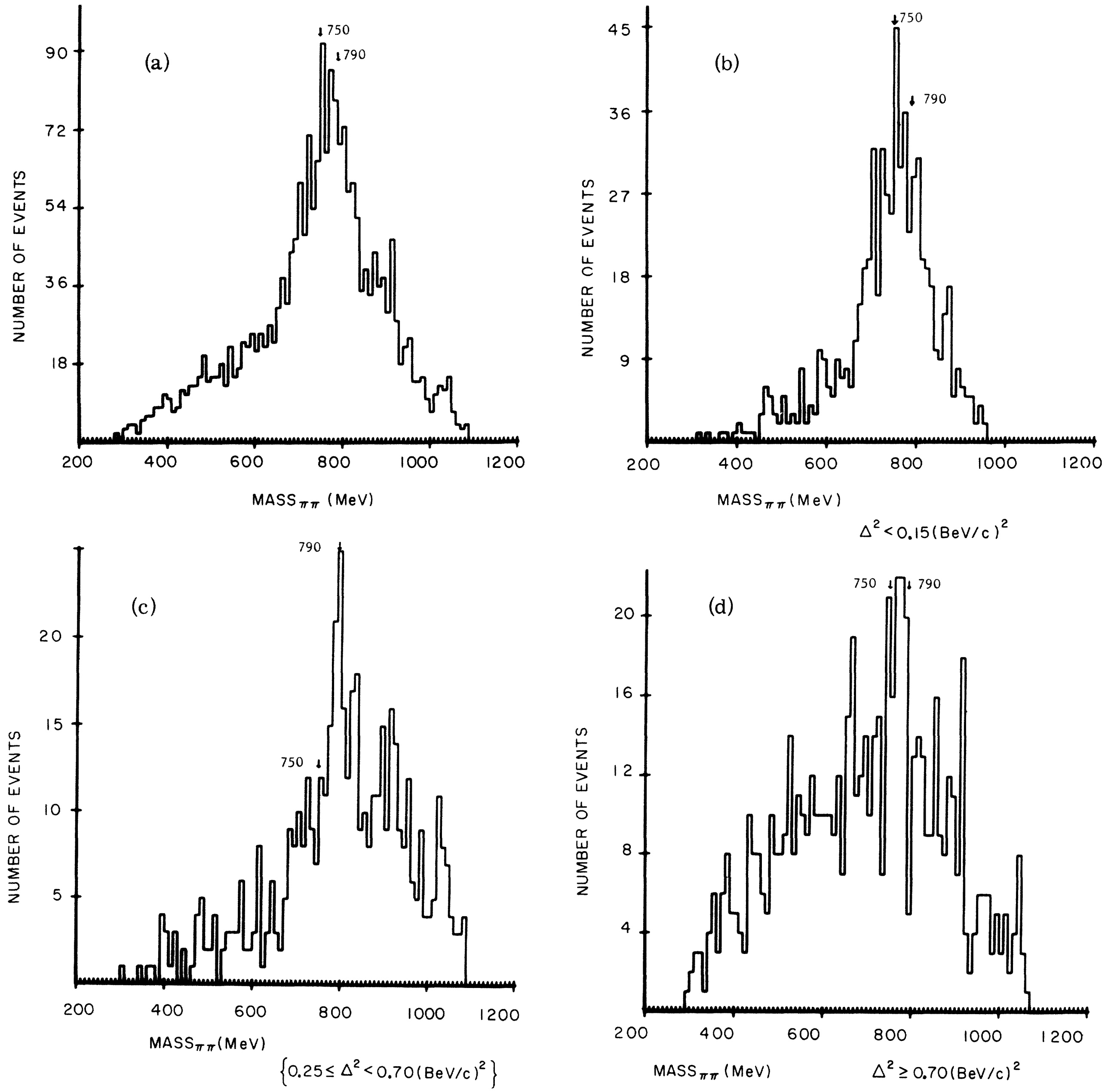

FIG. 2. Effective-mass distributions of the $\pi^+\pi^-$ system. Arrows indicate abscissas 750 MeV (ρ) and 790 MeV (ω). (a) All Δ^2; (b) $\Delta^2 < 0.15$ (BeV/c)^2; (c) $0.25 \leq \Delta^2 < 0.70$ (BeV/c)^2; (d) $\Delta^2 \geq 0.70$ (BeV/c)^2.

scanning staff.

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[5]These branching-ratio limits rest on the assumption that some other resonance (so far unknown) decaying into two charged pions does not also peak near 790 MeV.

NEW UPPER BOUND FOR THE HIGH-ENERGY SCATTERING AMPLITUDE AT FIXED ANGLE

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The requirements of unitarity and analyticity seem to impose rather strong constraints on the growth of scattering amplitude at high energy. The first significant result in this respect was that of Froissart[1] who obtained upper bounds for the high-energy scattering amplitude from Mandelstam representation and unitarity. He found that the scattering amplitude $f(s, \cos\theta)$ (for scalar particles of equal mass) satisfies the inequalities

$$|f(s,\cos\theta)| < C_1 s(\log s)^2, \quad \text{for } \theta = 0 \text{ or } \pi, \qquad (1)$$

$$|f(s,\cos\theta)| < C_2 s^{3/4}(\log s)^{3/2}, \quad \text{for } \theta \neq 0 \text{ or } \pi, \qquad (2)$$

for very large s, where s and θ are the square of the total energy and the scattering angle in the center-of-mass system. f is normalized here in a relativistic way, so that

$$\sigma_{\text{tot}} \sim \frac{1}{s}\text{Im}f(s,1), \quad \frac{d\sigma_{\text{el}}}{d\Omega} \sim \frac{1}{s}|f|^2.$$

It was recognized later by one of us[2] that it is not necessary to make use of the full analyticity assumed in the Mandelstam representation to obtain the bounds (1) and (2). It is sufficient to assume that $f(s, \cos\theta)$ be analytic in an ellipse E_ρ in complex $\cos\theta$ plane, with foci at +1 and -1 and semimajor axis of length $\rho \equiv 1 + \alpha/k^2$ (α a positive constant, k the center-of-mass momentum), and that f be uniformly bounded in this ellipse by some power of s.[3]

We may then ask the following questions:

(i) Is it possible, with the weak assumptions just mentioned, to improve the bounds (1) and (2)?

(ii) Will it be possible to improve them if more analyticity is assumed?

As to the first question, it is easy to see that the answer is negative: It is, namely, possible to find counter examples.

The purpose of this note is to give a partial answer to the second question. We shall show that, if $f(s, \cos\theta)$ is analytic and uniformly bounded by a power of s in a domain D_s defined below, it is, in fact, possible to improve bound (2) and replace it by

$$|f(s,\cos\theta)| < C_3(\log s)^{3/2}, \quad \text{for } \theta \neq 0 \text{ or } \pi. \qquad (3)$$

From this we obtain for the elastic differential cross section a bound

$$\frac{d\sigma_{\text{el}}}{d\Omega} < C_4 \frac{(\log s)^3}{s}, \qquad (4)$$

which decreases rapidly as s increases. [In contrast (2) gives a bound which increases with s.] So far we have not succeeded in improving the forward-backward bound (1). We do not know whether this improvement is possible under our assumptions.

In the following we put $\cos\theta = z$.

We assume that $f(s, z)$ is analytic and bounded by s^N (N is independent of s and z) in a domain D_s of the z plane [see Fig. 1(a)]. D_s is an intersection of a domain containing the ellipse E_ρ [shown by a dotted curve in Fig. 1(a)] and the z plane with cuts from $-\infty$ to $-\rho$ and ρ to ∞. The shape of D_s depends on s. Let us assume, however, that the distance between the boundaries of D_s and E_ρ is larger than some positive number