70 and 200 events, respectively, to separate the cases $p_{KN\Lambda}=\pm1$ by one standard deviation. With 10^6 1-BeV/c pions per second in the incident beam and a 1-atm He^4 discharge chamber giving 20-cm path length in He^4 for half the recoil neutrons, the rate would be 10 events per hour.

Possible applications of the deuteron method for obtaining polarized protons, in addition to the determination of the $KN\Lambda$ and $KN\Sigma$ parities, would be studies of gross polarization effects in nucleon-nucleon scattering, and study of the validity of the spectator model itself. Raising the polarization analyzer counting efficiency substantially above 3×10^{-4} would make possible refined polarization measurements. The method is applicable at all incident momenta greater than 1 BeV/c, and if anything, its reliability should improve with increasing momentum.

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[1]G. F. Chew and G. C. Wick, Phys. Rev. 85, 636 (1952).
[2]G. F. Chew and M. L. Goldberger, Phys. Rev. 87, 778 (1952).
[3]A. P. Batson et al., Proc. Roy. Soc. (London) A251, 233 (1959).
[4]L. Hulthén and K. V. Laurikainen, Rev. Mod. Phys. 23, 1 (1951).
[5]R. Kraemer et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 273.
[6]Neutron Cross Sections, compiled by D. J. Hughes and R. Schwartz, Brookhaven National Laboratories publication BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed., pp. 75, 82.
[7]K. W. Brockman, Jr., Phys. Rev. 110, 163 (1958).
[8]G. Charpak and L. Massonet (to be published).
[9]J. W. Cronin and O. E. Overseth, Phys. Rev. 129, 1795 (1963).
[10]J. A. Anderson et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 271.

DIFFRACTION SCATTERING*

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Recent experiments in Brookhaven[1] have revealed that the diffraction peak in pion-nucleon scattering has a constant shape in the energy range from 7 to 20 BeV. In the same energy interval the proton-proton diffraction pattern shrinks logarithmically, perhaps at a somewhat decreasing rate near the high-energy end of the interval.

It is the purpose of this note to explain the relevance for the discussion of diffraction scattering of a theorem by one of us (R.O.)[2] concerning the absence of fixed Regge poles. This theorem also excludes a large class of other fixed singularities in the complex angular momentum plane. In addition we propose models which could give shrinking and nonshrinking diffraction patterns for different reactions, and we discuss some of their experimental consequences.

Let us consider first the nonshrinking diffraction peak in π^-p scattering. This behavior of the differential cross section, together with an essentially constant total cross section, corresponds very much to what one would expect on the basis of a simple optical model, where we have an invariant amplitude with the asymptotic form

$$F(s,t)\sim id(s)t, \quad \text{for } t\to\infty \text{ and } s\leq 0. \tag{1}$$

Here $\sqrt{t}$ is the total c.m. energy in the πp channel and $(-s)^{1/2}$ is the momentum transfer; we have ignored spin variables. We will show in the following that the optical model Ansatz (1) is by no means natural in a causal relativistic dispersion theory.

In dispersion theory we assume that the amplitude $F(s,t)$ has no essential singularities in the physical sheet such that it satisfies a dispersion relation in t with a finite number of subtractions. We can use this representation in order to define an analytic function $F(s,\lambda)$ which uniquely interpolates the partial-wave amplitudes $F_l(s)$ with $l>1$ for the reaction $\pi\bar{\pi}\to p\bar{p}$. Then the high-energy behavior of the π^-p scattering amplitude is given by the leading singularity of $F(s,\lambda)$, at least as far as the absorptive part $A_t(s,t)$ is concerned. The complete amplitude may have contributions from elementary poles[3] in the s channel. But if we want to have $\sigma_{\rm tot} \sim$ constant and $F(0,t)$ to become purely imaginary for $t\to\infty$, then we expect an asymptotic form corresponding to Eq. (1), which would imply a fixed pole of $F(s,\lambda)$ at $\lambda=1$. However, we know from the theorem of Oehme[2] that no fixed pole is allowed.[4] More generally, the theorem excludes all fixed singularities in the complex λ plane which are such that the function $F(s,\lambda)$ becomes unbounded[5]; this includes logarithmic branch points. We note that an earlier argument by Gribov[6] excludes an asymptotic form like Eq. (1) only for s in the interval $4m_\pi^2 < s < 16\,m_\pi^2$, and hence it is not applicable in the physical region $s\le 0$.[7]

In an earlier publication we have considered the high-energy limit of $F(s,t)$ assuming the existence of elementary vector mesons.[8] We found an asymptotic form like

$$F(s,t)\sim B(s)t+C(s)t^{\alpha(s)}, \qquad (2)$$

where $B(s)$ is real for real $s\le 0$ and where $\alpha(s)$, with $\alpha(0)=1$, is the induced Pomeranchuk trajectory with $C(s)=-b(s)\times[1+e^{-i\pi\alpha(s)}]$. The coefficient $B(s)$ vanishes for $s\to-\infty$ like an inverse power, but it may well have an approximately exponential form for small values of $|s|$ in order to give a nonshrinking diffraction pattern. However, since $B(0)$ is real, such a model would imply $\chi(0,t)>1$, where

$$\chi(s,t)=16\,\pi\frac{d\sigma/ds}{\sigma_{\rm tot}^{\ 2}}. \qquad (3)$$

Present experiments[1] do not seem to extrapolate to values of $\chi(0,t)$ which are much larger than one, but the evaluation of the measurements for $|s|<0.2$ (BeV/c)^2 has not yet been completed.

Let us now discuss models with $\chi(0,t)=1$. We find that an allowed fixed singularity at $\lambda=1$ alone cannot give a constant total cross section in the high-energy limit. If we ignore $\ln\ln t$ factors, we can at best obtain $\sigma_{\rm tot}(t)\sim O[(\ln t)^{-1-\epsilon}]$. However, the situation may be different if we consider fixed branch points together with a pole trajectory $\lambda=\alpha(s)$ with $\alpha(0)=1$. Then we have a situation which is to some extent similar to the one encountered in potential scattering with a potential which behaves like gr^{-2} for $r\to 0$.[9] In this case we have a pair of fixed square-root branch points[10] in the λ plane at $\lambda=-\frac{1}{2}\pm\sqrt{g}$ and related branch points of the Regge trajectories $\lambda=\alpha(E)$ in the E plane; we assume here that the potential is attractive for larger values of r. The physical significance of these singularities has been discussed in reference 9. They describe the transition between a normal bound state of the system and a situation where it "collapses" into the center. The position of the fixed branch points is therefore determined by the balance between centrifugal and interaction forces. In the general case the possibility of particle production prevents a collapse, but we may assume that the fixed branch points are nevertheless present.[11] Then we can propose two models:

(1) In the s channel corresponding to the reaction $\pi\bar{\pi}\to p\bar{p}$ we have attractive forces at small distances which give rise to a square-root branch point at $\lambda=1$ with positive signature; in addition we have a Pomeranchuk trajectory $\alpha(s)$. For $s\le 0$ the amplitude has the asymptotic form

$$F(s,t)\sim id(s)t(\ln t)^{-3/2}+C(s)t^{\alpha(s)}, \qquad (4)$$

where we assume that $d(s)$ is given by an expression like

$$d(s)\approx d(0)\exp\left[\frac{s}{\pi}\int_{s_0}^{\infty}ds'\frac{\delta(s')}{s'(s'-s)}\right], \qquad (5)$$

which approximates an exponential for small values of s but vanishes for $s\to-\infty$ like an inverse power. Whether the form (4) gives rise to a shrinking or nonshrinking diffraction pattern depends mainly upon the relative magnitude of the cut and the pole terms. We have made numerical calculations for the case of π^-p scattering. If the slow decrease of the total π^-p cross section between 7 and 17 BeV/c is fitted by the expression $\sigma_{\rm tot}\sim d(0)(\ln t)^{-3/2}+b(0)$ with $C(0)=ib(0)$, then it is very difficult to choose the cut term sufficiently large in order to explain the absence of shrinking. One may also make use of the discontinuity[9] of the trajectory $\alpha(s)$ at $s=0$ in order to suppress the Pomeranchuk term for $s\lesssim -0.2$ (BeV/c)^2, say. In this case we still have $\chi(0,t)=1$, but $\chi(s,t)$ has a kink, and it is essentially a linear function for $s<-0.2$ (BeV/c)^2 which should extrapolate below the optical point. This does not seem to agree with the experimental data.[1]

(2) The forces at small distances are repulsive. In analogy to the potential problem we may then assume that $F(s,\lambda)$ has two conjugate branch points on the line $\mathrm{Re}\lambda=1$ which are connected by a cut.[12] This cut induces a branch line in the Pomeranchuk trajectory $\alpha(s)$ for real $s_L\le s\le 0$, where $s_L\to-\infty$ for a sufficiently strong but finite repulsion. Then we can have $\mathrm{Re}\alpha(s)=1$ for $s_L\le s\le 0$,

whereas $\mathrm{Im}\alpha(s)$ varies such that $\alpha(s)\neq 1$. Since $F(s,t)$ has no left-hand branch point for $t\to\infty$, we see that both Regge poles $\lambda=\alpha(s)$ and $\lambda=\alpha^*(s)$ contribute to the high-energy limit such that the asymptotic form for $s\leq 0$ given by

$$F(s,t)\sim\tfrac{1}{2}[C(s+i0)t^{\alpha(s+i0)}+C(s-i0)t^{\alpha(s-i0)}] + O[t(\ln t)^{-3/2}], \tag{6}$$

where the logarithmically decreasing terms are due to the fixed branch points and where $\mathrm{Re}\alpha(s) =1$. The expression (6) gives rise to a differential cross section of the form

$$\frac{d\sigma}{ds}\sim(\sinh\alpha_i)^{-2}\{|b|^2(1-e^{\pi\alpha_i})^2+|b|^2(1-e^{-\pi\alpha_i})^2 -2(1-e^{\pi\alpha_i})(1-e^{-\pi\alpha_i})[(\mathrm{Re}b^2)\cos 2\alpha_i\ln t -(\mathrm{Im}b^2)\sin 2\alpha_i\ln t]\}+O[(\ln t)^{-3/2}], \tag{7}$$

for $s\leq 0$ with $\alpha_i=\mathrm{Im}\alpha(s+i0)$, and $|b(s)|$ being an expression like $d(s)$ in Eq. (5). This could give a nonshrinking diffraction peak for πp scattering provided we can choose α_i such that the cosine term does not introduce disturbing oscillations, which seems to be possible. The fixed branch cut in $F(s,\lambda)$ gives a contribution to the total cross section proportional to $d(0)(\ln t)^{-3/2}$; it could play the role formerly attributed to the Pomeranchino (P' trajectory) in connection with the pion-nucleon dispersion relations.[13]

In pp scattering we have a shrinking diffraction peak corresponding to an undisturbed Pomeranchuk particle. Therefore we must assume that in the crossed s channel ($p\bar{p}\to p\bar{p}$) there is no short-range repulsion and hence no fixed cut in the λ plane which distorts $\alpha(s)$. It is important to note here that in our dispersion theoretic models for diffraction scattering the leading terms in the high-energy limit of the nucleon-nucleon amplitude are the same as those for the corresponding nucleon-antinucleon amplitude. This is simply a consequence of the fact that in both cases we have related crossed channels, and hence the same singularities with positive signature determine the high-energy limits of both amplitudes near the forward direction, independent of the character of these singularities. The measurement of the differential $p\bar{p}$ cross section in the asymptotic region is therefore of greatest importance for dispersion theory.

Even within the framework of dispersion theory we can, of course, construct many other models which give an approximately constant diffraction peak, especially if we consider singularities at $\lambda=1$ which change their character as a function of s such that there is no contradiction with the unitarity condition for real $s\geq 4m_\pi^2$. These models, however, look very artificial because they have no intuitive physical background. It is also possible that dispersion theory implies that the accumulations of singularities in the left half of the λ plane actually occur up to the line $\mathrm{Re}\lambda=1$.[14] These condensations could then be such that they approximate arbitrarily closely an asymptotic form corresponding to Eq. (1), but do not contradict the theorem of reference 2. They are caused, a priori, by fixed poles of $\mathrm{disc}F(s+i0,\lambda)$ along the left-hand cut at negative integer values of λ.[15] In $F(s,\lambda)$ these poles must be compensated for $s \geq 4m_\pi^2$ in order not to contradict the unitarity condition.

As we have seen, it does not appear that an asymptotic form corresponding to Eq. (1) is very easy to obtain within the framework of causal dispersion theory. One may wonder, therefore, whether this could be an indication that the dispersion scheme is too narrow, especially as far as the assumption of a polynomial bound at infinity is concerned. If we have an essential singularity of $F(s,t)$ at infinity in the physical sheet of the t plane, then we cannot write a dispersion relation with a finite number of subtractions. In this case no unique analytic interpolation of the partial-wave amplitudes $F_l(s)$ is possible, and the theorem of reference 2 is not applicable. It is plausible that such an essential singularity corresponds to a violation of microscopic causality in small dimensions[16] characterized by a length a^{-1}.[17] A most naive Ansatz would be of the form

$$F(s,t)\sim\left[\exp\left(-\frac{stu}{a^6}\right)-1\right]+\Phi(s,t), \tag{8}$$

where $\Phi(s,t)$ satisfies the usual dispersion relations.[18] For simplicity we consider an amplitude with complete crossing symmetry. The function $\Phi(s,t)$ now could have an asymptotic form like $\Phi(s,t)\sim id(s)t$, where $d(s)$ must vanish for $a^2\to\infty$. But then we would expect that the asymptotic total cross section $\sigma_{\mathrm{tot}}\sim d(0)$ contains a^{-1} as a characteristic length, whereas the actual magnitude of $\sigma_{\mathrm{tot}}(\pi^-p)$ at about 30 BeV/c is rather characteristic for a length of the order m_π^{-1}. For these and other reasons it appears superficially that the puzzles of diffraction scattering in the energy range presently available are not related to a possible smallest length. The problem is rather to understand the simple optical model within the framework of dispersion theory.

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[1]K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 10, 376 (1963). See also A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters 9, 111 (1962); C. C. Ting, L. W. Jones, and M. L. Perl, Phys. Rev. Letters 9, 468 (1962).

[2]R. Oehme, Phys. Rev. Letters 9, 358 (1962).

[3]R. Oehme, Phys. Rev. 130, 424 (1963).

[4]We note that the dispersion theoretic model for nonshrinking diffraction scattering by Y. Nambu and M. Sugawara [Phys. Rev. Letters 10, 304 (1963)] implies $\alpha(t) \equiv 1$. It therefore appears that this model is excluded by the theorem of reference 2.

[5]R. Oehme (unpublished); a complete discussion of these extensions of the theorem of reference 2, and its applicability in the case of the continued unitarity condition for reactions like $\pi\bar{\pi} \to N\bar{N}$, will be given in the lecture notes by R. Oehme, Scottish Universities' Summer School in Physics, 1963 (in preparation), and in those by P. G. O. Freund, Summer Institute of Physics, University of Colorado, 1963 (in preparation).

[6]V. N. Gribov, Nucl. Phys. 22, 249 (1961).

[7]A counter example is given by $F(s,t) \sim id(s)t + C(s)t^{\gamma(s)}$ with $\mathrm{Re}\gamma(s) < 1$ for real $s \leq 0$ and $\mathrm{Re}\gamma(s) > 1$ for real $s \geq 4m_\pi^2$. Such an asymptotic form is excluded by reference 2, but not by Gribov's argument.

[8]P. G. O. Freund and R. Oehme, Phys. Rev. Letters 10, 199,315(E) (1963).

[9]R. Oehme, Nuovo Cimento 25, 183 (1962).

[10]These fixed branch points should not be confused with possible moving branch points in the λ plane. Concerning the problem of branch-point trajectories, see reference 3 and R. Oehme (to be published).

[11]Fixed branch points can also be obtained in perturbation theory if one sums ladder graphs with couplings which give rise to more singular "potentials." See J. D. Bjorken and T. T. Wu, Phys. Rev. (to be published); R. Sawyer (private communication).

[12]See reference 9, Sect. 3; in potential scattering the branch points are at $\lambda = -\frac{1}{2} \pm i(|g|)^{1/2}$. On the line $\mathrm{Re}\lambda = -\frac{1}{2}$ the centrifugal potential is real, and we have "collapse" for $\lambda = -\frac{1}{2} + i\lambda_i$ with $\lambda_i^2 > |g|$. In field theory the position of these branch points may well be changed[11] such that they are on the line $\mathrm{Re}\lambda = 1$, which is, of course, an upper limit.

[13]K. Igi, Phys. Rev. Letters 9, 76 (1962).

[14]Ya. I. Azimov, Phys. Letters 3, 195 (1963).

[15]V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters 2, 239 (1962).

[16]R. Oehme, Phys. Rev. 100, 1503 (1955).

[17]W. Heisenberg, Z. Physik 101, 533 (1936).

[18]We have also considered product Ansätze, which may appear more plausible in connection with the problem of causality. They raise further problems, but seem to have the fairly general characteristic feature that they give rise to oscillations in the Coulomb interference term, the frequency of which increases with a power of the energy.

RECOIL PHOTONS FROM SCATTERING OF STARLIGHT BY RELATIVISTIC ELECTRONS*

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The processes which have been proposed[1] for the cosmic production of high-energy photons, recently detected above the atmosphere,[2,3] depend on the presence of particles and fields whose densities are themselves poorly known or unknown. One additional process, however, does not share this particular uncertainty. Collisions between the electrons which are known to produce the nonthermal radio emission (in particular, the galactic halo emission) and the photons of starlight have long been recognized as contributors to electron energy degradation, but the observable radiation they must produce has not been considered. Yet such observations are of astrophysical interest, for unlike synchrotron radiation, which depends strongly on B, this inverse Compton process depends only on the rather smoothly distributed starlight density and on the electron density, and must occur wherever fast electrons are found in space.

We can calculate roughly the photon flux due, say, to electrons in the halo, and compare it with observations. We use units convenient for astrophysical purposes: magnetic fields in microgauss, energies in MeV or eV, frequencies in Mc/sec, volume densities in cm^{-3}, and distances in light years.

Some results pertinent to the halo synchrotron