

change in parity for dipole transitions. Double-quantum transitions would occur with no change in parity. In asymmetric molecules, however, these selection rules will not be strict and both single- and double-quantum transitions are possible to the same level by different components of the dipole operator.

A correlation is shown in Table I between the absorption cross sections for the  $A \rightarrow {}^1L_a$  single-quantum transitions and the observed intensity of fluorescent light. We feel that this correlation of double-quantum excitation with single-quantum absorption probabilities is reasonable, since it is highly likely that there are many levels to which double-quantum transitions are allowed in the absorption bands of these complex molecules. Consequently, we suggest that such a two-quantum excitation is taking place in these molecules and that this phenomenon should be a general one for organic molecules of this type.

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## ABSORPTION OF ELECTROMAGNETIC WAVES IN QUANTUM AND CLASSICAL PLASMAS\*

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Recently some calculations of the absorption of electromagnetic waves in a plasma have been given. The absorption in classical plasmas has been treated with an elementary model by Dawson and Oberman<sup>1</sup> and by Oberman, Ron, and Dawson<sup>2</sup> via the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy. Reference 2 gives a complete classical derivation of the high-frequency conductivity of a plasma, taking into account properly collective effects. Another approach to the classical problem has been given by Perel' and Eliashberg<sup>3</sup> via a quantum-mechanical diagram technique. Although the latter approach can be systematized, most of the results are in error, because of the nonsystematic treatment of the ion role, and thus differ from the results of references 1 and 2.

The purpose of the present Letter is to give results of a systematic study of the absorption problem in both classical and quantum plasmas. We study the problem using the same temperature-dependent Green's function method as that em-

ployed in reference 2. We are, however, able to correct their procedure to give a consistent treatment of multispecies quantum and classical systems. In the classical limit complete agreement with reference 2 is obtained. Furthermore, we get an exact expression for the absorption coefficient valid for all temperatures, which covers both quantum and classical domains.

The outline of the calculation is as follows: We start from Kubo's<sup>4</sup> expression for the conductivity in terms of the autocorrelation function of the current operators. We then evaluate the leading asymptotic contribution to the conductivity by applying the well-known diagram technique of the temperature-dependent Green's function, taking into account the diagrams given in Fig. 1. These diagrams represent the exact contribution to the conductivity in quantum (classical) plasma, when the number of particles in the Bohr (Debye) sphere is large, the frequency is higher than the collision frequency, and the wavelength of the in-

cident field is larger than the Bohr (Debye) radius. The solid lines in the diagrams represent the  $s$ -species free propagators, and the wavy lines the effective interaction. Our general result is given by

$$\begin{aligned} \sigma(\omega) = & i \sum_s \frac{e_s^2 n_s}{\omega} + i \frac{4\pi}{3\omega^3} \sum_{ss'} e_s^2 e_{s'}^2 \frac{e_s}{m_s} \left( \frac{e_s}{m_s} - \frac{e_{s'}}{m_{s'}} \right) \frac{1}{(2\pi)^3} \int_0^\infty dk k^4 \\ & \times \frac{1}{2\pi i} \text{P} \int_{-\infty}^\infty dx (\epsilon^{\beta x} - 1)^{-1} \{ U_{\vec{k}}^+(x) U_{\vec{k}}^+(x + \hbar\omega) [Q_{\vec{k}}^+(x + \hbar\omega, s) - Q_{\vec{k}}^+(x, s)] \\ & \times [Q_{\vec{k}}^+(x + \hbar\omega, s') - Q_{\vec{k}}^+(x, s')] - U_{\vec{k}}^-(x) U_{\vec{k}}^+(x + \hbar\omega) [Q_{\vec{k}}^+(x + \hbar\omega, s) - Q_{\vec{k}}^-(x, s)] [Q_{\vec{k}}^+(x + \hbar\omega, s') - Q_{\vec{k}}^-(x, s')] \}, \end{aligned} \tag{1}$$

where  $e_s$ ,  $n_s$ , and  $m_s$  are respectively the charge, particle density, and mass of the  $s$ th species,  $\beta$  the inverse temperature in energy units,  $\text{P}$  denotes the principal value,  $\omega$  the frequency,

$$Q_{\vec{k}}^\pm(\omega, s) = \frac{1}{\hbar} \frac{1}{(2\pi)^3} \int d\vec{p} \frac{n_{\vec{p} + \frac{1}{2}\vec{k}}(s) - n_{\vec{p} - \frac{1}{2}\vec{k}}(s)}{\vec{p} \cdot \vec{k} - \omega \mp i\epsilon}, \tag{2}$$

with  $n_p(s)$  the distribution function of the  $s$ th species, and

$$U_{\vec{k}}^\pm(x) = \frac{4\pi}{k^2} \left[ 1 - \frac{4\pi}{k^2} \sum_s e_s^2 Q^\pm(x, s) \right]^{-1} \tag{3}$$

is the effective potential. The classical limit of Eq. (1) for a hydrogenic plasma leads to results identical to reference 2.

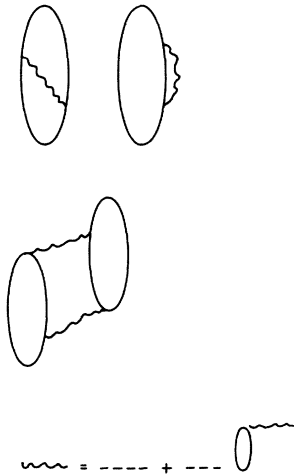


FIG. 1. The diagrams which contribute to the conductivity.

An interesting new result is the conductivity of a quantum system of electrons moving in a field of heavy scatterers. In this case Eq. (1) yields

$$\sigma(\omega) = i \frac{e^2 n}{m\omega} \left\{ 1 - \frac{2}{3\pi} \frac{e^2}{m\omega^2} \int dk k^2 \left[ \frac{1}{D(k, 0)} - \frac{1}{D(k, \omega)} \right] \right\}, \tag{4}$$

where  $e$ ,  $n$ , and  $m$  are respectively the charge, particle density, and mass of the electrons (properly renormalized in the case of a crystal), and

$$D(k, \omega) = 1 - \frac{4\pi e^2}{\hbar k^2} \frac{1}{(2\pi)^3} \int d\vec{p} \frac{n_{\vec{p} + \frac{1}{2}\vec{k}} - n_{\vec{p} - \frac{1}{2}\vec{k}}}{\vec{p} \cdot \vec{k} - \omega - i\epsilon} \tag{5}$$

is the dielectric constant of the system. This result can be applied to the study of the impurity contribution to the absorption of electromagnetic waves in doped semiconductors,<sup>5</sup> when the applied frequencies embrace the plasma frequency,  $4e^2 n/m$ , of the conducting electrons.

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