(14) will differ slightly in this case and the details will be given in reference 2.

The author wishes to express his thanks to Professor J. R. Oppenheimer for the hospitality and the support of the Institute for Advanced Study.

---

*Work supported by the National Science Foundation.

[1]For a complete list of references, see the review talk by S. D. Drell, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), pp. 897-911.

[2]N. N. Khuri (to be published).

[3]A. Martin, Phys. Rev. Letters 9, 410 (1962).

[4]R. Oehme, Phys. Rev. Letters 9, 358 (1962).

[5]N. N. Khuri, Phys. Rev. 130, 429 (1963).

[6]In all the present discussions, one assumes that $\beta_i(x)$ vanish faster than $x^{-1/2}$ as $x \to \infty$.

RELATIVE WEIGHTS OF THE DECAYS OF CERTAIN RESONANCES IN THEORIES WITH BROKEN SYMMETRY*

C. Dullemond
Department of Physics, University of Washington, Seattle, Washington

and

A. J. Macfarlane and E. C. G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York

(Received 5 March 1963)

(I) The principal aim of this work is to derive, in illustration of a general procedure,[1] relationships that obtain among the relative weights[2] of the decays of various resonances into baryon-meson states in the Ne'eman–Gell-Mann theory[3] of strong interactions, when exact invariance under SU_3 suffers a first-order perturbation by interactions invariant under only isospin and strangeness transformations. In particular, we deal with those resonances[4] which Glashow and Sakurai[5] have associated with the irreducible representation (IR) of SU_3 with highest weight[6] (3, 0).

(II) We may best explain our procedure by considering first a much simpler situation involving the decay of a particle (of isospin I with z component ν) into two particles (of isospins I_1 and I_2 with z components ν_1 and ν_2) in a theory whose charge independence suffers a first-order perturbation by an interaction (e.g., the electromagnetic interaction) which commutes with the operator I_z but not with $\vec{I}^2$ and hence may be taken to transform under R_3 like the $I_z = 0$ component of a vector operator. We denote the matrix element for the decay by $\langle I_1\nu_1 I_2\nu_2 | T | I\nu\rangle$ and consider the sums

$$P_1(\nu) = \sum_{\nu_2} |\langle I_1\nu - \nu_2 I_2\nu_2 | T | I\nu\rangle|^2, \qquad (1)$$

$$P_2(\nu_1) = \sum_{\nu_2} |\langle I_1\nu_1 I_2\nu_2 | T | I\nu_1 + \nu_2\rangle|^2. \qquad (2)$$

We easily prove that $P_1(\nu)$ is (A_1) independent of ν to zeroth order and (B_1) of the form $(\alpha + \beta\nu)$ to first order in the perturbation, and similarly that P_2 is (A_2) independent of ν_1 and (B_2) of the form $(\gamma + \delta\nu_1)$. The result (A_1) states the equality of the total weight for all decays for different charge states of the decaying particle; result (A_2) is the Shmushkevich theorem[7] for the decay. Results (B_1) and (B_2) are new. The proof of these results involves only simple facts regarding R_3 including properties of Clebsch-Gordan coefficients, the Wigner-Eckart theorem, and the fact that $C(j1j, m0m)$ is proportional to m for fixed j.

We illustrate using the decays of the well-known 3-3 nucleon resonance N^* into nucleon plus pion states. As an aid to the application of the above results, we draw up a Shmushkevich table (Table I) for the allowed processes. We see that results (A_1) and (B_1) give

$$\Gamma_1 = \Gamma_2 + \Gamma_3 = \Gamma_4 + \Gamma_5 = \Gamma_6, \qquad (3.A_1)$$

$$\Gamma_1 + \Gamma_2 + \Gamma_4 = \Gamma_3 + \Gamma_5 + \Gamma_6, \qquad (3.A_2)N$$

$$\Gamma_1 + \Gamma_3 = \Gamma_2 + \Gamma_5 = \Gamma_4 + \Gamma_6, \qquad (3.A_2)\pi$$

and hence we have the complete solution

$$2\Gamma_1 = 3\Gamma_2 = 6\Gamma_3 = 6\Gamma_4 = 3\Gamma_5 = 2\Gamma_6, \qquad (4)$$

in the zeroth order of the perturbation. In the first order of the perturbation, we fail to get a complete solution, but only the identities

$$\Gamma_1 - (\Gamma_2 + \Gamma_3) = (\Gamma_2 + \Gamma_3) - (\Gamma_4 + \Gamma_5) = (\Gamma_4 + \Gamma_5) - \Gamma_6, \qquad (5.B_1)$$

$$(\Gamma_1 + \Gamma_3) - (\Gamma_2 + \Gamma_5) = (\Gamma_2 + \Gamma_5) - (\Gamma_4 + \Gamma_6). \qquad (5.B_2)\pi$$

(III) We now generalize the discussion of para-

Table I. Shmushkevich table for $N \to N+\pi$.

Process	Relative weight	N^{*++}	N^{*+}	N^{*0}	N^{*-}	p	n	π^+	π^0	π^-
$N^{*++} \to p+\pi^+$	Γ_1	1				1		1		
$N^{*+} \to p+\pi^0$	Γ_2		1			1			1	
$N^{*+} \to n+\pi^+$	Γ_3		1				1	1		
$N^{*0} \to p+\pi^-$	Γ_4			1		1				1
$N^{*0} \to n+\pi^0$	Γ_5			1			1		1	
$N^{*-} \to n+\pi^-$	Γ_6				1		1			1

graph (II) to the situation mentioned in paragraph (I).

We use the notation[6] (λ, μ) for an IR of SU_3, the integers λ, μ being the components of its highest weight. In a given (λ, μ) we may introduce a basis $|(\lambda, \mu)I\nu Y\rangle$, where ν and Y (hypercharge) give the eigenvalues of the operators of the Cartan subalgebra of SU_3 and I belongs to the SU_2 subgroup. Knowing the (I, Y) content of the (λ, μ), one can assign to them appropriate[3,5,6,8] multiplets of particles or resonances. Let us now consider the decay weight

$$|\langle(\lambda_1, \mu_1)I_1\nu_1Y_1, (\lambda_2, \mu_2)I_2\nu_2Y_2|T|(\lambda, \mu)I\nu Y\rangle|^2 \quad (6)$$

and form sums over such quantities, $Q_1(I, \nu, Y)$ and $Q_2(I_1, \nu_1, Y_1)$, analogous to those appearing in Eqs. (1) and (2), i.e., respectively, over I_1, I_2, ν_2, Y_2 for fixed I, ν, Y and over I_2, I, ν_2, Y_2 for fixed I_1, ν_1, Y_1. Since the properties of Clebsch-Gordan coefficients of R_3 used in deriving (A_1) and (A_2) above generalize[1] to SU_3, we find (C_1) that $Q_1(I, \nu, Y)$ is, in fact, independent of I, ν, and Y; (C_2) that $Q_2(I_1, \nu_1, Y_1)$ is independent of I_1, ν_1, Y_1 to zeroth order in the perturbation. Result (C_1) states that the total widths of all decays for different members of the decaying multiplet are equal and (C_2) is the generalized Shmushkevich theorem for the situation.

Following Okubo,[8] we assume that the interactions which break exact symmetry under SU_3 are invariant under isospin and strangeness transformations. It follows[9] that we may take them to transform under SU_3 as does the $I=\nu=Y=0$ component of a tensor operator which transforms irreducibly under the regular representation (1, 1) of SU_3. This is in direct analogy to the simpler case, the vector representation of R_3 being the regular representation. If we now assume that this perturbation of SU_3 invariance is a first-order effect, we may proceed by an application of the Wigner-Eckart theorem[10] for SU_3 to the results, (D_1) and (D_2), that the sums $Q_1(I, \nu, Y)$ and $Q_2(I_1, \nu_1, Y_1)$ are now of the forms

$$\alpha+\beta Y+\gamma[I(I+1)-\tfrac{1}{4}Y^2],$$

$$\xi+\eta Y_1+\xi[I_1(I_1+1)-\tfrac{1}{4}Y_1^2], \quad (7)$$

respectively. Here we have used Eq. (A.8) of Okubo's paper.[8]

We now illustrate using the decays of the resonances N^*, Y_1^*, Ξ^*, and Ω, associated[5] with the IR (3, 0) of SU_3 into baryon plus meson states. Baryons and mesons are assigned to identical octet IR's (1, 1) of SU_3. Decays allowed by $\vec{I}$ and Y conservation correspond to the vertices $(N^*N\pi)$, $(N^*\Sigma K)$, $(Y_1^*N\bar{K})$, $(Y_1^*\Lambda\pi)$, $(Y_1^*\Sigma\pi)$, $(Y_1^*\Sigma\eta)$, $(Y_1^*\Xi K)$, $(\Xi^*\Lambda\bar{K})$, $(\Xi^*\Sigma\bar{K})$, $(\Xi^*\Xi\eta)$, $(\Xi^*\Xi\pi)$, and $(\Omega\Xi\bar{K})$ with η some unspecified $I=Y=0$ meson. We shall deal with the sums of the relative weights for all decays of resonance into baryon plus meson within each of these groups, denoting these sums in the order given by $w_1, w_2, \cdots, w_{12}$. We may now draw up the Shmushkevich table (Table II). Since we have not distinguished between different charge complexions within any group of decays (but, rather, summed over them), we must take account of this in the applications of the results $(C_1)\cdots(D_2)$ by dividing by a factor $(2I+1)$ when an (I, Y) multiplet is involved. Also, to abbreviate the results that ensue, let us use the invariance of decay weights under interchange of baryon and meson octets to give

$$w_1=w_2,\ w_3=w_7,\ w_4=w_6,\ w_8=w_{10},\ w_9=w_{11}, \quad (8)$$

valid in both the zeroth and first order of perturbation. We obtain, in the zeroth order of perturbation, the independent identities

$$\tfrac{1}{4}(2w_1)=\tfrac{1}{3}(2w_3+2w_4+w_5)=\tfrac{1}{2}(2w_8+2w_9)=w_{12}, \quad (9.C_1)$$

$$\tfrac{1}{2}(w_1+w_3)=w_4+w_8=\tfrac{1}{3}(w_1+w_4+w_5+w_9), \quad (9.C_2)$$

Table II. Shmushkevich table for $(3,0) \to (1,1)+(1,1)$.

Decay	N^*	Y_1^*	Ξ^*	Ω	N	Λ	Σ	Ξ	K	η	π	$\bar{K}$
w_1	1				1						1	
w_2	1						1		1			
w_3		1			1							1
w_4		1				1					1	
w_5		1					1				1	
w_6		1					1			1		
w_7		1						1	1			
w_8			1			1						1
w_9			1				1					1
w_{10}			1				1	1		1		
w_{11}			1					1			1	
w_{12}				1				1				1

and, in the first order,

$$\tfrac{1}{4}(2w_1)+\tfrac{1}{2}(2w_8+2w_9)=\tfrac{2}{3}(2w_3+2w_4+w_5)$$
$$=2(2w_8+2w_9)-2w_{12}, \qquad (10.\mathrm{D}_1)$$

$$(w_1+w_3)+(w_3+w_8+w_9+w_{12})=3(w_4+w_8)$$
$$+\tfrac{1}{3}(w_1+w_4+w_5+w_9). \qquad (10.\mathrm{D}_2)$$

A full solution of the zeroth-order problem does not follow from Eqs. (8) and (9). However, we can use the consequences of charge independence [obtained as in paragraph (II)] to tell what fractions of each w_α belong to the different charge complexions of the αth group of decays, and hence use the consequences[11] of invariance under the Weyl reflections of SU_3 to derive the complete solution

$$3w_1=12\,w_3=8w_4=12\,w_5=12\,w_8=12\,w_9=6w_{12}. \qquad (11)$$

In the first-order problem, we have Eqs. (8) and (10) and no others. It is thus possible to express all remaining w_α in terms of the experimentally accessible quantities w_1, w_4, w_5, and w_9, but not possible to obtain any identity involving only these quantities. Hence, although we can make predictions using our broken unitary symmetry theory, we cannot actually test it.

We conclude by remarking that our methods generalize[1] in each of the situations discussed to scattering and production reactions.

---

*Research supported by the U. S. Atomic Energy Commission.

[1]This is discussed in a forthcoming publication by the authors, which contains proof of certain statements made below.

[2]Relative weight converts into relative reduced width by multiplication by appropriate kinematic factors.

[3]Y. Ne'eman, Nucl. Phys. 26, 222 (1961). M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

[4]For recent information, see S. L. Glashow and A. H. Rosenfeld, University of California Radiation Laboratory Report UCRL 10579, 1962 (unpublished).

[5]S. L. Glashow and J. J. Sakurai, Nuovo Cimento 25, 237 (1962); 26, 622 (1962).

[6]See, e.g., R. E. Behrends et al., Rev. Modern Phys. 34, 1 (1962).

[7]A discussion of Shmushkevich's method is given by R. E. Marshak and E. C. G. Sudarshan, in Introduction to Elementary Particle Physics (Interscience Publishers, Inc., New York, 1961), p. 185. The original paper is I. M. Shmushkevich, Dokl. Akad. Nauk S.S.S.R. 103, 235 (1955).

[8]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 944 (1962).

[9]B. Diu (unpublished).

[10]J. Ginibre (unpublished).

[11]C. A. Levinson, H. J. Lipkin, and S. Meshkov, Nuovo Cimento 23, 236 (1962).