If the lines became narrower by a factor of ten (i.e., if the width was reduced from several cm^{-1} to several tenths of a cm^{-1}), as might reasonably be expected in a 10% solution of $PrCl_3$ in $LaCl_3$, then no increase in path length or exposure time would be necessary to obtain lines of the same strength on the plate as those obtained by us for the pure material.

We wish to thank Dr. B. P. Stoicheff for his help throughout the course of this work.

*On leave of absence from Atomic Energy Establishment, Bombay, India.

[1]R. J. Elliott and R. Loudon, Phys. Letters 3, 189 (1963).
[2]E. V. Sayre, K. M. Sancier, and S. Freed, J. Chem. Phys. 23, 2060 (1955); 29, 242 (1958).
[3]G. H. Dieke and R. Sarup, J. Chem. Phys. 29, 741 (1958).

LIFETIME AND BEAM SIZE IN A STORAGE RING

C. Bernardini, G. F. Corazza, G. Di Giugno, and G. Ghigo
Laboratori Nazionali del Sincrotrone, Frascati, Roma, Italy

and

J. Haissinski and P. Marin
Laboratoire de l'Accelerateur Lineaire, Orsay, France

and

R. Querzoli
Laboratori Nazionali del Sincrotrone, Frascati, Roma, Italy

and

B. Touschek
Istituto Nazionale de Fisica Nucleare, Roma, Italy
(Received 1 April 1963)

We want to report on measurements of the lifetime of stored beams carried out with the Frascati e^+e^- storage ring (AdA) at the Laboratoire de l'Accelerateur Lineaire of the Science Faculty at Orsay. The design parameters of the ring have already been published[1] as well as preliminary results about operation at a low stored intensity.[2] The use of the Orsay linac as injector allowed the storage of a big enough number of electrons to observe the following effects.

With a small number of stored electrons (less than 10 000) the lifetime is of the order of 50 hours corresponding to a residual (air) pressure of $\sim 5 \times 10^{-10}$ Torr. The vacuometer reading was 3.5×10^{-10} Torr.

The lifetime τ depends on the number N of particles in the beam. It can be fitted by

$$1/\tau = \alpha(E)N + 1/\tau_0, \tag{1}$$

where $\alpha(E)$ is a strongly energy-dependent parameter. A best fit illustrating (1) is shown in Fig. 1.

The energy dependence of $\alpha(E)$ is shown in Fig. 2. In the interval $100 < E < 207$ MeV this curve can be fitted by

$$\alpha(E)E^{5.5} = \text{const.} \tag{2}$$

$\alpha(E)$ has a maximum at about 70 MeV. Below this energy the lifetimes increase very rapidly. Lifetime measurements below 50 MeV were not carried out because of the difficulties in revealing the synchrotron radiation at such low energies.

The lifetime of a given beam is independent of the presence of the other, so that the effect described must be interpreted as a "self-interaction" of the particles in each bunch.

A theoretical explanation of the effect can be given in the following manner. We observe that in a "natural beam," i.e., in a beam the dimensions of which are defined solely by the fluctuations of the synchrotron radiation, the momentum distribution is subject to the inequality

$$\delta q \gg \delta q_z \approx \delta q_e. \tag{3}$$

(The latter equality is one of order of magnitude.) Here δq is the rms of the radial momentum, δq_z the rms of the vertical momentum, and δq_e the rms of the longitudinal momentum, the latter being measured in the system in which the bunch is at rest. The inequality (3) holds above a certain critical energy (of the order of about 30 MeV in AdA). The Møller scattering between two elec-

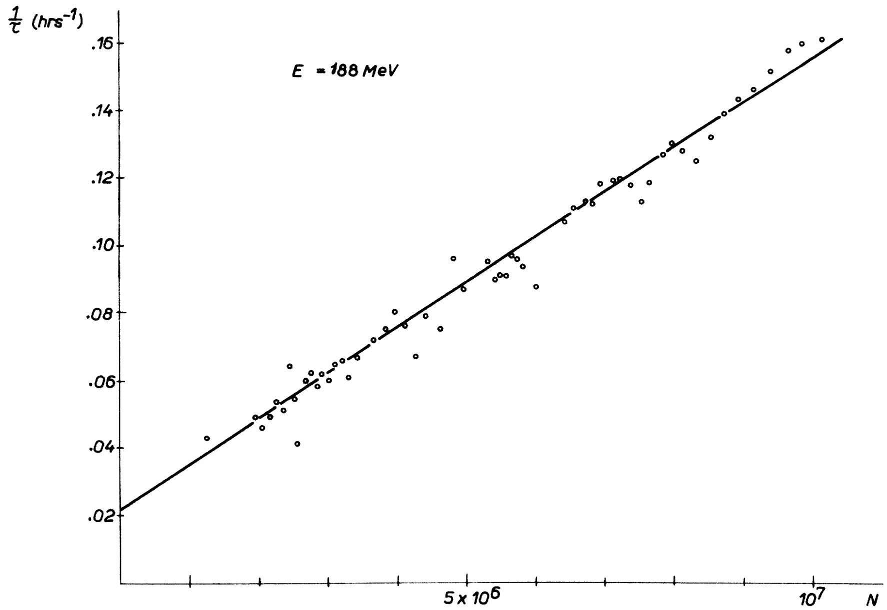

FIG. 1. Lifetime τ versus N, the number of stored particles in a beam, at the energy of 188 MeV.

trons in the bunch can now lead to a transfer of radial momentum into longitudinal momentum, and it is immediately seen that if the acquired longitudinal momentum is bigger than $\Delta q = \Delta p/\gamma$, where Δp is the (central) momentum acceptance of the radio frequency in the lab system and $\gamma = E/mc^2$, any such scattering process will lead to the loss of two particles. For, after the scattering process, the forward scattered electron will have too much and the backward scattered electron too little energy to be contained with the stable region of the radio frequency.

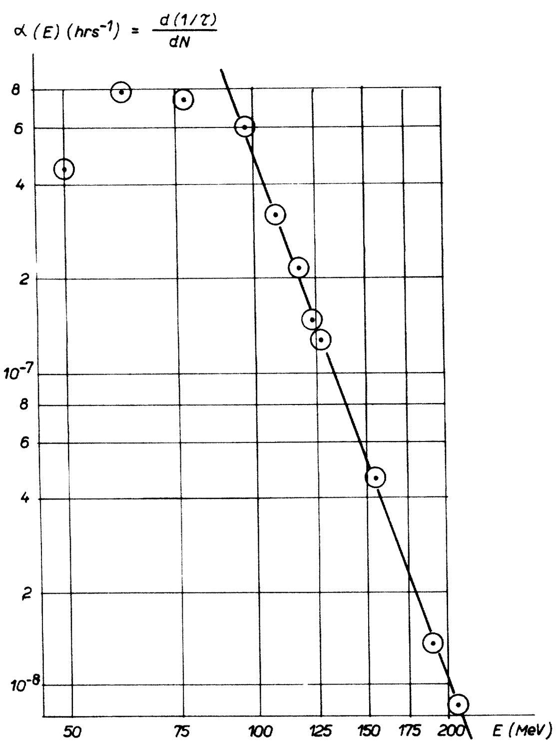

FIG. 2. Plot of the rate $\alpha(E)$, as defined in Eq. (1) of the text, versus the energy of electrons in a beam.

Taking advantage of the fact that the motion of the particles in the system in which the bunches are at rest is nonrelativistic—at least for moderately small storage rings—$\alpha(E)$ can be determined explicitly, and one obtains

$$\alpha(E) = (2\pi^{1/2} r_0^2 c/kV\Delta p^2 \delta q)L(x). \tag{4}$$

Here $r_0 = 2.8\times10^{-13}$ cm is the Lorentz radius, c the velocity of light, V is the volume of the bunch (measured in the lab system), k is the "harmonic index" of the radio frequency. The parameter x is defined by

$$x = (\Delta p/\gamma\delta q)^2, \tag{5}$$

and $L(x)$ is a function of the radial momentum distribution. If one assumes—incorrectly—that this distribution is Gaussian, one obtains for $x \to 0$

$$\lim_{x\to 0} L(x) = \tfrac{1}{2}[\log(1/x) - C - \tfrac{3}{2}], \tag{6}$$

where $C = 0.577$ is the Euler constant. Inserting the most probable values of the parameters and choosing for V the "natural" volume (i.e., about 5×10^{-4} cm^3 at an energy of 200 MeV), one obtains a value for $\alpha(E)$ which is about 40 times larger than the values shown in Fig. 1. It can also be shown that in the region $E > 100$ MeV [where (6) is certainly valid], the empirical equation (2) should be replaced by

$$\alpha(E)E^{+4.5} = \text{const.} \tag{2a}$$

This can be seen as follows: The "natural" volume varies as $E^{2.5}$, δq as E^2, Δp^2 as E, and the factor L can be approximated in this region by const $\times E$.

This qualitative disagreement can be completely removed if one assumes that the actual volume of the bunches is about 40 times bigger than the natural one and that this enlargement of the volume is due to a coupling between radial and vertical betatron oscillations. The last assumption makes sure that (2) holds and not (2a), since in the case of coupling V varies as $E^{3.5}$ and not as $E^{2.5}$.

The function $L(x)$ also gives a qualitative description of the deviation from (2) for energies below 100 MeV. This deviation is due to the fact that with decreasing energy the width of the momentum distribution also decreases (x goes as E^{-5}). For $x = 1$ (at about 38 MeV in AdA) the longitudinal momentum transferred in the majority of collisions is too small to lead to the loss of particles. A detailed comparison with experiment is rendered difficult by the following circumstances: The momentum distribution of the natural beam is not Gaussian; if $x \approx 1$, it is further modified by the fact that it is no longer true that most Coulomb collisions lead to the loss of particles. The behavior of $\alpha(E)$ at high energies is not subject to these uncertainties, since all the processes which lead to the loss of electrons come from the center—and not as in the case of low energies from the tail—of the momentum distribution.

There is further supporting evidence for assuming that the actual volume is considerably larger than the natural one. At about 200 MeV the natural height of the beam should be about 2 μ. Multiple electron-positron scattering should bring this to about 10 if the ring is charged with two beams containing 10^7 particles each. Since we have shown that $\alpha(E)$ is insensitive to the presence of the other beam, it follows that the effective height of a single beam must be considerably larger than 10 μ. This is also borne out by the—as yet not conclusive—evidence on the frequency of 2γ annihilations, which strongly suggests an effective beam height of more than 25 μ.

Our thanks are due to Dr. F. Lacoste for his help in installing AdA at Orsay and his collaboration during the first runs of the machine. We want also to acknowledge the help of the linac crew and particularly of Dr. B. Millman and Dr. L. Burnod. Finally we thank Professor A. Blanc Lapierre for his hospitality and interest.

[1]C. Bernardini, G. F. Corazza, G. Ghigo, and B. Touscheck, Nuovo Cimento 18, 1293 (1960).

[2]C. Bernardini, U. Bizzarri, G. F. Corazza, G. Ghigo, R. Querzoli, and B. Touscheck, Nuovo Cimento 23, 202 (1962).

DETERMINATION OF THE RELATIVE Σ - Λ PARITY*

H. Courant,† H. Filthuth, P. Franzini,‡ R. G. Glasser,‖ A. Minguzzi-Ranzi, A. Segar,⊥ and W. Willis**
CERN, Geneva, Switzerland

and

R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Sakitt,†† B. Sechi-Zorn, N. Seeman, and G. A. Snow‡‡
University of Maryland, College Park, Maryland and U. S. Naval Research Laboratory, Washington, D. C.
(Received 11 April 1963)

To determine the relative Σ - Λ parity, we have measured the invariant mass spectrum of Dalitz pairs from the decay of unpolarized Σ^0 hyperons, $\Sigma^0 \to \Lambda^0 + e^- + e^+$. This method has been suggested by Feinberg[1] and Feldman and Fulton.[2] The problem is to establish the decay as an electric (odd Σ - Λ parity) or a magnetic (even parity) dipole transition. Under the even-parity hypothesis, the radiative matrix element is proportional to the momentum of the Dalitz pair, whereas with odd parity the matrix element is independent of the pair momentum. This has the consequence that for odd parity more Dalitz pairs exhibiting large invariant mass would be expected to occur