16 $I=1$ combinations, K^0K^- and $\bar{K}^0K^+$ produced in the reactions $K^- + p \to \Sigma^- + K^+ + \bar{K}^0$ and $K^- + p \to \Sigma^+ + K^- + K^0$. Further, if one assumes $I=1$, the triangular inequality relating these reactions to their neutral counterparts $K^- + p \to \Sigma^0 + K^0 + \bar{K}^0$ and $\Sigma^0 + K^+ + K^-$, $[\sigma(\Sigma^+)]^{1/2} + [\sigma(\Sigma^-)]^{1/2} \geq 2[\sigma(\Sigma^0)]^{1/2}$, is violated to the extent $(3 \pm 3)^{1/2} + (0 \pm 1)^{1/2} \geq 2(22 \pm 5)^{1/2}$.

[17]We estimate, for example, that $\varphi \to 2\pi$ via an electromagnetic transition is $\sim 2 \times 10^{-3}$ less frequent than $\varphi \to K\bar{K}$. The 3π rate is dominated by $\varphi \to \rho + \pi$ because only two-body phase space is involved.

[18]Due to these uncertainties the ratio β provides essentially no new information on the φ spin.

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7- TO 20-BeV/c $\pi^- + p$ AND $p + p$ ELASTIC SCATTERING AND REGGE POLE PREDICTIONS*

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This Letter reports measurements of the elastic differential cross section from 7 to 20 BeV/c incident momentum for $p+p$ and 7 to 17 BeV/c for $\pi^- + p$ over the t range 0.2 to 1.0 (BeV/c)^2, where t is the negative square of the Lorentz-invariant four-momentum transfer. This experiment is part of a program to study basic strong interactions in the energy range available at the Brookhaven AGS, i.e., ~10-20 BeV. These measurements are of great current interest due to the striking predictions made by the Regge pole theory and allow a critical evaluation of the theory. If the energy is sufficiently high that the vacuum or Pomeranchuk Regge trajectory dominates, one would expect as predicted by Chew and Frautschi[1] and others[2] that for any incident particle

$$\frac{d\sigma}{dt} = \left[\frac{d\sigma}{dt}\right]_{\text{opt}} F(t) \left[\frac{s}{s_0}\right]^{2\alpha_p(t)-2}, \qquad (1)$$

where $[d\sigma/dt]_{\text{opt}}$ is the minimum value of $d\sigma/dt$ at $t=0$ predicted from the optical theorem, s is the Lorentz-invariant square of the total center-of-mass energy, and s_0 is usually taken to be $2m_p^2$. This prediction obviously gives a shrinkage of $d\sigma/dt$ (logarithmic with s) corresponding at low t in a semiclassical way to a growth of the radius of interaction. Previous measurements[3] were of insufficient accuracy to establish such behavior above 10 BeV/c. The present $p+p$ scattering results do show a shrinkage of this type consistent with the Regge pole prediction but the $\pi^- + p$ scattering shows no shrinkage, thus contradicting the assumption of dominance of the strong interactions by a single vacuum pole. Our results are also inconsistent with the predictions of the three-pole model of Hadjioannou et al.[4] and Drell,[5] which was developed to explain the total cross-section data – which are inconsistent with single-pole prediction. In the three-pole model F in Eq. (1) is a relatively weak function of s as well as a function of t but shrinkage of $d\sigma/dt$ is still predicted – with a modified $\alpha_p(t)$ which slightly overestimates shrinkage effects for $p+p$. Therefore, we will analyze as though the vacuum pole alone were important and then relate our results to the three-pole model.

Figure 1 shows the experimental arrangement. A $4\frac{1}{2}°$ secondary beam from the Brookhaven AGS was momentum analyzed and focussed at scintillator S_3, a circular counter 2 in. in diameter. This beam had a measured angular divergence of ±1.5 mrad and a momentum resolution of $\pm 1\frac{1}{2}\%$ with the mean momentum known to better than 0.3%. The desired particle was selected by the scintillator telescope $S_1S_2S_3$ and a high-resolution differential gas Cherenkov counter C previously described.[6] The liquid hydrogen target was 4 in. in diameter by 20 in. long. The scattered particle was detected by the scintillation counter hodoscope, H_s, consisting of 12 vertical counters (2 in. wide by 12 in. high) and 12 horizontal counters (1 in. high by 24 in. long), defining 144 equivalent intersectional counter areas which covered the t range 0.2 to beyond 1.0 (BeV/c)^2. The recoil proton from the target was detected by two hodoscopes, the target screen, H_t, 12 vertical 2-in. wide counters alongside the target, and the recoil screen, H_r, 22 vertical and 22 horizontal counters all $2\frac{1}{2}$ in. wide by 60 in. long defining 484 equiva-

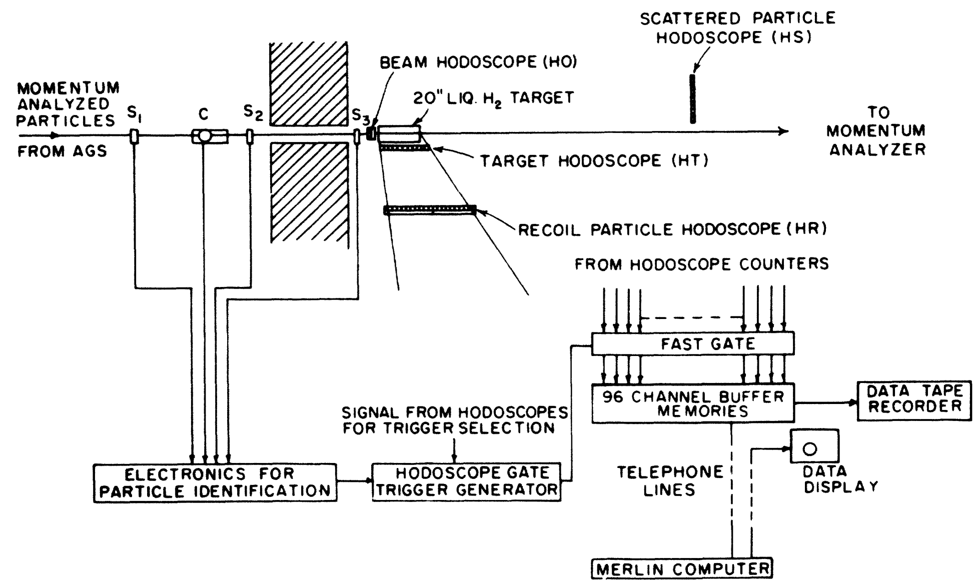

FIG. 1. The experimental arrangement.

lent $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. areas. A 4-by-4 hodoscope, H_0, located in front of the hydrogen target, defined the lateral and vertical position of the incident particle to $\frac{1}{2}$ in.

The outputs of the counters were fed to 96 fast (30-nanosecond) discriminator-gates. Parallel outputs from the vertical elements of H_r, H_t and H_S were fed into the trigger selecting circuit such that a coincidence between the Cherenkov telescope signal and at least one pulse from each hodoscope generated a trigger signal to open the 96 gates. The 96 bits of information thus were transferred to a digital data handler[7] having a magnetic core buffer memory with a capacity of 32 such events. This unit, which is located in a data-center trailer at AGS, could accept events 5 μsec apart. Between AGS bursts these data were recorded on magnetic tape and simultaneously transmitted over telephone lines to the Brookhaven Merlin computer[8] for on-line processing. As each event was received, the computer classified it according to whether it had one or more triggers per hodoscope. For those events having only one particle per hodoscope, the space angles of both scattered and recoil particles were calculated. The event was then stored in memory; 24 different distributions of recoil angle were stored, coplanar and noncoplanar events each being separated into 12 groups according to the horizontal location in H_S involved. An oscilloscope display of these distributions was returned to the AGS data-center trailer over additional telephone lines. A peak at the kinematically expected recoil angle appeared in each of the coplanar event distributions and none in the noncoplanar. At the end of each data-taking period the computer totaled the number of scattered particles in each bin and calculated and subtracted a background of inelastic events. This background was one to a few percent for most of the t range but, for a few cases at the highest t values, it became as high as 30%. The computer calculated the absolute cross section $d\sigma/dt$, its statistical error, and each of the 12 mean t values.

The effective mean scattering polar angle resolution varied from ±2 mrad at 20 BeV/c to ±5 mrad at 7 BeV/c incident. The recoil polar angle resolution was ±15 mrad and the azimuthal angles were measured to ±10 mrad on the recoil side and varied from ±30 mrad at $t=0.2$ to ±15 mrad at $t=1.0$ for the scattered particle. Before the run, the processing of data by the Merlin computer and programming method was cross-checked against an IBM 7090 and gave identical results. Hodoscope H_S was moved perpendicular to the beam and other systematic checks indicated that all counters were operating with uniform efficiency. The results were also found to be independent of the effective coplanarity resolution which was varied in the computer program. The results are shown in Figs. 2 and 3. The vertical scale is

$$\left[\frac{\sigma_{\text{tot}}(20\ \text{BeV}/c)}{\sigma_{\text{tot}}(P)}\right]^2 \frac{d\sigma}{dt}$$

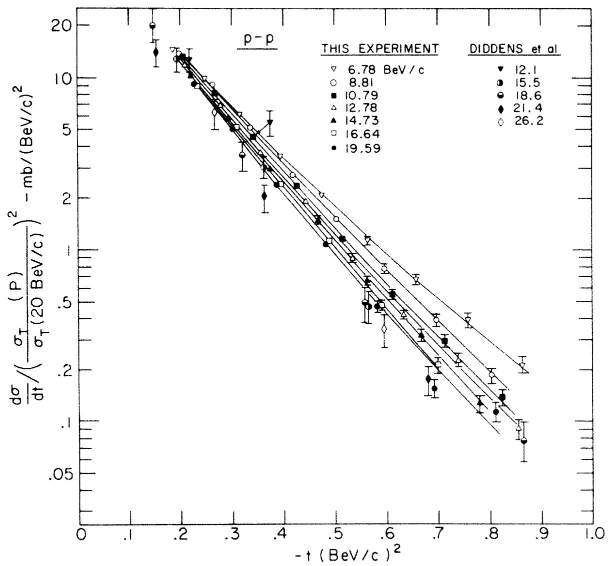

FIG. 2. Differential cross section for elastic $p+p$ scattering. The different symbols represent different incident laboratory momenta. The data of Diddens et al. are from reference 3. The 12.1-BeV/c point of Diddens et al. at $t=-0.3$ (BeV/c)^2 should be superimposed on our point at 10.79 BeV as indicated by the arrow.

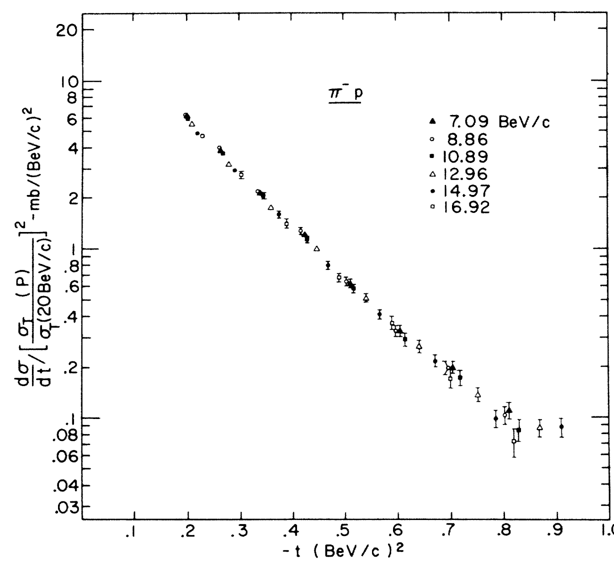

FIG. 3. Differential cross section for elastic π^-+p scattering. The different symbols represent different incident laboratory momenta.

in mb/(BeV/c)^2. From an analysis of the total cross-section data[6] we used $\sigma_{tot}(p+p)=39.5$ mb for 11 to 20 BeV/c; at 9 and 7 BeV/c we used 40.2 mb and 41.2 mb, respectively. For π^-+p the least squares fit to the total cross section $\sigma_{tot}=24.37+24.94/P$ from a previous publication[9] was used. Using the shape of the differential cross sections the mean value of t, shown as the abcissa, was calculated; allowance was made for beam shape, beam momentum spread, finite target size and finite counter sizes. The errors shown include compounded estimates of relative errors which would affect conclusions regarding shrinkage. These include statistical errors, relative efficiency errors, relative normalization errors, uncertainty in background subtraction and relative errors introduced in calculating mean t values due to momentum and angle uncertainties.

The absolute normalization constant or scale factor is uncertain by 10% for $p+p$ and 15% for π^-+p. These values will be improved in further processing of absolute corrections to the data. The largest t value shown is approximately -0.8 (BeV/c)^2 since at the high momenta the peak-to-background ratio fell below ≈3 to 1 for larger t. At each momentum we analyzed typically 15 000 elastic events.

The curves in Fig. 2 are computer least-squares fits of the form $d\sigma/dt=\exp(a+bt+ct^2)$. It is clear from the separation of the individual curves and the small errors in Fig. 2 that a shrinkage of the diffraction scattering curve $d\sigma/dt$ witn increasing s is definitely established.

The results of previous $p+p$ elastic scattering experiments by Diddens et al.[3,10] in the momentum range 12 to 26 BeV/c and t range 0 to 1 (BeV/c)^2 are also shown on Fig. 2. The bulk of these data, though perhaps suggestive of shrinkage at high t, were, within the errors, reasonably consistent with no shrinkage. Combining the results of Diddens et al. with the low-energy experiments from other laboratories yielded evidence of shrinkage between ~4 BeV/c and the average of 12 to 26 BeV/c. One would hardly, however, consider 4 BeV/c as in the asymptotic region in the sense of Regge pole theory, since all total cross sections are still strongly energy-dependent there.

In the case of π^-+p (Fig. 3) the data fit well on a single curve, independent of incident momentum; a computer fit to all 50 points of the form $d\sigma/dt=\exp(a+bt+ct^2)$ gave a χ^2 of 51. Clearly there is either no shrinkage of the π^-+p elastic scattering curve or, at most, it is a very small effect compared to the $p+p$ case.

A better way to illustrate Regge pole effects is to take the logarithm of both sides of Eq. (1), giving

$$\log\left[\frac{d\sigma}{dt}\Big/\frac{d\sigma}{dt_{\rm opt}}\right]=\log F+[2\alpha_p(t)-2]\log(s/s_0). \qquad (2)$$

In the vacuum pole case F is a function of t only, whereas in the three-pole model, the effects of the additional poles can be represented by introducing an additional weak dependence of F on s, which we shall temporarily ignore.

Figures 4 and 5 show the cross sections at fixed t plotted versus $\log_{10}s$. The least-squares fits previously obtained were used to interpolate to constant t values as the original data points were at slightly different values of t for different incident momenta.

The points in Figs. 4 and 5 have been least-squares fitted to straight lines which, according to Eq. (2) have slopes $2\alpha(t)-2$. Values of $\alpha(t)$ obtained using all $p+p$ data are shown in Fig. 6. The values of $\alpha(t)$ obtained for $p+p$ data at >10 BeV/c incident momentum are also shown. This distinction was made because above 10 BeV/c the $p+p$ total cross sections are constant. Also, the momentum variation of the total cross sections and differences between particle and antiparticle cross sections have become small enough so that, for example, the three-pole model has been thought to apply. For all $p+p$ data combined the least-squares fitted straight line extrapolates to $\alpha_p(0)=1.07\pm0.03$ and would pass through $\alpha_p=0$ at $t\approx-1.3$ (BeV/c)^2. For the data above 10 (BeV/c)^2 only, the straight line goes through $\alpha_p(0)=1.0\pm0.05$ and through $\alpha_p=0$ at $t\approx-1.5$ (BeV/c)^2. Therefore, there may be a tendency for somewhat more shrinkage when 7- and 9-BeV/c points are included but the error limits overlap.

The Regge pole theory predictions would certainly be verified by these $p+p$ data. The vacuum pole analysis certainly fits and so would a simple three-pole analysis following the method of Hadjioannou et al.[4] The fact that the slope is smaller than the original value proposed[1] is not significant, since this was merely a reasonable value. In a previous $p+p$ (12-26 BeV/c, $t=0$-1) experiment and analysis, there is a log-log plot equivalent to our Fig. 4. (Reference 3, p. 113, Fig. 2).

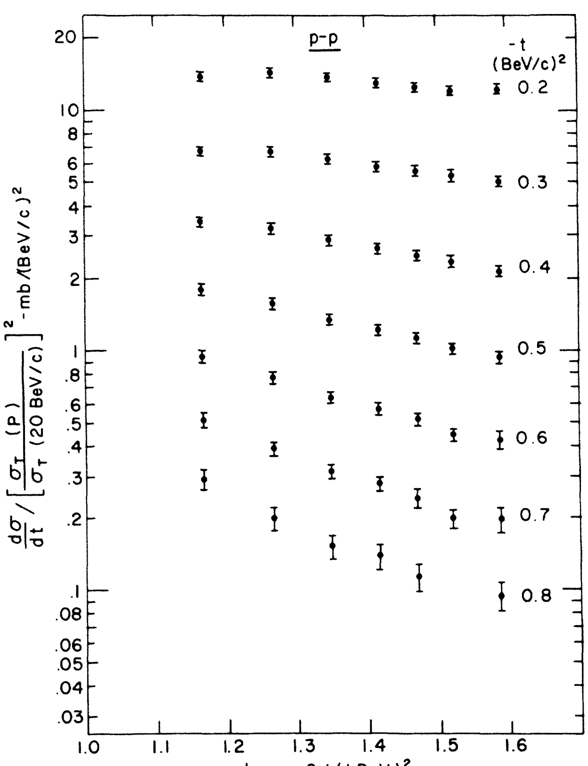

FIG. 4. Differential cross section for elastic $p+p$ scattering as a function of $\log_{10}s$. These data are obtained by interpolation from those of Fig. 2, the errors being taken from the nearest point.

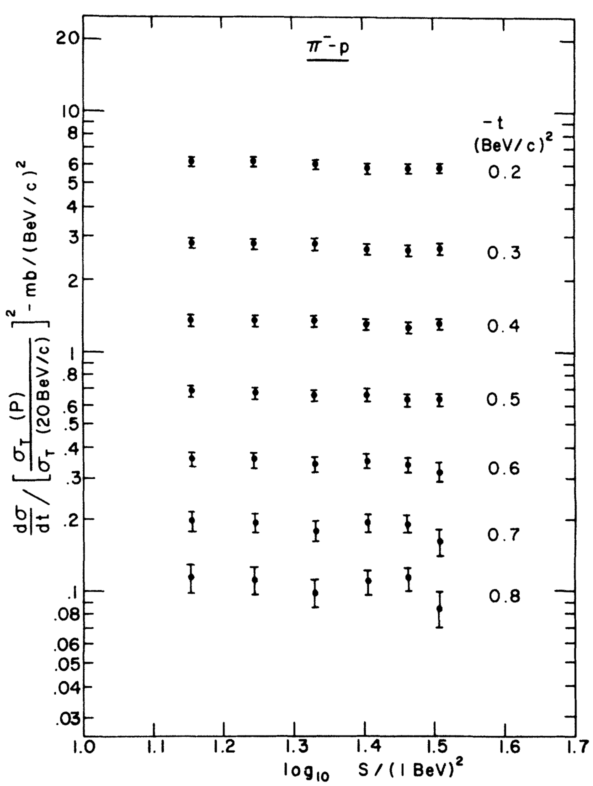

FIG. 5. Differential cross section for elastic π^-+p scattering as a function of $\log_{10}s$. These data were obtained by interpolation from those of Fig. 3, the errors being taken from the nearest point.

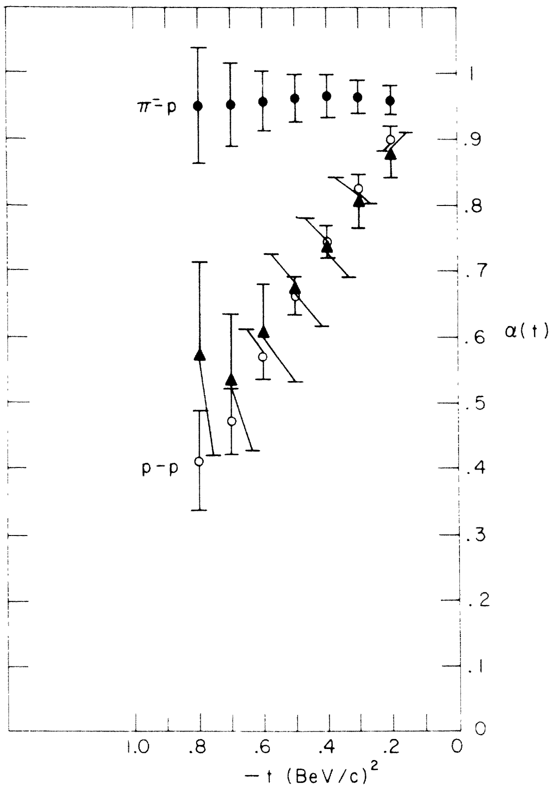

FIG. 6. $\alpha(t)$ vs t. The solid circles are the results for all the $\pi^- + p$ data. The open circles are the results for all the $p + p$ data. The solid triangles are the results using only the $p + p$ data above 10-BeV/c incident momentum.

Considering the experimental errors, parallel lines were consistent with the high-energy data but when weighted results of the experiment are compared to low-energy results from 4 BeV/c, it was concluded that $\alpha(t)$ went through zero at $t \approx 1.0$ BeV/c. See reference 3 for evidence of the behavior of $\alpha(t)$ at higher t.

As mentioned above, for the $\pi^- + p$ data all points at all momenta fit on a single curve with a reasonable χ^2. We found that no detectable change (except widening the error limits) is introduced by treating $\pi^- + p$ data for >10 BeV/c separately, and so all data combined are shown in the $\alpha_p(t)$ plot in Fig. 6. When a least-squares computer fit to these data was made, we obtained a line which had intercept $\alpha_p(0) = 0.96 \pm 0.03$ and slope $= 0.008 \pm 0.080$ (BeV/c)$^{-2}$.

The ratio

$$\frac{d\alpha_p}{dt}(\pi^- + p) \Big/ \frac{d\alpha_p}{dt}(p + p) = 0.01 \pm 0.15.$$

It is therefore, from the foregoing, definitely established that within errors there is no evidence for any shrinkage of the diffraction pattern of $\pi^- + p$ with increasing s (7-17 BeV/c) in the t range 0.2-1, and errors limit the possible shrinkage to a small fraction of $p + p$ cases.

The fact that the best-fit lines for both $\pi^- + p$ and $p + p$ do not pass through 1.0 when $t = 0$ is of no great consequence since this may simply imply some small s dependence at $t = 0$ such as could be due to extrapolation to $t = 0$, the fact that we ignored the three-pole s variation of F, or the fact that Regge pole theory is not a completely satisfactory parametrization for the problem. Obviously, the above results completely rule out sufficient dominance of the vacuum pole to neglect others.

As previously mentioned, considering only vacuum and vector meson trajectories, the three-pole model of Hadjioannou et al. was able to explain satisfactorily all total cross-section data; our $p + p$ differential data also could be fitted by it.

This three-pole model was originally a four-pole model: P (Pomeranchuk vacuum pole), P' [2nd vacuum trajectory $\alpha(0) \simeq 0.5$], ω, and ρ. The ρ was originally included, since it also has spin 1 and a priori might have been as important as the ω. The coupling of ρ, however, was assumed to be weak to explain small total cross-section differences between $\pi^- + p$ and $\pi^+ + p$ and also $p + p$ and $n + p$. The assumption of straight-line parallel Regge trajectories without spin effects (i.e., averaged over spins) was made. Hadjioannou et al.[4] showed that in the three-pole model one could write

$$\frac{d\sigma}{dt} = F(s, t)\left[\frac{s}{s_0}\right]^{2\alpha_p(t) - 2}$$

where for $p + p$ the s dependence of F is relatively weak above 10 BeV/c; and that, by neglecting it and making a one-pole analysis, one merely somewhat overestimated $1 - \alpha_p$ for the Pomeranchuk trajectory. In the case of $\pi^- + p$ we find that the three-pole model reduces to a two-pole model P and P', since the ω meson has the wrong G-parity of -1 and cannot contribute.[11] Since P and P' are both vacuum poles and the amplitude ratios at $t = 0$ are fixed by the momentum variation of the total cross section, presently developed simple ideas on Regge trajectories would predict considerable shrinkage in the $\pi^- + p$ case. Therefore, we find that our data contradict the three-pole model since we cannot simultaneously explain the $p+p$ and π^-+p results. It is possible that certain departures from simple Regge pole theory might in the future explain the data. It is difficult to see how agreement could be obtained without considerable complexity. However, if one allowed cuts, more poles, important spin effects, arbitrary distortions of Regge pole trajectories, it is probable some agreement could be obtained.

It appears safe, however, to conclude that Regge pole theory will not, as originally hoped, serve as a simple and definite prescription for this incident momentum range. Proposals would be further restricted by forthcoming results on $\pi^+ + p$, $K^\pm + p$ and $\bar{p}+p$ which we have also obtained but not finished processing yet. In any event one can, of course, always say that the energy is not high enough for asymptotic theorems. The striking difference in behavior of the πp and pp systems is, nevertheless, an interesting physical fact from any theoretical point of view.

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They also wish to thank the Instrumentation Division for generous cooperation in electronic problems and operation of the Merlin computer.

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[7]The electronic design was by W. Higinbotham and D. Potter and the unit was originally built for us in Brookhaven Instrumentation Division.

[8]A computer, similar to the Los Alamos Maniac II, built several years ago by the Brookhaven Instrumentation Division.

[9]S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 352 (1961).

[10]The errors shown are those referred to as "absolute errors" in Diddens et al., since the "relative errors within one momentum" also given are clearly inappropriate for drawing conclusions about shrinkage. The pattern of data and experimental errors do not seem to justify a definite conclusion on shrinkage for these data. The 18.6-GeV/c points are corrected as published later in an erratum, [Phys. Rev. Letters 10, 71(E) (1963)].

[11]The ρ has the right G-parity but its contribution must be small due to the small (π^-+p)-(π^++p) and pp-np total cross-section difference.

LEPTONIC DECAY OF THE Ξ^- HYPERON*

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(Received 21 March 1963)

We have analyzed 353 production events of the types

$$K^- + p \to \Xi^- + K^+, \tag{1a}$$

$$K^- + p \to \Xi^- + \pi^0 + K^+, \tag{1b}$$

$$K^- + p \to \Xi^- + \pi^+ + \bar{K}^0, \tag{1c}$$

where in each case the decay kink of the Ξ^- was required to be present. In Reactions (1a) and (1b), the decay Λ was seen; in (1c), the decay Λ and/or the $\bar{K}^0$ was seen. These events were produced in an exposure of a 1.8-BeV/c and 1.95-BeV/c K^- beam in the Alvarez 72-inch hydrogen bubble chamber.

We wish to report here on an unambiguous example of leptonic decay via the mode

$$\Xi^- \to \Lambda + e^- + \bar{\nu}. \tag{2}$$

The event (Fig. 1 and Table I) fits the production Reaction (1c), and both the $\bar{K}^0$ and the Λ from the subsequent decay are visible. The χ^2 is 6.4 for four degrees of freedom. The miss-