role of isotopic spin in calculation of the sign of A. Thanks are due the staffs of the 60-in. and 90-in. cyclotrons for their unfailing cooperation, and Professor E. Wichmann of the University of California for several helpful discussions.

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*Alfred P. Sloan Foundation Fellow.

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TECHNIQUE FOR ELIMINATING INTERFERENCE EFFECTS AND BIASES FROM OVERLAPPING RESONANCES

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We consider the case of a reaction with three particles (1, 2, 3) in the final state where two resonances A and B occur, one in the system of particles 1, 2 and the other in the system of particles 2, 3, and with invariant masses which cross on the Dalitz plot [Fig. 1(a)] for the reaction. The events on the Dalitz plot fall into any one of the four categories listed in Table I. For each event which occurs in the overlap region AB, no distinction can be made as to which process it represents (α, β, γ or δ). As a result, physicists have either (a) ignored the events in region AB or (b) considered the events in region AB to be all of one type α (or β) when studying the production and decay properties of resonance A (or B).

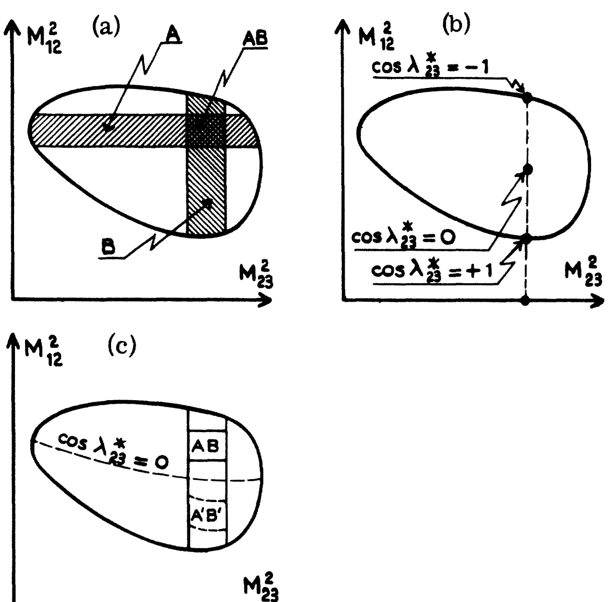

FIG. 1. Typical Dalitz plot (a) showing resonance bands A and B, (b) showing relationship between $M_{12}{}^2$ and $\cos\lambda_{23}{}^*$, (c) showing overlap region AB and its conjugate $A'B'$. Dashed line indicates line for $\cos\lambda_{23}{}^* = 0$.

We have considered the biases introduced by both (a) and (b) and found a technique to eliminate them – a technique which has not yet been described in the literature, to the best of our knowledge. We illustrate the method by applying it to a study of the properties of resonance B in process β.

We introduce and define the following quantities; $\hat{p}_2$, $\hat{p}_3$, $\hat{p}_B$, $\hat{p}_i$ are unit vectors in the directions

Table I. Possible configuration of particles 1, 2, 3 in final state.

Process	Configuration in final state	Position on Dalitz plot for event of this type
α	$A+3$, $A \to 1+2$	In band A
β	$B+1$, $B \to 2+3$	In band B
γ	Interference between processes α and β	In the overlap region AB of bands A and B
δ	$1+2+3$ Nonresonating	Anywhere within the ellipse

of particle 2, particle 3, resonant "particle" B, and the incident particle, respectively, in the over-all center-of-mass system (c.m.s.). An asterisk is used to denote these quantities in the center-of-mass of resonance B. Then the production plane normal, $\hat{n}=(\hat{p}_i\times\hat{p}_B)/|\hat{p}_i\times\hat{p}_B|$, and the decay angles λ_{23}^* (angle between particle 3 in the B c.m.s. and the direction of B in the over-all c.m.s.), and η_{23}^* (angle between $\hat{n}$ and particle 3 in the B c.m.s.) are defined by $\cos\lambda_{23}^*=\hat{p}_3^*\cdot\hat{p}_B$ and $\cos\eta_{23}^*=\hat{n}\cdot\hat{p}_3^*$, respectively.

Now the kinetic energy of particle 3 in the over-all c.m.s., T_3, is linearly related to M_{12}^2, the invariant mass squared of the system of particles 1 and 2. Moreover, T_3 is also a linear function of $\cos\lambda_{23}^*$ for a specific value M_{23}^2 and given incident beam energy, so that M_{12}^2 is a linear function of $\cos\lambda_{23}^*$; that is, $M_{12}^2=F+G\cos\lambda_{23}^*$, where the coefficients F and G are functions of incident beam energy and M_{23}^2. Therefore, each point along the line of constant M_{23}^2 in the Dalitz plot corresponds to a unique value of $\cos\lambda_{23}^*$ [Fig. 1(b)]. The points where this line intersects the kinematical limit (so-called Dalitz "ellipse") correspond to $\cos\lambda_{23}^*=\pm1$. In our example, $\cos\lambda_{23}^*=-1$ represents the point where the line of constant M_{23}^2 intersects the upper part of the "ellipse" (i.e., where M_{12}^2 is a maximum for the particular M_{23}^2). Also, $\cos\lambda_{23}^*=0$ represents the point on the line of constant M_{23}^2 midway between the maximum and minimum values of M_{12}^2 allowed by kinematics. Thus, it is seen that the overlap region AB corresponds to a particular range of values of $\cos\lambda_{23}^*$; therefore, (a) if one ignores the region AB in studying process β, then this implies that a specific region of $\cos\lambda_{23}^*$ has been depopulated, and hence the resulting angular distribution is biased, and (b) if one includes the region AB and considers all events there as process β events, then a specific region of $\cos\lambda_{23}^*$ has been overpopulated with events from processes α and γ; hence the resulting distribution is biased. These biases affect the Adair angular distribution, since for events of process β in which resonance B is produced in the forward direction in the over-all c.m.s. (i.e., with small momentum transfer), the Adair angle is the same as angle λ_{23}^*. Furthermore, the $\cos\eta_{23}^*$ distribution (normal to the plane of production) will also be distorted as a result of these biases. All other distributions may be similarly distorted.

In order to eliminate such biases, we suggest the following technique. We consider that the B resonance from process β decays in a given state of parity. This means that in the decay of B in its own rest frame, the intensity for particle 2 in one direction and particle 3 in the opposite direction is the same as the intensity for the directions of particles 2 and 3 interchanged. Therefore, under the condition that resonance B is of the above type, one gets equivalent statistical samples for process β from the true events and from the conjugate events obtained by interchanging particles 2 and 3 in their own c.m.s. This operation of particle interchange does not affect M_{23}^2, but it does change $\cos\eta_{23}^*$ into $-\cos\eta_{23}^*$ and $\cos\lambda_{23}^*$ into $-\cos\lambda_{23}^*$. Also, because of the linear relation between M_{12}^2 and $\cos\lambda_{23}^*$, this operation changes M_{12}^2.

An important consequence of the foregoing is that for each experimental event of process β on the Dalitz plot with a given M_{23}^2 in region B and a given M_{12}^2 (corresponding to a given $\cos\lambda_{23}^*$), there should also occur an event with the same M_{23}^2 but with a different M_{12}^2, corresponding to $-\cos\lambda_{23}^*$. We refer to such events as the conjugate of one another. If the overlap region AB does not include $\cos\lambda_{23}^*=0$ [i.e., if AB lies completely within the upper or lower half of the "ellipse" in Fig. 1(c)], then all the events in AB which represent the production of resonance B in process β have their conjugate events lying outside AB in an area $A'B'$ [Fig. 1(c)], and vice versa.

While resonance A may produce in the region AB a background which makes analysis impossible for process β, its influence in the region $A'B'$ is generally very small since the region $A'B'$ is located outside the band A. Therefore if one ignores the experimental events in region AB and instead repopulates this region with conjugate events of the true events lying in region $A'B'$, then these fictitious conjugate events represent a statistical sample of process β events in region AB, as if the coupling constant of producing resonance A were turned off. Thus, if one then makes any distribution from the events in band B, including the fictitious events instead of the true events in region AB, one gets the true unbiased production and decay distributions of resonance B from process β, provided the background from process δ is small. If, however, the background contamination from process δ is not negligible, but is uniformly distributed in phase space, then it is the same in regions AB and $A'B'$; that is, the contamination from process δ is unaffected by the above technique. It can be subtracted without too much difficulty.[1]

The following procedure is recommended for analysis of process β: (1) Define a region AB on the Dalitz plot where the resonance A may distort the analysis. (2) Check that AB does not include any point corresponding to $\cos\lambda_{23}^* = 0$ [see Fig. 1(c)]. (3) Define a C region large enough to contain the $A'B'$ region. (4) For each event included in C, go to the c.m.s. of particles 2 and 3, interchange their direction, and make thereby a fictitious conjugate event. (5) Ignore the fictitious conjugate events which do not fall into AB. (6) Remove the true events from region AB and repopulate this region with the fictitious conjugate events.

One should remember when making the χ^2 fits that the sample of fictitious events is highly correlated statistically to the sample of true events from which the former were constructed.

The same method can be applied to study the process α provided that AB does not include the points where $\cos\lambda_{12}^* = 0$ in the A c.m.s.

Finally, if one makes an analysis of what is in AB region alone and subtracts from the histograms the contributions of β and α, represented by the two kinds of fictitious events in the B c.m.s. and in the A c.m.s., respectively, one can then study the interference (i.e., process γ) by itself. One should remember, however, that in such an operation, one has subtracted the background (process δ) twice.

An alternate technique to the one proposed above would be to use a rotation, in the c.m.s. of the resonance being studied, of 180° around the normal $\hat{n}$ to the plane of production (instead of the spatial parity operation used above). This is equivalent to the product of a spatial parity operation and a reflection with respect to the production plane (a good operation if over-all parity is conserved). With that alternate technique, one should remember that the conjugation inverts the components of the decay particle spins in the plane of production.

[1]Two tests to check the uniformity of the background from process δ are (1) symmetry in $\cos\lambda_{23}^*$ for those events in band B but not in AB or $A'B'$, (2) symmetry with respect to the quantity $(\hat{n}\times\hat{p}_B)\cdot\hat{p}_3^*$ (i.e., left-right symmetry about the direction of B) using the same sample of events as in test (1).

ISOBAR MODEL ANALYSIS OF SINGLE PION PRODUCTION IN PION-NUCLEON COLLISIONS BELOW 1 BeV†

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In this paper we show that an improved (3, 3) isobar model for single pion production in pion-nucleon collisions can account for the majority of the observed mass spectra and the ratio of π^0 to π^+ production in π^+-p collisions from 350 MeV to 1 BeV. This is in contrast to earlier analyses using the isobar model.[1-4] The essential new feature of the present analysis is the inclusion of the P-wave decay of the (3, 3) isobar.

Although the rough features of the shape of the mass spectra predicted by the isobar model of Lindenbaum and Sternheimer[1-3] (called LS model hereafter) have been observed in many experiments,[5-8] the LS model fails to describe the π - π mass spectra. The main approximations of the LS model are that (a) the interference between the two isobar diagrams (see Fig. 1) can be neglected, (b) the isobar is produced in a S state and (c) the isobar decays isotropically. Bergia, Bonsignori, and Stanghellini[4] (called BBS model hereafter) modified the LS model to include the interference between the two isobar diagrams, still, however, making approximations (b) and (c). They show that interference effects modify the results of the LS model greatly.

We have extended the work of Bergia, Bonsign-

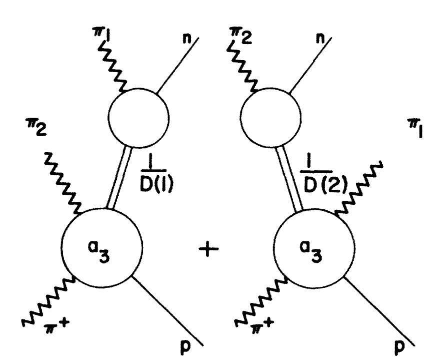

FIG. 1. Two isobar diagrams for the reaction $\pi+p \to \pi_1+\pi_2+N$.