Green's function approach we used above is sufficiently powerful and simple to allow us to treat this problem in detail without making the quasiparticle approximation.

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# INTERNAL MAGNETIC FIELD IN THE de HAAS – van ALPHEN EFFECT IN IRON*

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The de Haas–van Alphen effect has recently been observed in iron by Anderson and Gold,[1] who find the striking result that the field quantity entering the expression for the Lorentz force on a conduction electron is $\vec{B}=\vec{H}+4\pi\vec{M}_S$. In particular they report that the effective field is $\vec{H}+\vec{H}_0$, where $\vec{H}_0=21.4\pm 1.9$ kG in one orientation and $21.9\pm 1.7$ kG in another; the value of $4\pi M_S$ is 21.8 kG. By definition $\vec{B}$ is the average magnetic field over the volume of a specimen, weighting all volume elements equally, on the assumption that the magnetic carriers are not shifted in position by the test charge.[2] With this assumption $\vec{B}$ is the field seen by a cosmic-ray particle. But it is not at all clear why a conduction electron should sample all volume elements equally, and for this reason the experimental result is unexpected, and, in fact, other values had been predicted. We consider the theory below and find that the theoretical internal field $H_0$ is, in fact, just $4\pi M_S$ for the model used.

We consider a model with one conduction electron in a ferromagnetic crystal; at each lattice point of a cubic lattice there is a rigidly bound magnetic moment

$$\vec{\mu}_i=(ge\hbar/2mc)\vec{S}_i. \quad (1)$$

We neglect the spin of the conduction electron, but look only for interactions of the bound spins $\vec{S}_i$ with the momentum $\vec{p}$ of the conduction electron. In lowest order the perturbation from the (orbit)-(other spin) interaction[3] is

$$H'=\frac{e}{mc}\sum_i \vec{\mu}_i\cdot\frac{[\vec{p}\times\vec{r}_i]}{r_i^3}, \quad (2)$$

where $\vec{r}_i$ is the vector to the conduction electron from the $i$th lattice point. If we neglect the off-diagonal component of the magnetic moment and set $\mu=\mu_i^z$ for all $i$,

$$H'=\frac{e\mu}{mc}\sum_i\frac{p_x y_i-x_i p_y}{r_i^3}. \quad (3)$$

We consider a specimen in the form of a thin slab parallel to the $yz$ plane; the specimen is magnetized along the $z$ axis. By symmetry $\sum(y_i/r_i^3)=0$. The sum $\sum(x_i/r_i^3)$ can be evaluated by its electrostatic analog: It is the $x$ component of the electric field vector inside a lattice slab bearing a unit positive charge on each lattice point. We have, for $n$ lattice points per unit volume

$$\sum\frac{x_i}{r_i^3}=4\pi nx+\sum_{\vec{G}\neq 0}C_{\vec{G}}^x e^{i\vec{G}\cdot\vec{x}}, \quad (4)$$

where the contribution $4\pi nx$ represents the effect of a uniform distribution of charge; the oscillatory term is periodic in the reciprocal lattice. The coefficients $C_{\vec{G}}$ may be calculated, but we do not need their values.

With $n\mu \equiv M_S$, we have

$$H' = -4\pi M_s\left(\frac{e}{mc}\right)xp_y - \left(\frac{e\mu}{mc}\right)p_y\sum_{\vec{G}\neq 0} C_{\vec{G}}^x e^{i\vec{G}\cdot\vec{x}} + \cdots. \quad (5)$$

The oscillatory part of $H'$ is invariant under a lattice translation and may therefore be added to the unperturbed Bloch Hamiltonian to give eigenfunctions of the Bloch form. The usual spin-orbit interaction (not considered in $H'$) is oscillatory and is disposed of in the same way. The remainder $-4\pi M_S(e/mc)xp_y$ of $H'$ is equal to the $\vec{A}\cdot\vec{p}$ term in the kinetic energy

$$\frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}\right)^2 = \frac{1}{2m}p^2 - \frac{e}{mc}\vec{A}\cdot\vec{p} + \frac{e^2}{2mc^2}A^2, \quad (6)$$

provided

$$\vec{A} = (0, 4\pi M_s x, 0). \quad (7)$$

This is just the vector potential of a magnetic field $H_0^z = 4\pi M_S$.

The oscillatory contribution to $\vec{A}$ is of the order $\mu_B/a^2 \sim 10^{-4}$ cgs; the order of magnitude of the smooth contribution is $4\pi M_S R_C \sim 10^{-1}$ cgs, where the cyclotron radius $R_C$ has been taken as $10^{-5}$ cm in a 50-kG applied field. Thus it appears that we may neglect in $A^2$ the terms arising from the square of the oscillatory component of $\vec{A}$ and also terms in the product of the smooth and oscillatory components. In any event the oscillatory parts of $A^2$ are to be treated as part of the Bloch Hamiltonian. Our estimate $A_{\text{osc}} \sim 10^{-4}$ cgs may underestimate an $s$-state component; we might rather take $A_{\text{osc}}$ as of the order of a Compton wavelength times a hyperfine field, or $10^{-10}\times 10^{7} \approx 10^{-3}$ cgs, which is still small compared to $4\pi M_S R_C$.

Our result is that the contribution of the magnetization to the effective internal magnetic field acting on the momentum of a conduction electron is $4\pi M_S$, very closely, provided that the interaction process is elastic.[4] The value is essentially independent of the details of the conduction electron eigenstates.

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[4]Electron-electron inelastic collisions typically are rather infrequent because of effective screening of the Coulomb interaction. It is possible, however, that $d$-$d$ or $s$-$d$ collisions (enhanced by a high density of states of $d$ electrons at the Fermi level) may be sufficiently frequent in some ferromagnetic metals to make it impossible to observe a de Haas – van Alphen effect except at very low temperatures.

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# LYMAN ALPHA PRODUCTION IN PROTON-RARE GAS COLLISIONS*

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Nonresonant charge-exchange cross sections for proton-rare gas collisions have been measured by several investigators.[1] Almost without exception the cross sections versus energy exhibit a single, rather broad maximum. We have measured the emission of Lyman alpha radiation from collisions of 1- to 25-keV protons with rare gas atoms. These measurements show two prominent maxima in the Lyman alpha-producing reactions for $H^+ + Ne$, $H^+ + Ar$, and $H^+ + Kr$. The cross section for $H^+ + Xe$ has a peak at 10 keV and gives an indication of another at very low proton energy. The $H^+ + He$ cross section does not show two distinct peaks in the energy range of our apparatus, but there does seem to be a shoulder on the high-energy side of the maximum. Figs. 1 and 2 are a presentation of the measured cross section versus incident proton energy in the lab system for the five rare gases. The argon curve is the average of 5 runs with two different gas samples. The helium and krypton curves are the average of 4 runs each; the xenon, 3 runs; and the neon, 2 runs. All curves are reproducible to within ±5%.

The basic apparatus and detection scheme are described elsewhere.[2] Modifications include the