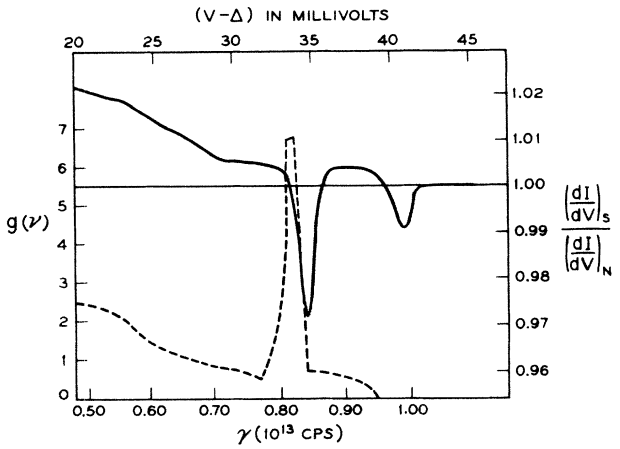

FIG. 3. $(dI/dV)_S/(dI/dV)_N$ vs V (measured from Δ) for an Al-Pb junction at 0.40°K compared with the phonon spectrum for aluminum.

Also shown in Walker's[7] Al phonon spectrum (recalculated by Phillips[7]). It will be seen that the prominent longitudinal peak at 34 mV, as well as the end point of the spectrum at 39 mV, are reflected in the tunnel characteristic. Structure at lower biases was masked by the Pb but an Al-Al sandwich will be investigated.

Figure 2 (curve C) also shows d^2I/dV^2 vs V for a Sn-Sn sandwich and a large amount of structure is observed, some near the Debye energy of 17 mV. The rather surprising low-energy structures were at first thought to be due to Pb impurity in the Sn films but were exactly reproduced after thorough cleaning of the evaporation system and use of a high-purity tin source. The fundamental structure in tin is only as strong as the sum and harmonic structure in lead (curve B) and therefore about $\frac{1}{10}$ the magnitude of the lead fundamental peaks.

We wish to acknowledge valuable discussions with J. R. Schrieffer, D. J. Scalapino, and J. W. Wilkins, and the assistance of L. Kopf in the preparation of the tunnel junctions and of J. Klein with the differentiation techniques.

[1]G. M. Eliashberg, Zh. Eksperim. i Teor. Fiz. 38, 966 (1960) [translation: Soviet Phys. -JETP 11, 696 (1960)].

[2]P. Morel and P. W. Anderson, Phys. Rev. 125, 1263 (1962).

[3]The exact expressions will be given in the following Letter [J. R. Schrieffer, D. J. Scalapino, and J. W. Wilkins, Phys. Rev. Letters 10, 336 (1963)].

[4]I. Giaever, H. R. Hart, Jr., and K. Megerle, Phys. Rev. 126, 941 (1962).

[5]J. M. Rowell, A. G. Chynoweth, and J. C. Phillips, Phys. Rev. Letters 9, 59 (1962).

[6]B. N. Brockhouse, T. Arase, G. Caglioti, K. R. Rao, and A. D. B. Woods, Phys. Rev. 128, 1099 (1962).

[7]J. C. Phillips, Phys. Rev. 113, 147 (1959); C. B. Walker, Phys. Rev. 103, 547 (1956).

[8]P. W. Anderson, J. R. Schrieffer, and D. J. Scalapino (to be published). These discontinuities do not appear in the normal-metal-superconductor theory of reference 3, but only in the two-superconductor characteristic; this is in agreement with our observations.

[9]D. M. Ginsberg and M. Tinkham, Phys. Rev. 118, 990 (1960); D. M. Ginsberg, P. L. Richards, and M. Tinkham, Phys. Rev. Letters 3, 337 (1959); P. L. Richards, Phys. Rev. Letters 7, 412 (1961).

EFFECTIVE TUNNELING DENSITY OF STATES IN SUPERCONDUCTORS*

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(Received 15 March 1963)

Recent tunneling experiments[1,2] involving superconducting metals exhibit structure in the I - V characteristic which has been interpreted in terms of electron-phonon processes. In the preceding Letter[3] Rowell, Anderson, and Thomas present the results of improved experiments which more clearly resolve this structure. Below we summarize the results of a theoretical determination of the tunneling characteristic which is in good agreement with these experiments.

To include dynamic interactions between phonons and electrons in a consistent manner, it is necessary to extend the conventional expression for the tunneling current.[4,5] We take the point of view of Bardeen and of Cohen, Falicov, and Phillips who characterize the tunneling process by an effective one-body Hamiltonian

$$H_T = \sum_{kk's} \{T_{kk'} c_{ks}^{\ b\dagger} c_{k's}^{\ a} + \text{H.c.}\}. \qquad (1)$$

Here $c_{ks}^{\ a}$ and $c_{ks}^{\ b\dagger}$ destroy and create electrons in Bloch states of momentum k, energy ϵ_k measured relative to the chemical potential μ, and

spin orientation s in metals a and b, respectively. At zero temperature the transition probability per unit time for an electron to tunnel from a to b is given to lowest order in T by ($\hbar = 1$)

$$w_{a\to b} = 2\pi \sum_{n_a n_b} \Big| \sum_{kk's} T_{kk'} \langle n_b | c_{ks}^{b\dagger} | 0_b \rangle \times \langle n_a | c_{k's}^{a} | 0_a \rangle \Big|^2 \delta(W_n^b + W_n^a - V). \quad (2)$$

Here $|n_a\rangle$ and $|n_b\rangle$ represent exact energy eigenstates of metals a and b in the absence of both H_T and the applied potential V (measured in electron volts). The excitation energy W_n^a is given by the difference in energy of the states $|n_a\rangle$ and $|0_a\rangle$; W_n^b is defined in a similar manner. For $V \ll \mu$ the dominant contribution in (2) comes from states k and k' near the Fermi surface. In this case we can approximate $T_{kk'}$ by its average value in this region and reduce (2) to the form

$$w_{a\to b} = \text{const} \int_0^\infty d\omega\, N_{T+}^{b}(\omega) N_{T-}^{a}(V-\omega). \quad (3)$$

Here $N_{T\pm}^{i}(\omega)$ is the effective tunneling density of states for metal i defined by

$$N_{T\pm}^{i}(\omega) = N^i(0) \int_{-\infty}^{\infty} d\epsilon_k\, \rho_i^{\pm}(k,\omega), \quad (4)$$

where

$$\rho_i^{+}(k,\omega) = \sum_{n_i} \Big| \langle n_i | c_{ks}^{i\dagger} | 0_i \rangle \Big|^2 \delta(W_n^i - \mu^i - \omega), \quad (5a)$$

$$\rho_i^{-}(k,\omega) = \sum_{n_i} \Big| \langle n_i | c_{ks}^{i} | 0_i \rangle \Big|^2 \delta(W_n^i + \mu^i - \omega), \quad (5b)$$

and $N(0)$ is the density of Bloch states in energy at the Fermi surface. By definition $\rho^{\pm}(k,\omega)$ are the spectral weight functions for the one-electron Green's function $G(k,\omega)$, and hence it follows from (4) that

$$N_{T\pm}(\omega) = \mp \frac{N(0)}{\pi} \int_{-\infty}^{\infty} d\epsilon_k\, \mathrm{Im} G(k, \pm\omega). \quad (6)$$

Thus, a knowledge of $G(k,\omega)$ suffices to determine the tunneling current under the above conditions. (In the above derivation a spherical Fermi surface with an effective mass has been assumed.)

To determine G it is convenient to use the formalism of Nambu[6] and write

$$G(k,\omega) = \frac{\omega + \tilde{\epsilon}(k,\omega)}{\omega^2 - \tilde{\epsilon}^2(k,\omega) - \Delta^2(k,\omega) + i0^+} \quad (7)$$

where $\tilde{\epsilon}(k,\omega) = \epsilon_k / Z(k,\omega)$ is the renormalized Bloch energy and $\Delta(k,\omega) = \Phi(k,\omega)/Z(k,\omega)$ is the complex energy gap parameter. By using the fact that Z and Δ are essentially independent of k, (6) and (7) may be combined to give

$$\frac{N_T(\omega)}{N(0)} = \mathrm{Re}\left\{ \frac{|\omega|}{[\omega^2 - \Delta^2(\omega)]^{1/2}} \right\}. \quad (8)$$

For Δ independent of ω, this expression agrees with the conventional BCS density of states $d\epsilon_k/dE = E/(E^2 - \Delta^2)^{1/2}$ for $|E| > \Delta$ and zero otherwise. Note, however, if Δ varies with ω, the effective density of states appropriate to the tunneling process $N_T(\omega)/N(0)$ differs from the standard quasiparticle form $[dE(\epsilon_k)/d\epsilon_k]^{-1}$, where $E(\epsilon_k)$ is the real part of the pole of G on the second sheet. This difference, essential in understanding the structure in the I-V characteristic, is due to the fact that $c_{ks}^{a}|0_a\rangle$ and $c_{ks}^{b\dagger}|0_b\rangle$ are not quasiparticle eigenstates of metals a and b as they are within the simple BCS approximation. Thus, one cannot use the standard quasi-particle density-of-states expression which is appropriate, for example, in calculating the electronic specific heat. Equation (8) is easily generalized to finite temperature.

We calculate $\Delta(\omega)$ and $Z(\omega)$ by including both electron-phonon and Coulomb interactions. The phonons are characterized by a frequency distribution $F(\omega_q)$ and are coupled to the electrons by an interaction strength $\alpha(\omega_q)$. The screened Coulomb interaction is replaced by a pseudopotential U defined to include interactions between electrons outside a band of energies $|\omega| < \omega_C$, which is large compared with the Debye energy. The integral equation determining the complex gap parameter is then

$$\Delta(\omega) = \frac{1}{Z(\omega)} \int_{\Delta_0}^{\omega_C} d\omega' \,\mathrm{Re}\left[\frac{\Delta'}{(\omega'^2 - \Delta'^2)^{1/2}} \right] \left\{ \sum_\lambda \int d\omega_q\, \alpha_\lambda^2(\omega_q) \times F_\lambda(\omega_q) [D_q(\omega' + \omega) + D_q(\omega' - \omega)] - U \right\}, \quad (9)$$

where $D_q(x) = (x + \omega_q - i0^+)^{-1}$, $\Delta_0 \equiv \Delta(\Delta_0)$ is the gap parameter at the edge of the energy gap, and λ labels the phonon polarization. Notice that Z does not enter the integral in (9); it is reduced to quadrature, once $\Delta(\omega)$ is known, by

$$[1 - Z(\omega)]\omega = \int_{\Delta_0}^{\omega_C} d\omega' \,\mathrm{Re}\left[\frac{\omega'}{(\omega'^2 - \Delta'^2)^{1/2}} \right] \sum_\lambda \int d\omega_q\, \alpha_\lambda^2(\omega_q) \times F_\lambda(\omega_q) [D_q(\omega' + \omega) - D_q(\omega' - \omega)]. \quad (10)$$

Equations (9) and (10) have been solved by an

on-line computer facility[7] for a simplified model devised to represent the phonon spectrum of Pb. The distribution of longitudinal and transverse phonons was approximated by the sum of two Lorentzians, centered at frequencies $\omega_1^l = 8.5 \times 10^{-3}$ eV and $\omega_1^t = 4.4 \times 10^{-3}$ eV and having half-widths $\omega_2^l = 0.5 \times 10^{-3}$ eV and $\omega_2^t = 0.75 \times 10^{-3}$ eV. The coupling parameter $\alpha_\lambda(\omega_q)$ was taken to be a constant independent of both q and the polarization λ; this is a reasonable approximation since the dominant part of the phonon interaction involves umklapp processes. The Coulomb pseudopotential was adjusted to $U = 0.11$, the value we believe appropriate to lead. The cutoff ω_c was taken equal to $7\omega_1^t + \Delta_0$. While the spherical Fermi surface approximation is a fairly good one for Pb,[8] we need not rely on this fact in comparing with the experimental results[3] since these were performed with dirty superconductors.

In Fig. 1, Δ_1 and Δ_2, the real and imaginary parts of Δ, are plotted as a function of energy. The effective density of states (8) is plotted in Fig. 2 along with the result of the simple BCS model. The gross structure of $N_T(\omega)$ can be understood by expanding $N_T(\omega)$ to first order in Δ^2; thus

$$N_T(\omega)/N(0) = 1 + [\Delta_1^2(\omega) - \Delta_2^2(\omega)]/(2\omega^2).$$

At energies $\omega_1^t + \Delta_0$ and $\omega_1^l + \Delta_0$, the strong increase of the phonon emission rate produces a rapid increase of Δ_2, accompanied by a decrease of Δ_1. Both changes produce a sharp drop of $N_T(\omega)$ near these phonon emission thresholds.

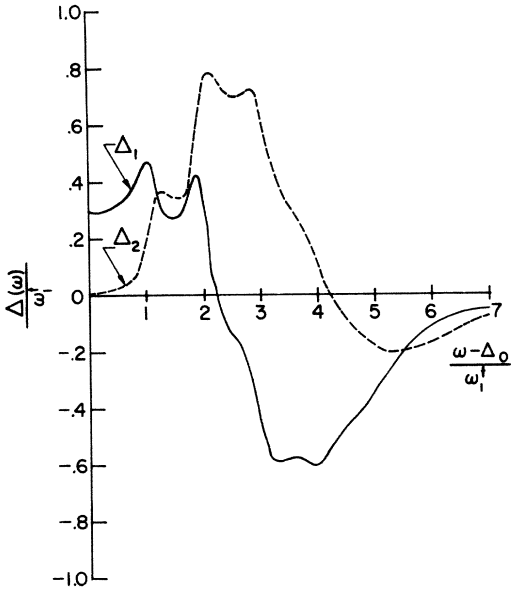

FIG. 1. Plot of the real (solid) and imaginary (dashed) parts of $\Delta(\omega)/\omega_1^t$ vs $(\omega - \Delta_0)/\omega_1^t$. Here $\omega_1^t = 4.4 \times 10^{-3}$ eV and $\Delta_0 = 1.34 \times 10^{-3}$ eV.

The experimental verification of the structure in $N_T(\omega)$ can be obtained from the I-V characteristics for tunneling between a normal metal (say Al) and lead in the superconducting and in the normal states, as described in the preceding Letter. It follows from (2) that the ratio of the differential conductances in the two states is

$$\frac{dI_s(V)}{dV} \Big/ \frac{dI_n(V)}{dV} = \frac{N_T(V)}{N(0)} = \mathrm{Re}\left\{ \frac{V}{[V^2 - \Delta^2(V)]^{1/2}} \right\}. \quad (11)$$

The experimental data of Rowell, Anderson, and Thomas, also shown in Fig. 2, are in remarkably good agreement with the theoretical curve, considering the simplicity of the model we used.

We would like to emphasize that the electron-phonon coupling is so strong for lead that the quasi-particle picture is meaningless over much of the energy spectrum. Nevertheless, the

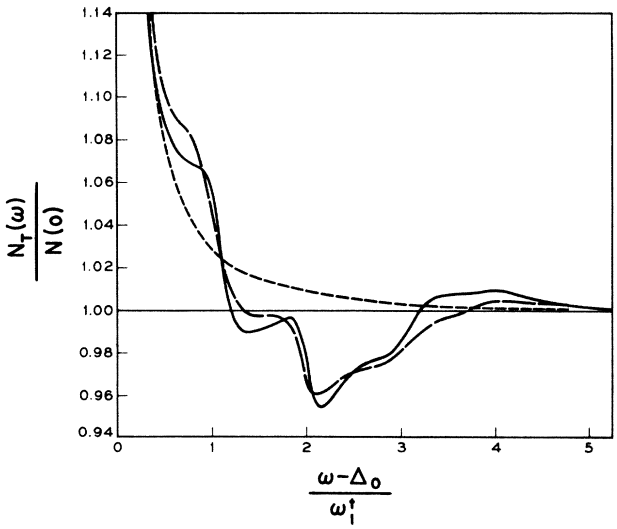

FIG. 2. The effective tunneling density of states $N_T(\omega)/N(0)$ vs $(\omega - \Delta_0)/\omega_1^t$ (solid) and the density of states of the simplified BCS model $\omega/(\omega^2 - \Delta_0^2)^{1/2}$ (short dash). The ratio of the differential conductance of Pb in the superconducting to that in the normal state,

$$\frac{dI_s(\omega)/d\omega}{dI_n(\omega)/d\omega},$$

is plotted (long dash) as a function of $(\omega - \Delta_0)/\omega_1^t$ for $T = 1.3$°K. These data were obtained from the tunneling experiments reported by Rowell, Anderson, and Thomas.

Green's function approach we used above is sufficiently powerful and simple to allow us to treat this problem in detail without making the quasiparticle approximation.

We are indebted to Thompson-Ramo-Wooldridge, Inc. for making the on-line computing facility available to us free of charge and to George Boyd for aid with the computations. We are also grateful to J. M. Rowell, P. W. Anderson, and D. E. Thomas for several stimulating discussions regarding the model used to represent the phonon spectrum of Pb and for prepublication use of their tunneling curves.

*A contribution of the Laboratory for Research on the Structure of Matter, University of Pennsylvania, covering research sponsored by the Advanced Research Projects Agency.

[1]I. Giaever, H. R. Hart, Jr., and K. Megerle, Phys. Rev. 126, 941 (1962).

[2]J. M. Rowell, A. G. Chynoweth, and J. C. Phillips, Phys. Rev. Letters 9, 59 (1962).

[3]J. M. Rowell, P. W. Anderson and D. E. Thomas, preceding Letter [Phys. Rev. Letters 10, 334 (1963)].

[4]J. Bardeen, Phys. Rev. Letters 6, 57 (1961).

[5]M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Letters 8, 316 (1962).

[6]Y. Nambu, Phys. Rev. 117, 648 (1960).

[7]C. J. Culler and B. C. Fried, Proceedings of Pacific Computer Conference, 1963 (unpublished).

[8]A. V. Gold, Phil. Trans. Roy. Soc. London A251, 85 (1958).

INTERNAL MAGNETIC FIELD IN THE de HAAS – van ALPHEN EFFECT IN IRON*

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(Received 18 March 1963)

The de Haas–van Alphen effect has recently been observed in iron by Anderson and Gold,[1] who find the striking result that the field quantity entering the expression for the Lorentz force on a conduction electron is $\vec{B}=\vec{H}+4\pi\vec{M}_S$. In particular they report that the effective field is $\vec{H}+\vec{H}_0$, where $\vec{H}_0=21.4\pm1.9$ kG in one orientation and 21.9 ± 1.7 kG in another; the value of $4\pi M_S$ is 21.8 kG. By definition $\vec{B}$ is the average magnetic field over the volume of a specimen, weighting all volume elements equally, on the assumption that the magnetic carriers are not shifted in position by the test charge.[2] With this assumption $\vec{B}$ is the field seen by a cosmic-ray particle. But it is not at all clear why a conduction electron should sample all volume elements equally, and for this reason the experimental result is unexpected, and, in fact, other values had been predicted. We consider the theory below and find that the theoretical internal field H_0 is, in fact, just $4\pi M_S$ for the model used.

We consider a model with one conduction electron in a ferromagnetic crystal; at each lattice point of a cubic lattice there is a rigidly bound magnetic moment

$$\vec{\mu}_i=(ge\hbar/2mc)\vec{S}_i. \tag{1}$$

We neglect the spin of the conduction electron, but look only for interactions of the bound spins $\vec{S}_i$ with the momentum $\vec{p}$ of the conduction electron. In lowest order the perturbation from the (orbit)-(other spin) interaction[3] is

$$H'=\frac{e}{mc}\sum_i\vec{\mu}_i\cdot\frac{[\vec{p}\times\vec{r}_i]}{r_i^3}, \tag{2}$$

where $\vec{r}_i$ is the vector to the conduction electron from the ith lattice point. If we neglect the off-diagonal component of the magnetic moment and set $\mu=\mu_i{}^z$ for all i,

$$H'=\frac{e\mu}{mc}\sum_i\frac{p_x y_i-x_i p_y}{r_i^3}. \tag{3}$$

We consider a specimen in the form of a thin slab parallel to the yz plane; the specimen is magnetized along the z axis. By symmetry $\sum(y_i/r_i^3)=0$. The sum $\sum(x_i/r_i^3)$ can be evaluated by its electrostatic analog: It is the x component of the electric field vector inside a lattice slab bearing a unit positive charge on each lattice point. We have, for n lattice points per unit volume

$$\sum\frac{x_i}{r_i^3}=4\pi nx+\sum_{\vec{G}\neq0}C_{\vec{G}}{}^x e^{i\vec{G}\cdot\vec{x}}, \tag{4}$$

where the contribution $4\pi nx$ represents the effect of a uniform distribution of charge; the oscillatory term is periodic in the reciprocal lattice. The coefficients $C_{\vec{G}}$ may be calculated, but we do not