DETECTION OF AN INFINITE ANAOMALOUS THRESHOLD IN THE REACTION $K^- + p \to \Sigma + \pi + \pi$

R. Aaron
Indiana University, Bloomington, Indiana
(Received 6 December 1962)

Recent progress in the theoretical treatment of strong interactions has leaned heavily on the analytic properties of diagrams in perturbation theory. In the case of production processes, these analytic properties are quite complex, and because of the presence of anomalous thresholds and other complications, any serious calculation requires the aid of a large computer. It would indeed increase our confidence in the validity of this diagrammatic approach, and perhaps encourage more realistic calculations, if some dramatic physical effect could be shown to be unequivocally due to the near presence of an anomalous singularity. This possibility is offered in a recent Article by Landshoff and Treiman,[1] and we shall now apply the methods discussed by the aforementioned authors to a well-known elementary-particle reaction, namely, $K^- + p \to \Sigma + \pi + \pi$.

It was shown by LT that, for example, the reduced diagram of Fig. 1(a) gives rise to an infinite anomalous singularity (of the inverse square root type) in the amplitude for multiple pion production in pion-nucleon collisions. As a function of the incoming center-of-mass energy, W, the anomalous threshold can move to distances of order μ^2 (μ = pion mass) from the physical region in the complex $s = \omega^2$ plane (where ω is the energy of the two pions in their own center-of-mass system). However, despite the smallness of the effective energy denominator, the extreme weakness of the amplitude for the reaction $\pi + n \to \bar{n} + d$ causes the effect of this diagram to be swamped by the pole terms, etc. All other examples given by LT suffer from similar difficulties, the reason being that the effect is extremely sensitive to the values of the internal and external masses in the reduced triangle diagram, and if we restrict ourselves to the realm of known "stable" particles, it is difficult to find any reasonable reactions that will strongly exhibit the effect of the infinite singularity discussed above. Let us, however, be permitted to include the ω^0 (3π system) in our family of stable particles; we shall then give arguments that the reduced diagram of Fig. 1(b) should give rise to a pronounced peaking in the differential cross section $(d\sigma/d\omega)_{,T=0}$ $(K^- + p \to \Sigma + \pi + \pi)$, in the vicinity of maximum ω, for center-of-mass energy, W, approximately equal to $W_n \equiv (M_{\omega^0} + M_\Lambda)$.[2] The path of the anomalous threshold relative to the physical region in the W-s plane is shown in Fig. 2. For W greater than W_n, the anomalous singularity falls steeply into the lower half of the complex s plane, and within 30 to 50 MeV the peaking of the amplitude essentially disappears.

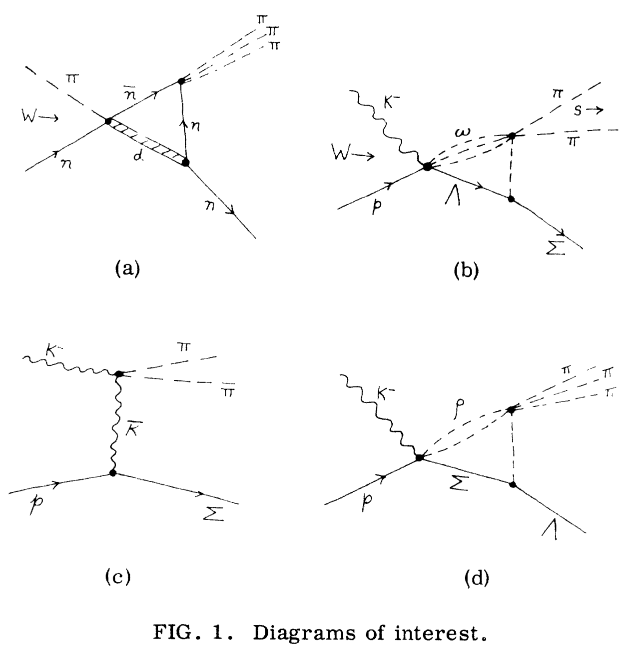

FIG. 1. Diagrams of interest.

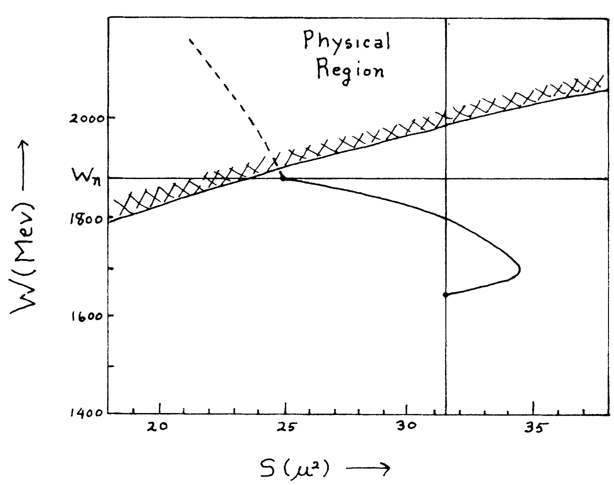

FIG. 2. The solid curve represents the path of the anomalous singularity in the W-s plane. The dashed line is the path of the real part of the singularity. $W_n = (M_{\omega^0} + M_\Lambda)$ and $s_n = (\mu + M_{\omega^0})^2$ are the normal thresholds.

We may write the contribution to the amplitude for the above process resulting from the reduced three-point diagram of Fig. 1(b) in the form

$$A_3(s) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\rho(s')ds'}{s'-s}, \qquad (1)$$

where s_0 is the anomalous threshold, and the weight function $\rho(s)$ is given by[3]

$$\rho(s) = \int d^4q_1\delta_+(q_1^2-m_1^2)\delta_+(q_2^2-m_2^2) \times \delta_+(q_3^2-m_3^2)\prod_{i=1}^{3}A_i. \qquad (2)$$

The quantities q_1, q_2, q_3 and m_1, m_2, m_3 are the internal momenta and masses, respectively, and the A_i are the amplitudes at the three vertices. Landshoff and Treiman take the amplitudes to be constants. We do the same thing, with an exception—we assume that the double pion production takes place mostly through the formation of an intermediate ρ meson, and therefore, that the amplitude corresponding to $\pi+\omega^0\rightarrow\pi+\pi$ is strongly weighted in the vicinity of 750 MeV. We simulate the above effect by taking the amplitude to be of the form $A(\pi+\omega^0\rightarrow\rho\rightarrow2\pi)=1$ for 700 MeV $\leq\omega\leq$ 800 MeV, and 0 if ω is not in this range. It is important to note that this weighting greatly enhances the peaking effect discussed above. The amplitude $A_3(s)$ then takes the form

$$A_3(s) = \frac{4F}{\pi}\int_{s_0}^{x}\frac{1}{[K(s')]^{1/2}}\frac{ds'}{s'-s} \equiv FI_3(s), \qquad (3)$$

where F is a constant representing the average effect of $\prod A_i$ in Eq. (2), and $\sqrt{x}=800$ MeV.

$$K(s') = (s_2-s')(s'-s_1), \qquad (4)$$

with

$$s_2 = W + M_{\Sigma},$$

$$s_1 = W + M_{\Sigma}. \qquad (5)$$

The integral of Eq. (3) can be performed analytically.[4] The amplitude, A_3, is then calculated (assuming $M_{\omega^0}=766$ MeV) for incident K^- lab momenta of 1.18 (corresponding to $W=W_n$), 1.15, and 1.12 BeV/c, and the results are shown graphically in Fig. 3(a). It is interesting to note how quickly the effect diminishes as the incoming momentum falls from 1.18 to 1.12 BeV/c.[5]

In order to actually estimate the effect of the triangle singularity of Fig. 1(b), we must know the size of the contribution to the full amplitude

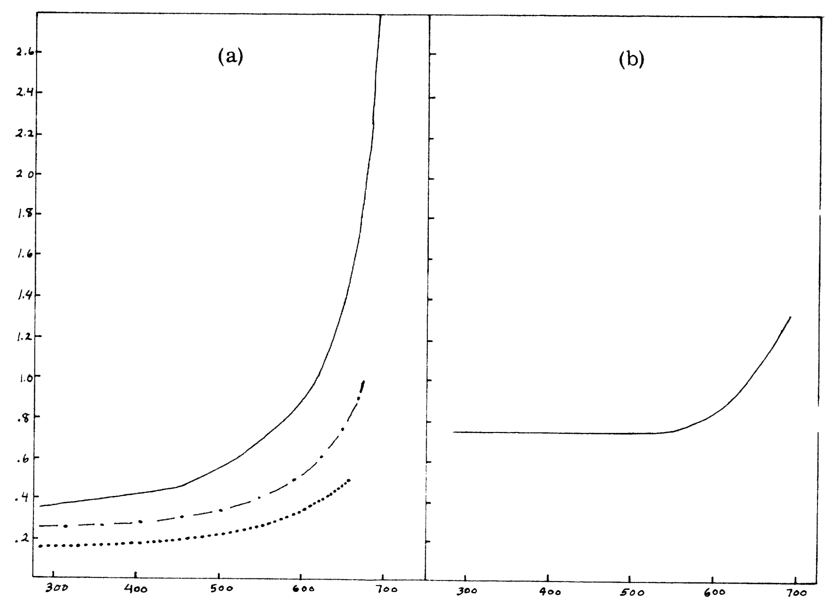

FIG. 3. (a) The amplitude $A_3(s)$ is plotted in arbitrary units as a function of ω for incident K^- lab momenta of 1.18 (solid curve), 1.15 (dash-dot curve), and 1.12 BeV/c (dotted curve). (b) On the same scale as used in Fig. 3(a), the amplitude $A_0(s)$ is plotted as a function of ω.

of the triangle diagram relative to other possible contributions. We give a crude order-of-magnitude argument by considering the contribution to the full amplitude of the pole term of Fig. 1(c), which we call A_{pole}. It is reasonable to assume that this term gives a significant contribution to the amplitude in question. In the region in which the amplitude is changing rapidly as a function of ω, we may write[6]

$$\frac{A_3}{A_{\text{pole}}}=\frac{\lambda f_{\pi\omega^0\rho}f_{\pi\lambda\Sigma}}{f_{K\bar{K}\rho}f_{KN\Sigma}}\left[\frac{(P_p-P_\Sigma)^2-M_K^2}{I_3(s)}\right]. \quad (6)$$

The quantity in brackets is ~2, and if we believe qualitatively either the vector meson theory of Sakurai,[7] or the unitary symmetry scheme of Gell-Mann,[8,9] we may take the ratio $f_{\pi\omega^0\rho}f_{\pi\lambda\Sigma}/f_{K\bar{K}\rho}f_{KN\Sigma}\sim 1$. The remaining parameter, λ, represents in some way the average effect of the $K^- + p \to A + \omega^0$ amplitude, and $|\lambda|^2$ is the relative probability that the process occurs.[10] We believe a reasonable choice for $|\lambda|^2$ to be 0.1-0.2; such a choice leads to the conclusion that A_3/A_{pole} is of $O(1)$. There are, of course, other contributions to the full amplitude, in addition to the pole and triangle terms considered, about which we can say nothing. We believe, however, that the above discussion does give an order-of-magnitude estimate of the effect of the triangle singularity—in fact, indicates that it might be quite large—and furthermore, sufficient motivation for the ensuing discussion, in which we make certain assumptions concerning the relative sizes of amplitudes.

We now write the total amplitude, $A(s)$, in the form

$$A(s)=A_0(s)+A_3(s), \quad (7)$$

where $A_0(s)$ represents all contributions except that of the triangle diagram. [We have already given plausibility arguments that $A_3(s)$ may be relatively quite large in the energy region in question.] For illustrative purposes we arbitrarily choose $A_0(s)$ as shown in Fig. 3(b). The peaking in the high-energy region is to simulate the fact that we are on the shoulder of the ρ resonance. This peaking is not sufficient to overcome phase-space factors, and there is no appreciable peaking in the differential cross section if we consider $A_0(s)$ to constitute the entire amplitude. We wish to emphasize that the shape of $A_0(s)$ is chosen to be reasonable, and the size is chosen completely arbitrarily. We now calculate the differential cross section using the amplitude of Eq. (7) with our assumed $A_0(s)$ and the appropriate phase-space and flux factors. In Fig. 4, assuming $M_{\omega^0} = 766$ MeV, we then plot $d\sigma/d\omega$ at K^- lab momenta of 1.18, 1.15, and 1.12 BeV/c. If we use the higher ω^0 mass (782 MeV), the entire set of graphs would be shifted 15 MeV to the right, and the analogous curves would now correspond to incident K^- momenta of 1.21, 1.18, and 1.15 BeV/c, respectively.

We see, therefore, that within the framework of our present assumptions, there should be a noticeable peaking in the high-energy end of the π-π spectrum in the reaction $K^- + p \to \Sigma + \pi + \pi$ performed at a K^- lab momentum of 1.18 to 1.21 BeV/c, assuming that the mass of the ω_0 meson lies between 766 and 783 MeV. The effect pre-

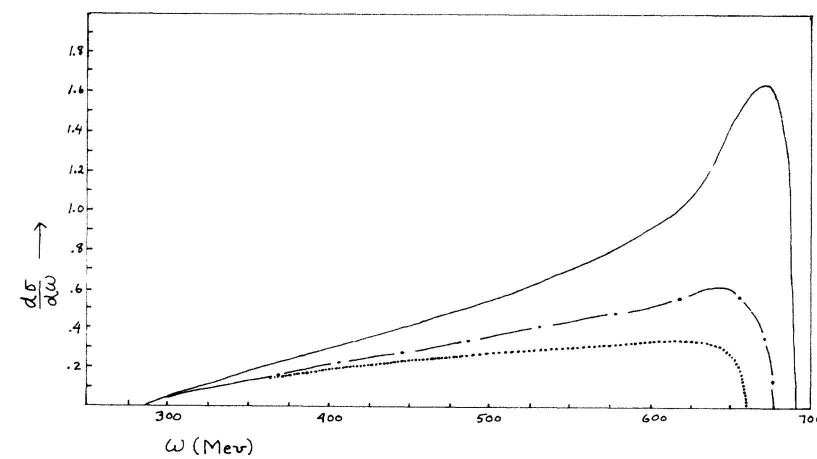

FIG. 4. The differential cross section for the process $K^- + p \to \Sigma + \pi + \pi$ is plotted as a function of ω for incident K^- lab momentum of 1.18 (solid curve), 1.15 (dash-dot curve), and 1.12 BeV/c (dotted curve).

dicted is very sensitive to the value of the lab momentum, and the better the resolution of the incoming beam, the more dramatic the effect. We believe that this type of effect merits further investigation, for in addition to strengthening our faith in the present form of perturbation theory, it might turn out that many of the newly discovered "resonances," which mysteriously appear and disappear in different experiments,[11] are kinematical manifestations of the type discussed above.

The author wishes to acknowledge helpful discussions with Professor M. Ross, Professor M. T. Vaughn, and Dr. P. K. Srivastava.

---

[1]P. V. Landshoff and S. B. Treiman, Phys. Rev. 127, 649 (1962). We hereafter refer to this work as LT.

[2]In our calculations we use $M_{\omega^0} = 766$ MeV as given by E. L. Hart, R. I. Louttit, and T. W. Morris, Phys. Rev. Letters 9, 133 (1962). If we use $M_{\omega^0} = 782$ MeV as given by C. Alff et al., Phys. Rev. Letters 9, 322 (1962), our results must be trivially modified. The appropriate changes will be noted at the proper time.

[3]R. E. Cutkosky, J. Math. Phys. 1, 429 (1960).

[4]B. O. Peirce and R. M. Foster, A Short Table of Integrals (Ginn and Company, New York, 1956), 4th ed., p. 29.

[5]If we use $M_{\omega^0} = 782$, the corresponding K^- lab momenta will be 1.21, 1.18, and 1.15 BeV/c. The qualitative features of the graphs would be the same.

[6]$I_3(s)$ is the quantity defined by Eq. (3). $f_{\pi\Lambda\Sigma}$ and $f_{KN\Sigma}$ are the pseudovector coupling constants, and $f_{\pi\omega^0\rho}$ and $f_{K\bar{K}\rho}$ are defined in reference 8. P_p and P_Σ are the particle four-momenta.

[7]J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).

[8]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[9]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[10]It is amusing to note that if the LT effect is not obscured by particle width, the amplitude for the process $K^- + p \rightarrow \Lambda + \omega^0$ may also be large because of the effect of the diagram of Fig. 1(d).

[11]For example, see P. L. Bastien et al., Phys. Rev. Letters 8, 114 (1962).

SCATTERING OF MUONS FROM HYDROGEN AT LARGE MOMENTUM TRANSFER*

G. E. Masek, T. E. Ewart, J. P. Toutonghi, and R. W. Williams
Physics Department, University of Washington, Seattle, Washington
(Received 19 November 1962)

We have measured the elastic scattering cross section of 1.2-BeV/c negative muons on free protons using a beam from the Bevatron of the Lawrence Radiation Laboratory, University of California, and a pair of large liquid hydrogen targets with spark chamber detectors. The range of momentum transfers was from 5 F^{-2} to 18 F^{-2} (450 MeV/c to 850 MeV/c), and our results are therefore directly comparable with electron-proton scattering in a region in which the proton form factors have reduced the scattering cross section by about a factor of five.

Previous measurements of muon scattering[1,2] have involved complex nuclei and lower momentum transfers; to lift these restrictions one must overcome a low-counting-rate problem [the cross section is expected to be $(2-50)\times10^{-32}$ cm^2 sr^{-1}] and a possible systematic error due to pion contamination, with a cross section perhaps 10^4 times greater. A high counting rate was realized by employing two large (9 in. in diameter and 54 in. long) liquid hydrogen targets; large solid angle spark chamber detectors (typical azimuthal coverage, ~20%); and a direct negative beam, from an internal target, which contained a large muon flux and whose pion content was minimized by judicious adjustment of the initial beam-forming magnets. Typically, at the hydrogen targets, the beam consisted of 4000 muons and 15 000 pions per burst. The pion contamination was reduced to tolerable levels by electronic means, and the effective pion flux at the target (measured by direct observation of the scattering distribution as described below) was about 10^{-2} per burst.

The electronic rejection of pions was accomplished by a combination of accurate momentum analysis with magnets and velocity discrimination by threshold Cherenkov counters [see Fig. 1(a)]. Four gas-filled Cherenkov counters, similar to counters we have used previously,[2,3] were part of the beam-selecting system, and a coincidence between all four was required to verify the presence of a muon. Bending magnets in the beam served both to select the required particle momentum and to sweep out knock-on electrons which otherwise would interfere with the independence of the four velocity measurements by the Cherenkov counters. A subthreshold pion