that we cannot distinguish between a decay in flight and a decay at rest for a residual momentum of less than 60 MeV/c, i.e., we do not count the time spent in the last segment of track corresponding to this momentum, about 2.5 microns.

The result we obtained using the Bartlett maximum likehood method[6] is

$$\tau({}_{\Lambda}\mathrm{He}^{4,5}) = 1.4^{+1.8}_{-0.5} \times 10^{-10} \text{ second.}^{7}$$

It should be pointed out that there may exist in the scanning a bias against finding decays in flight. However, due to the fact that we restricted ourselves to mesonic decays, and that the plates were thrice scanned, we expect such a bias to be small.

The authors are grateful to Dr. E. J. Lofgren, Dr. J. Murray, and the Bevatron staff for making the exposure possible. Thanks are due to A. Bernardi for his assistance in making the measurements, and to Mr. J. Goodridge, Mrs. M. Ciccolo, Mrs. M. Ponton, and Mrs. Y. W. Kang for their careful scanning of the stack.

*Work supported by the U. S. Atomic Energy Commission.

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NONSHRINKING DIFFRACTION SCATTERING*

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The purpose of this note is to present arguments which suggest that the forward peak of high-energy elastic scattering does not shrink.

First, we discuss how one should analyze the data to obtain evidence concerning shrinkage. We show that both p - p [1] and π - p [2] scattering are not only consistent with, but rather suggestive of no shrinkage.[3]

Secondly, we present a simple argument which supports no shrinkage in terms of a complex, energy-dependent effective potential. We also construct a more specific model of high-energy elastic scattering and show explicitly how this model predicts no shrinkage and produces a finite (nonzero) total cross section in the limit of infinite energy.

Let $A(s,t)$ be the elastic amplitude as a function of s, the square of the c.m. total energy, and t, the negative of the square of the c.m. momentum transfer. Throughout this note, we assume that $A(s,t)$ behaves, when $s \to \infty$ and t is finite, as

$$A(s,t) \to \beta(t) s^{\alpha(t)} \exp[-\tfrac{1}{2} i\pi\alpha(t)], \qquad (1)$$

where $\alpha(t)$ and $\beta(t)$ are real functions of t and $\alpha(0) = 1$. According to a recent derivation[4] of (1), this behavior is expected as long as $A(s,t)$ is analytic in the sense of Mandelstam, and the phase $\delta(s,t)$ of $A(s,t)$ satisfies a certain condition.[5] Since this condition is sufficiently weak, we assume (1) and consider in this note how $\alpha(t)$ behaves as a function of t. If $\alpha(t)$ changes with t near $t = 0$, the forward peak of high-energy elastic scattering shrinks, while there is no shrinkage if $\alpha(t)$ does not vary.

We now discuss how one should analyze data to obtain evidence concerning shrinkage. Here the main point is that (1) is the asymptotic form, while the available data refer to finite energies. For finite s and t, $A(s,t)$ must be written as

$$A(s,t)=\beta(t)s^{\alpha(t)}\gamma(s,t)\exp[-\tfrac{1}{2}i\pi\alpha(t)], \qquad (2)$$

where $\gamma(s,t)$ satisfies for finite t

$$\lim_{s\to\infty}\gamma(s,t)=1. \qquad (3)$$

The total cross section is then given by

$$\sigma_{\text{tot}}(s)\propto \operatorname{Im}A(s,t=0)/s\propto \operatorname{Re}\gamma(s,0), \qquad (4)$$

while the c.m. differential cross section is expressed as

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{t=0}}=\frac{\beta^2(t)}{\beta^2(0)}\left|\frac{\gamma(s,t)}{\gamma(s,0)}\right|^2 s^{2[\alpha(t)-1]}. \qquad (5)$$

In both published analyses,[1,2] the factor in (5) which involves $\gamma(s,t)$ is assumed to be unity. However, $\sigma_{\text{tot}}(s)$ and thus $\operatorname{Re}\gamma(s,0)$ are not necessarily constant[6] in the energy regions concerned. Therefore, we must examine if the factor in question modifies the published conclusions.[1,2]

In the Michigan analysis,[2] (5) is plotted against t with s fixed. They retain those data which fit a pure exponential form of (5) as a function of t. They find[2] that this exponential form is independent of s. Now, (5) can, in fact, be written as an exponential function of t in a region near $t=0$. The exponent becomes $[\beta+\gamma(s)+2\alpha'(0)\ln s]t$, where a constant β is due to $\beta(t)$, a function $\gamma(s)$ is due to $\gamma(s,t)$, and $\alpha'(0)$ is the derivative of $\alpha(t)$ at $t=0$. The condition (3) implies that $\gamma(s)\to 0$ as $s\to\infty$. Thus, $\gamma(s)$ and $2\alpha'(0)\ln s$ cannot cancel each other because of different asymptotic behaviors, especially when s becomes large. Therefore, the energy independence of the exponential form must mean both $\gamma(s)=0$ and $\alpha'(0)=0$. This is why the Michigan analysis[2] could be valid evidence for no shrinkage in π-p scattering. To confirm this, however, it is necessary to find the same evidence also in the higher energy region.

We wish to point out that the high-energy p-p data[7] also have the energy-independent exponential form in a region of small t between 12 and 26 BeV of the lab incident nucleon momentum. Since $\sigma_{\text{tot}}(s)$ is constant[6] in this energy region, it is possible that $\gamma(s)$ defined above is zero. Then these data[7] are not only consistent with, but rather suggestive of no shrinkage in p-p scattering.

In the CERN analysis,[1] the logarithm of (5) is plotted against $\ln s$ with t fixed. They conclude shrinkage by fitting, as linear functions of $\ln s$, all the data with $|t|$ up to 1.1 $(\text{BeV})^2$ and $s/2M^2$ greater than 4, M being the nucleon mass. Now, the difficulty is that (5) never allows this linear fit unless the factor which involves $\gamma(s,t)$ is unity. Even if this factor approaches unity as $s\to\infty$ because of (3), this factor, in general, causes apparent shrinkage for finite s, which necessarily disappears as $s\to\infty$. This means that the data fall on curves which approach straight lines asymptotically. Furthermore, $\alpha(t)$ should be determined by the asymptotic slopes, but not by the over-all slopes, of these curves. The reported evidence[1] is, however, due to the over-all slopes. In fact, the experimental points[1] do not seem to fall on straight lines but rather on curves which clearly tend to horizontal lines as $s\to\infty$. Therefore, the data[1] actually suggest no shrinkage in p-p scattering.

We may argue also as follows. The above linear fit becomes legitimate as s becomes sufficiently large. Suppose that is the case in the region where $\sigma_{\text{tot}}(s)$ is constant.[6] This requires that only data with $s/2M^2\geq 14$ are used. The experimental points[1] then become consistent with no shrinkage. In fact, the experimental points[1] towards the high-energy end clearly fall on horizontal lines. Thus the data[1] suggest no shrinkage in p-p scattering.

The point in our argument is that we must eliminate the effect due to $\gamma(s,t)$ in (5) in order to see valid evidence for shrinkage. Possibly the best way to do this is to make a series of analyses using the data with increasing minimum energy and see how evidence for shrinkage changes as the minimum energy increases.

We now present our physical argument which supports no shrinkage. First we observe that $\alpha(t)$ is, according to (1), essentially the phase of $A(s,t)$ in the limit of $s\to\infty$. Therefore, no shrinkage implies that the phase of $A(s,t)$ in the limit of $s\to\infty$ does not vary in a region near $t=0$, and vice versa. We then suppose that high-energy elastic scattering is described at least qualitatively in terms of an effective potential which is complex and energy dependent. In the limit of $s\to\infty$, this effective potential must become pure imaginary due to dominant absorption. It then follows, independently of the details of the effective potential, that the scattering amplitude becomes pure imaginary as $s\to\infty$, not only in the forward direction but also in the directions corresponding to a finite momentum transfer. This is how our argument suggests no shrinkage.

To explain the above argument more explicitly, we write down the Schrödinger equation[8]

$$[\Delta + k^2 - V(r,k^2)]\psi(r) = 0, \tag{6}$$

where $V(r,k^2)$ is complex and energy dependent and is assumed to be spherically symmetric for simplicity. The scattering amplitude $f(\theta)$ is then given[9] by

$$f(\theta) = (k/i)\int_0^\infty J_0(k\theta b)\left\{e^{iX(b)} - 1\right\} bdb,$$
$$X(b) = -(1/2k)\int_{-\infty}^{+\infty} V(r,k^2)dz, \tag{7}$$

where θ is the scattering angle, $J_0(k\theta b)$ is the Bessel function of the zeroth order, and the integral in $X(b)$ is along a straight line parallel to the incident direction with the impact parameter b. The solution (7) is correct as long as $kd \gg 1$, where d is the distance within which $V(r,k^2)$ changes appreciably and $\theta \lesssim 1/kd \ll 1$. The last condition implies that the momentum transfer, $k\theta$, is allowed to be finite. We see by inspection that $f(\theta)$ becomes pure imaginary for all θ, as long as $V(r,k^2)$ becomes pure imaginary, independently of the details of $V(r,k^2)$.

Our specific model of high-energy elastic scattering is a Schrödinger potential model of (6) where $V(r,k^2)$ satisfies the conditions (a) that $V(r,k^2)$ becomes pure imaginary and does not vanish in the limit of $k^2 \to \infty$, and (b) that $V(r,k^2)$ is analytic in k^2 except for a finite number of poles and a single cut which extends from the inelastic threshold to $+\infty$ along the real axis.[10]

Because of conditions (a) and (b), $V(r,k^2)$ has a phase representation of the type in reference 4. If the phase $\delta(r,k^2)$ of $V(r,k^2)$ along the cut satisfies the condition of reference 5, $V(r,k^2)$ approaches, as $k^2 \to \infty$, a simple power behavior of k. The power is $2n - 2\delta(r,k^2=\infty)/\pi$, where n is an integer due to polynomials in the phase representation and $\delta(r,k^2=\infty)$ is $\frac{1}{2}\pi$ plus an integer multiple of π. Therefore, the only powers which do not make $V(r,k^2)$ vanish as $k^2 \to \infty$ are 1, 3, 5, etc. This means that $V(r,k^2)$ must diverge as $k^2 \to \infty$ at least linearly in k.

The total cross section $\sigma_{tot}(k^2)$ is given[9] in terms of $f(\theta)$ by $\sigma_{tot}(k^2) = (4\pi/k)\,\mathrm{Im}f(\theta=0)$. If we approximate $V(r,k^2)$ by an effective square-well potential[11] with the range R and the depth V, we obtain

$$\sigma_{tot}(k^2) = 2\pi R^2 \,\mathrm{Re}\left\{1 + 2\left(\frac{1}{a}+\frac{1}{a^2}\right)e^{-a} - \frac{2}{a^2}\right\},$$
$$a = iVR/k, \tag{8}$$

where Re stands for the real part. Since absorptivity implies that $\mathrm{Im}V < 0$, a becomes asymptotically a positive real number. Moreover, a cannot vanish as $k^2 \to \infty$ because of the divergence in $V(r,k^2)$. Therefore, R must remain finite asymptotically in order for $\sigma_{tot}(k\ )$ to remain finite as $k^2 \to \infty$. This is how our model, in fact, predicts no shrinkage, while it produces a finite total cross section as $k^2 \to \infty$.

We add a few remarks. First, it is plausible that $V(r,k^2)$ diverges only linearly in k. Then the dispersion relation for $V(r,k^2)$ indicates that $\mathrm{Re}V(r,k^2)$ remains finite asymptotically, provided $\mathrm{Im}V(r,k^2)$ approaches its limit sufficiently fast. This behavior of $\mathrm{Re}V(r,k^2)$ is quite reasonable. Secondly, our model does not seem to violate[12] the Mandelstam assumption. Finally, the analyticity of $V(r,k^2)$ is crucial in concluding that R is finite asymptotically. Otherwise, a could vanish as $k^2 \to \infty$, which then implies that R must diverge as $k^2 \to \infty$ in order for $\sigma_{tot}(k^2)$ to have a nonzero limit as $k^2 \to \infty$.

We have also completed[13] a formal analysis which, in fact, predicts no shrinkage. This analysis consists of assuming the double phase representation for the elastic amplitude, which is the two-dimensional generalization of the phase representation of reference 4. The essential assumptions made in this analysis are that the elastic amplitude has the analyticity due to Mandelstam and that the phase of the elastic amplitude has the simplest analyticity in the momentum-transfer variable. This analysis proves also that $\beta(t)$ in (1) is analytic everywhere except for the t cut and has no zeros except at infinity.

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*Work supported by the National Science Foundation and by the U. S. Atomic Energy Commission.

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EXAMPLE OF $\Sigma^+ \to p+\gamma$ IN A HYDROGEN BUBBLE CHAMBER*

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Several searches have been made for the decay $\Sigma^+ \to p+\gamma$ of which two[1,2] claim negative results and two[3,4] claim positive results. All of the previous attempts were done in emulsions. In this Letter we present evidence of a $\Sigma^+ \to p+\gamma$ decay where the Σ^+ hyperon was produced by a K^- meson at rest in a hydrogen bubble chamber.

As part of our study of stopping K^- mesons in the Alvarez 15-in. hydrogen bubble chamber, we have measured all Σ^+ hyperons which were produced and did not decay into a light track. The events were found by a double general scan in which all K^- mesons entering a specified fiducial region were followed. The events were processed through a combined program of PANG and KICK called PACKAGE, and then the output was sorted by an EXAMIN program.

In our event, the K^- came to rest and produced a Σ^+ which decayed in flight. The proton came off backwards and stopped in the chamber. This type of configuration could be due to either

$$K^- + p \to \Sigma^+ + \pi^-, \quad \Sigma^+ \to p+\gamma \tag{1}$$

or

$$K^- + p \to \Sigma^+ + \pi^-, \quad \Sigma^+ \to p+\pi^0. \tag{2}$$

The unfitted data for the tracks are shown in Table I.

The near colinearity of the Σ^+ and π^-, and the close agreement of the beginning momentum of the K^- with the value of momentum obtained by range, strongly suggest that one should try to fit the production of the Σ^+ assuming the K^- stops. This fit is a 3-constraint fit since the magnitude of the Σ^+ momentum, from curvature, is too poorly determined to be used in the fitting. After propagating the Σ^+ to the decay vertex, we did a 1-constraint fit. The results for our three measurements are shown in Table II. As an additional check, we tried fitting the over-all event as a 4-constraint fit on another measurement and obtained

$$\chi^2(\Sigma^+ + p + \pi^0) = 147.54, \quad \chi^2(\Sigma^+ \to p+\gamma) = 0.59.$$

We now consider various interpretations of the event as a $\Sigma^+ \to p+\pi^0$ decay which would give