

and (7). They serve to deduce the charge density fluctuations needed in Poisson's equation

$$-ik\epsilon_0 E = e(\delta n_i - \delta n_0), \quad (9)$$

which closes the system of equations and yields the dispersion formula

$$\frac{1}{i\omega(i\omega + \nu_i)m_i + k^2KT} + \frac{n_i/n_0}{i\omega(i\omega + \delta)m_0 + 5k^2KT/3} = -\frac{1}{i\omega'(i\omega' m_e'' + \nu_e m_e') + 5k^2KT/3} - \frac{\epsilon_0}{n_i e^2}. \quad (10)$$

Here

$$m_e' = m_e / [\cos^2\theta + m_e^2 \nu_e^2 / e^2 B^2]$$

and

$$m_e'' = m_e' \frac{(\cos^2\theta - m_e^2 \nu_e^2 / e^2 B^2)}{(\cos^2\theta + m_e^2 \nu_e^2 / e^2 B^2)}$$

represent effective electron masses which reflect the inhibition of electron motion by the magnetic field to the required accuracy ( $|\omega'| \ll \nu_e$ ;  $m_e \nu_e / eB$  is small compared with 1 but not necessarily with  $\cos\theta$ ).

Formula (10) allows the establishment of conditions at which damping changes to growth, i. e., under which  $\omega$  is real for real  $k$ . A very slight departure from these conditions suffices to produce growth, since the second term on the left, describing the dynamics and damping of neutral sound, is rendered unimportant by the smallness of  $n_i/n_0$ . Ignoring it, as well as terms of the order  $(\omega/\nu_i)^2$ , a comparison of the (dominant) imaginary parts in Eq. (10) leads to the determination of an angle at which undamped propaga-

tion can take place:

$$\frac{u_{\parallel}}{\omega/k} - 1 - \frac{\nu_i m_i \nu_e m_e}{(eB)^2} = \frac{\nu_i m_i}{\nu_e m_e} \cos^2\theta. \quad (11)$$

Such an angle exists subject to Eq. (2) and deviates very little from  $90^\circ$  for drifts which exceed the threshold by only moderate factors. Comparing real parts, and inserting the angle given by Eq. (11), leads to Eq. (3).

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<sup>1</sup>H. G. Booker, Radar Studies of the Aurora, edited by J. A. Ratcliffe (Academic Press, Inc., New York, 1960), Chap. VIII.

<sup>2</sup>A. M. Peterson, The Aurora and Radio Wave Propagation, edited by D. H. Menzel (Harvard University Press, Cambridge, Massachusetts, 1960), Chap. I.

<sup>3</sup>K. L. Bowles, R. Cohen, G. R. Ochs, and B. B. Balsley, *J. Geophys. Res.* **65**, 1853 (1960).

<sup>4</sup>K. L. Bowles, Observations reported at the Fifteenth Annual Gaseous Electronics Conference, Boulder, Colorado, October, 1962 (unpublished).

<sup>5</sup>J. P. Dougherty and D. T. Farley, *Proc. Roy. Soc. (London)* **A263**, 238 (1961).

<sup>6</sup>J. A. Fejer, *Can. J. Phys.* **39**, 716 (1961).

<sup>7</sup>T. Hagfors, *J. Geophys. Res.* **66**, 1699 (1961).

<sup>8</sup>E. E. Salpeter, *J. Geophys. Res.* **66**, 982 (1961).

<sup>9</sup>O. Buneman, *Phys. Rev. Letters* **1**, 8 (1958).

<sup>10</sup>M. N. Rosenbluth and N. Rostoker, *Phys. Fluids* **5**, 776 (1962).

<sup>11</sup>O. Buneman, *J. Nucl. Energy: Pt. C* **4**, 111 (1962).

<sup>12</sup>Claims that auroras are audible to the human ear reached the author after he developed this theory.

<sup>13</sup>D. L. White, *J. Appl. Phys.* **33**, 2547 (1962).

<sup>14</sup>J. J. Hopfield, *Phys. Rev. Letters* **8**, 311 (1962).

## THERMAL BOUNDARY RESISTANCE BETWEEN SOME SUPERCONDUCTING AND NORMAL METALS\*

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A thermal boundary resistance between a solid and liquid He<sup>4</sup> was first observed by Kapitza.<sup>1</sup> It has recently been found that this resistance between a metal and liquid He<sup>4</sup> is greater when the metal is in the superconducting state than when it is normal.<sup>2,3</sup> Little anticipated this result<sup>4</sup> and suggested<sup>5</sup> that the thermal boundary resistance between two different metals would

be different when one of the metals was in the superconducting state than when they were both normal. The purpose of this paper is to report measurements of the thermal resistance at the boundary between tin and copper and between lead and copper for cases in which the tin and lead are superconducting and normal.

Figure 1 shows the arrangement of the sample

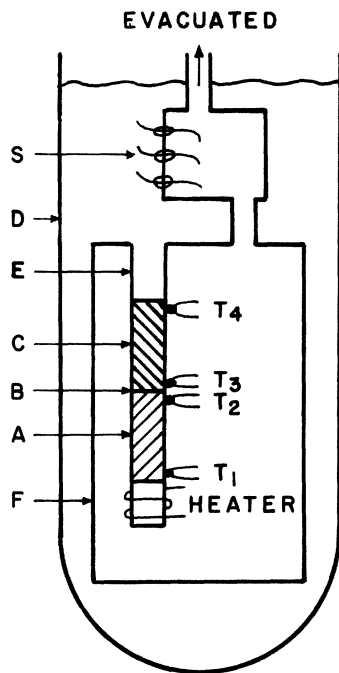


FIG. 1. Sample arrangement and schematic diagram of apparatus.

and gives a schematic diagram of the apparatus used. A tin-copper boundary  $B$  was formed by vacuum casting a cylindrical piece of very pure tin  $A$  onto a piece of copper  $C$ . Each was 1 in. long. This structure was turned to  $\frac{1}{4}$ -in. o.d. in a lathe. A short piece of copper with a heater attached to it was soldered to the lower end of the tin rod with Wood's metal.  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$  were carbon-resistor thermometers attached as shown. Each consisted of a  $\frac{1}{10}$ -watt, 48-ohm Allen Bradley carbon resistor cemented into a close-fitting thin-wall copper sleeve which was soldered to the sample. This assembly was soldered to the stainless steel tube  $E$  which was then mounted in the sample chamber  $F$ . Dewar  $D$  contained liquid helium which completely surrounded the sample chamber. Several glass-to-metal seals  $S$  admitted the necessary leads from the helium bath. The tin which was superconducting at the operating temperatures of 1.4 to 1.9°K could be changed to the normal state by applying a magnetic field. This was supplied by a coil of niobium-25% zirconium wire, superconducting at these temperatures, wound on a form surrounding the sample chamber  $F$ . To avoid flux trapping in the tin, all measurements were taken first with the tin superconducting followed by measurements with a tin in the normal states.

Temperature measurements with  $T_1$  and  $T_2$  and with  $T_3$  and  $T_4$  for a given heat current  $\dot{Q}$  determined the temperature gradients in the tin and copper, respectively. These gradients were used to extrapolate temperatures to the boundary  $B$  from which the discontinuity in temperature  $\Delta T$  at  $B$  was obtained. The thermal boundary resistance was determined from

$$R = A(\Delta T) / \dot{Q},$$

in which  $A$  is the cross-sectional area of the boundary. The four thermometers were calibrated after each run by allowing the sample assembly to come to thermal equilibrium with the bath helium at several different temperatures. These temperatures were determined by measuring the vapor pressure of the helium using the T55E scale.

The data for the thermal resistance  $R$  at the boundary between tin and copper from 1.4 to 1.9°K are presented in Fig. 2. Temperature values plotted here were obtained by extrapolating those measured by  $T_1$  and  $T_2$  to the boundary  $B$ . For some heat currents  $T_1$  was as much as 0.1°K above the helium bath temperature, and  $\Delta T$  was in the region of 0.020°K. It is seen that  $R$  varies as  $1/T^3$  with the tin in the superconducting state. When the tin was in the normal state,  $R$  was considerably smaller and only slightly dependent on temperature.

Assuming a perfect boundary between two dissimilar solids, that all heat flow was via lattice waves, and that the thermal resistance  $R$  at the boundary was due to acoustical mismatch, Little

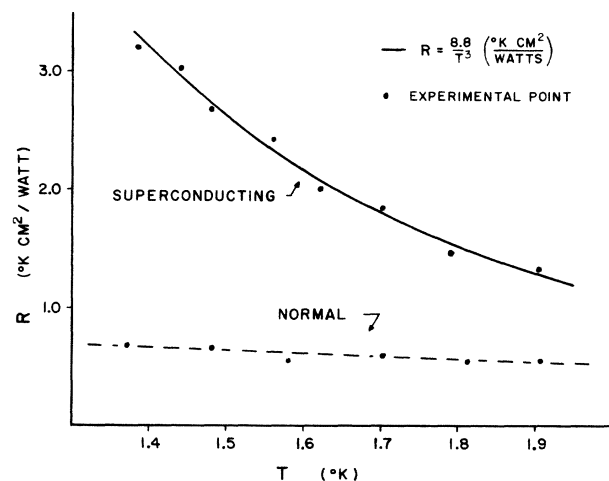


FIG. 2. Thermal resistance at the boundary between tin and copper as a function of temperature.

calculated<sup>4</sup>

$$R = \frac{1}{5.01 \times 10^9 [(\Gamma_l/C_l^2) + (\Gamma_t/C_t^2)] T^3} \frac{^\circ\text{K cm}^2}{\text{W}}.$$

In this expression  $\Gamma_l$  and  $\Gamma_t$  are, respectively, the longitudinal and transverse transmission coefficients of the boundary, and  $C_l$  and  $C_t$  are the longitudinal and transverse velocities of sound in the medium in which the heat current originates. By substituting appropriate values for tin and copper into this, one obtains  $R = 6.45/T^3$ . Agreement between this and the measured value of  $8.8/T^3$  for the case in which the tin was superconducting suggests that the thermal resistance at the boundary between a superconducting and normal metal is largely due to acoustical mismatch. The much smaller boundary resistance obtained when the tin was normal suggests that the electrons play the dominant role in the transfer of heat between normal metals. The more efficient electron transport processes involved in the transfer of heat across the boundary, when both metals were normal, effectively shorted out the resistance due to acoustical mismatch observed when the tin was superconducting. Such electron processes would not predominate when the tin was superconducting, since the energy gap associated with the transition of the tin to superconductivity would prevent most of the electrons from participating in these processes.

Similar measurements were made of the thermal resistance at the boundary between lead and copper. The sample was similar to the tin-copper sample, except that the tin was replaced by a specimen of very pure lead. Since the lead was very soft, liquid nitrogen was poured onto it during machining to prevent distortion. When the lead was superconducting, the boundary resistance between it and copper varied as  $11.0/T^4$  ( $^\circ\text{K cm}^2/\text{W}$ ). With the lead in the normal state, the boundary resistance was about  $0.2^\circ\text{K cm}^2/\text{W}$ , and it varied only slightly with temperature. The difference between the temperature dependences of the resistance observed for the lead-copper boundary and the tin-copper boundary may be characteristic of these boundaries. It

may also be due to variations in properties of an alloyed transition zone formed during vacuum casting by the dissolving of some tin or lead into the copper at the boundary. Measurements of the thermal resistance at the boundaries between other samples including those formed between two superconductors and between an ordinary dielectric and a superconductor are in progress.

In order to investigate the possibility of making use of such boundary resistance in constructing a thermal switch,<sup>5</sup> a lead rod was joined to a copper rod by means of a thin film of soft solder. The thermal resistance of this junction was measured. With the lead and solder superconducting, the resistance was approximately twice as great as that at the boundary between lead and copper when the two had been vacuum cast together. At  $1.2^\circ\text{K}$ , application of a magnetic field reduced this resistance to about  $\frac{1}{18}$  the value measured when the lead and solder were both superconducting. Further investigations are in progress of the switching properties of multiple junctions.

During the course of this work a cylinder of lead with thermometers attached was soldered into tube *E* of Fig. 1. An additional thermometer was suspended in the liquid helium in tube *E*. This permitted measurements of the Kapitza resistance at the boundary between a solid and liquid He<sup>4</sup>. In agreement with others<sup>2</sup> the resistance was greater by about a factor of three when the lead was superconducting than when it was normal.

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<sup>1</sup>P. Kapitza, J. Phys. (U.S.S.R.) 4, 181 (1941).

<sup>2</sup>L. J. Challis, Proc. Phys. Soc. (London) 80, 759 (1962).

<sup>3</sup>J. I. Gittleman and S. Bozowski, Phys. Rev. 128, 646 (1962).

<sup>4</sup>W. A. Little, Can. J. Phys. 37, 334 (1959).

<sup>5</sup>W. A. Little, Phys. Rev. 123, 435 (1961).