EXCITATION OF FIELD ALIGNED SOUND WAVES BY ELECTRON STREAMS*

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(Received 27 February 1963)

Radar reflections from polar auroras[1,2] and from equatorial electrojets[3,4] indicate that strong irregularities move across the earth's magnetic field, with equiphase planes aligned along the field (see Fig. 1). The phase velocities are of the order of the speed of sound. Propagation vectors (phase-plane normals) tend to be tilted toward the direction of the electron drift but do not necessarily coincide with it (moderate angles ϕ, Fig. 1).

These phenomena occur at heights (~100 km) where the degree of ionization is low (~10^{-8}). The motion of charged particles is dominated by collisions with neutrals and the fluctuation theory for collision-free plasmas[5-8] becomes inapplicable. So does the theory of the enhancement of such fluctuations by the two-stream mechanism[9,10] which, except in one case,[11] refers to streaming along, not across, the magnetic field.

At the 1962 Spring Union Radio-Scientifique Internationale (URSI) meeting the author suggested that the observed undulatory disturbances might be sound waves excited by the streaming electrons, the mechanism resembling the interaction of slow waves with beams in traveling-wave tubes. Unlike ion waves in collisionless plasmas (often misnamed "ion sound"), these waves are carried by elastic, as well as electric, restoring forces and couple directly to the purely elastic sound waves in the dominant neutral background.[12]

Excitation of sound by streaming electrons (or holes) is familiar in crystals.[13,14] The ionospheric situation would be analogous to phonon excitation by Hall currents, but the dynamics and coupling are somewhat different.

The coupling is provided by the ions which communicate with the neutral gas by collisions and with the electrons by macroscopic electrostatic fields. The causal cycle of wave excitation is illustrated in detail in Fig. 2.

Sound propagates very slowly compared with light. Therefore, the longitudinal electric waves accompanying the sound are decoupled from electromagnetic waves. Oscillating magnetic fields are effectively absent and there is no connection with hydromagnetic, Alfvén, or whistler modes.

The static magnetic field, however, plays an important part because it produces vastly different responses of ions and electrons to electric fields, since the ratio of the gyrofrequency to the collision frequency is less than one for ions and greater than one for electrons:

$$\nu_e m_e \ll eB \ll \nu_i m_i \tag{1}$$

$$(\nu_e \sim 10^5,\ eB/m_e \sim 10^7,\ \nu_i \sim 10^4,\ eB/m_i \sim 10^2).$$

This accounts for the appearance of dc Hall currents in the first place. The sharp dependence of electron mobility on angle, near the 90° inclination to the magnetic field, provides a necessary tunability of the electronic generator to the acoustic load. The wave fronts are field aligned, because the electrons can short out all electric fields along B. Unlike hydromagnetic processes, these phenomena depend critically on the conductivity being finite.

Viscous damping of sound in the neutral gas is considerable at 100 km. Hence the phase velocity does not coincide exactly with the gas-kinetic val-

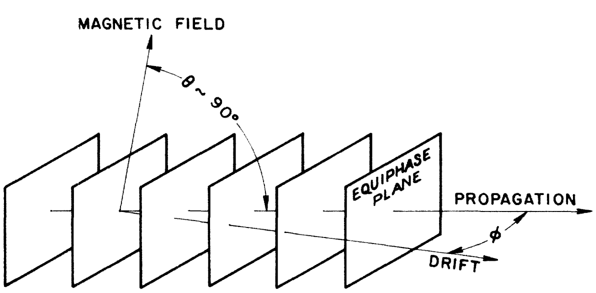

FIG. 1. Orientation of direction of propagation, equiphase planes, drift, and magnetic field.

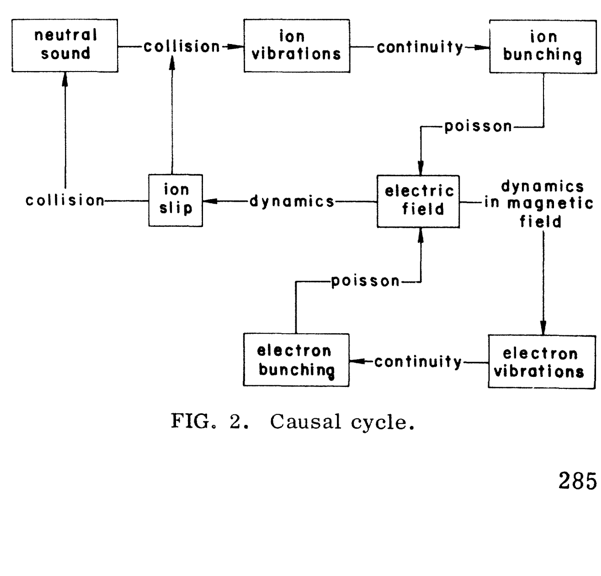

FIG. 2. Causal cycle.

ue $(5KT/3m_0)^{1/2}$. As with all low-Q systems, the frequency ω at a given wave number k depends strongly on the form of excitation. Since the electrons must "push" the wavefronts along, the component of drift in the direction of propagation, $u_{\parallel}$ $(=u\cos\varphi$, Fig. 1), must exceed ω/k. The source of feedback is the Doppler shift which causes the supersonic electrons to "hear" a negative frequency $\omega'=\omega-ku_{\parallel}$. The somewhat more stringent condition

$$u_{\parallel}>(\omega/k)(1+\nu_i m_i \nu_e m_e/e^2B^2) \tag{2}$$

is required by the theory outlined below: The drift has to exceed the phase velocity by a factor depending critically upon the collision frequencies, and, hence, on height.

The elastic and electric interactions of the ions impose further conditions which lead to the dispersion formula

$$\frac{\omega}{k}\left(1-\frac{\nu_i^2}{\omega_{pi}^2}-\frac{\nu_i^2 m_i m_e}{e^2B^2}\right)$$

$$=\frac{8KT}{3m_i}\frac{k}{\omega}-\frac{\nu_i}{\nu_e}\left[u_{\parallel}-\frac{\omega}{k}\left(1+\frac{\nu_i m_i \nu_e m_e}{e^2B^2}\right)\right], \tag{3}$$

where

$$\omega_{pi}^2=n_i e^2/m_i\epsilon_0\sim 10^5 n_i. \tag{4}$$

Normally n_i is large enough for the term ν_i^2/ω_{pi}^2 to become negligible. Similarly, $\nu_i^2 m_i m_e/e^2B^2$ is quite small.

Therefore the phase velocity ω/k adopts a value close to $(8KT/3m_i)^{1/2}$ when the drift just clears the threshold given by Eq. (2) and a lower value when the drift exceeds its threshold. Drifts are not uniform throughout the disturbed region and one actually sees a spread of phase velocities. The lower values, caused by the (presumably less prevalent) faster drifts, show up with less intensity. To account for appreciably lower than maximum phase velocities with only moderate excess drifts, one has to assume that ν_i/ν_e is not too small, and, perhaps, rather larger than the value $\frac{1}{10}$ indicated by the frequencies quoted after Eq. (1).

Equations (2) and (3) were deduced from a three-component gas mixture analysis, using a Navier-Stokes equation for each component (neutrals, ions, electrons) with the appropriate electric and magnetic body forces for the charged components as well as intercomponent collisional momentum-transfer terms. Various approximations lead to the following equations governing the longitudinal oscillatory velocities, designated for neutrals by subscript 0, for ions by subscript i, and for electrons by subscript e:

$$\left(i\omega+\frac{5k^2KT}{3i\omega m_0}+\delta\right)v_0=\frac{\nu_i n_i m_i(v_i-v_0)}{n_0m_0}, \tag{5}$$

$$\left(i\omega+\frac{k^2KT}{i\omega m_i}+\nu_i\right)v_i=\frac{e}{m_i}E+\nu_i v_0, \tag{6}$$

$$\left[\left(i\omega'+\nu_e\right)\frac{e^2B^2+(\nu_e+i\omega')^2m_e^2}{e^2B^2\cos^2\theta+(\nu_e+i\omega')^2m_e^2}+\frac{5k^2KT}{3i\omega'm_e}\right]v_e$$

$$=\frac{e}{m_e}E. \tag{7}$$

In Eq. (5) the term δ stands for viscous damping. The right-hand side represents collisional momentum transfer due to ion slip v_i-v_0. In Eqs. (6) and (7) viscous damping has been ignored. The ions obey a gas law with $\gamma=1$ rather than $\frac{5}{3}$, since their temperature fluctuations follow those of the neutrals. Electrons control their own temperature fluctuations: They do not exchange energy in elastic collisions with neutrals.

Magnetic effects on the ions have been ignored [see Eq. (1)]. The factor multiplying $i\omega'+\nu_e$ in the electron equation represents the magnetic effect on longitudinal components after eliminating the transverse components, remembering that there are no transverse elastic or electric restoring forces. A collisional acceleration $\nu_e v_0$ of the electrons, due to the vibrations of the neutral background, was found to be negligible; so were modulations of ν_e. Collisional momentum transfer from the electrons to the neutrals also is unimportant, the amplitudes of longitudinal electron motion being only of the same order of magnitude as the ion amplitudes.

Substitution of Eq. (6) into Eq. (7), with neglect of terms of the order ω/ν_i, shows that the neutrals experience, indirectly, the body force en_iE which moves the ions in the first place. This approximation may be substituted back into Eq. (6) to give explicitly v_i from E. The continuity equations for the density fluctuations δn_i and δn_e,

$$\delta n_i/n_i=kv_i/\omega,\quad \delta n_e/n_e=kv_e/\omega', \tag{8}$$

have already been implied in the evaluation of the pressure fluctuations on the left side of Eqs. (6)

and (7). They serve to deduce the charge density fluctuations needed in Poisson's equation

$$-ik\epsilon_0 E = e(\delta n_i - \delta n_0), \tag{9}$$

which closes the system of equations and yields the dispersion formula

$$\frac{1}{i\omega(i\omega+\nu_i)m_i + k^2KT} + \frac{n_i/n_0}{i\omega(i\omega+\delta)m_0 + 5k^2KT/3}$$

$$= -\frac{1}{i\omega'(i\omega' m_e'' + \nu_e m_e') + 5k^2KT/3} - \frac{\epsilon_0}{n_i e^2}. \tag{10}$$

Here

$$m_e' = m_e/[\cos^2\theta + m_e^2\nu_e^2/e^2B^2]$$

and

$$m_e'' = m_e' \frac{(\cos^2\theta - m_e^2\nu_e^2/e^2B^2)}{(\cos^2\theta + m_e^2\nu_e^2/e^2B^2)}$$

represent effective electron masses which reflect the inhibition of electron motion by the magnetic field to the required accuracy ($|\omega'| \ll \nu_e$; $m_e\nu_e/eB$ is small compared with 1 but not necessarily with $\cos\theta$).

Formula (10) allows the establishment of conditions at which damping changes to growth, i.e., under which ω is real for real k. A very slight departure from these conditions suffices to produce growth, since the second term on the left, describing the dynamics and damping of neutral sound, is rendered unimportant by the smallness of n_i/n_0. Ignoring it, as well as terms of the order $(\omega/\nu_i)^2$, a comparison of the (dominant) imaginary parts in Eq. (10) leads to the determination of an angle at which undamped propagation can take place:

$$\frac{u_\parallel}{\omega/k} - 1 - \frac{\nu_i m_i \nu_e m_e}{(eB)^2} = \frac{\nu_i m_i}{\nu_e m_e}\cos^2\theta. \tag{11}$$

Such an angle exists subject to Eq. (2) and deviates very little from 90° for drifts which exceed the threshold by only moderate factors. Comparing real parts, and inserting the angle given by Eq. (11), leads to Eq. (3).

The author would like to thank Professor V. R. Eshleman and Professor A. M. Peterson for suggesting the problem, for making available experimental data, and for helpful discussions.

*This work was supported in part by the Air Force Cambridge Research Laboratories and in part by the Office of Naval Research.

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THERMAL BOUNDARY RESISTANCE BETWEEN SOME SUPERCONDUCTING AND NORMAL METALS*

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(Received 21 February 1963)

A thermal boundary resistance between a solid and liquid He^4 was first observed by Kapitza.[1] It has recently been found that this resistance between a metal and liquid He^4 is greater when the metal is in the superconducting state than when it is normal.[2,3] Little anticipated this result[4] and suggested[5] that the thermal boundary resistance between two different metals would be different when one of the metals was in the superconducting state than when they were both normal. The purpose of this paper is to report measurements of the thermal resistance at the boundary between tin and copper and between lead and copper for cases in which the tin and lead are superconducting and normal.

Figure 1 shows the arrangement of the sample