Table I gives the velocities of the gamma rays in the θ and θ' directions for different positron energies, as calculated by classical vector addition.

The upper curve in Fig. 2 gives the spectrum of times recorded for annihilation at rest, i.e., recorded at 180°. The position of the peak is indicated by the dashed line on the right. If in the case of annihilation in flight the velocity of the gamma rays were constant, the peak would occur in the same place. The dashed line on the left marks the place where the peak would be if the velocity of the gamma rays added on to the positron velocity by classical vector addition. The lower curve gives the experimental results obtained for annihilation in flight.

In order to check that annihilation in flight was being measured and not annihilation at rest, a second measurement was performed without the Perspex and no peak was obtained at all. We also calculated that the number of counts fitted well with what is known of the number of annihilations in flight.[5,6] The experiment was repeated four times under four different sets of conditions (varying θ, θ', and channel number), and the same results were found.

It is clear from the graph that the velocity of the gamma rays is constant (±10%), in accordance with the second postulate of the special theory of relativity, and does not add on to the velocity of the source.

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HOT-ELECTRON PLASMA BY BEAM-PLASMA INTERACTION

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The object of this Letter is to report the current experimental progress of one application of beam-plasma interaction. Plasma-electron heating is considerably greater in this experiment than in previous experimental observations.[1,2] A 5-keV direct-current electron beam interacting with a plasma of deuterium has yielded x-ray photons of energy up to 250 keV from the plasma region. An electron temperature of 32 keV and an electron density of 4×10^{11} electrons cm^{-3} are inferred from the total photon flux and its energy distribution. The plasma existed in the steady state and was contained in a magnetic mirror system with the electron stream on the axis. A 3 to 1 mirror ratio was used with a 1500-gauss field at the midplane. The plasma $\beta = 8\pi nkT/B^2$, calculated at the midplane from the inferred electron temperature and density, was 0.2.

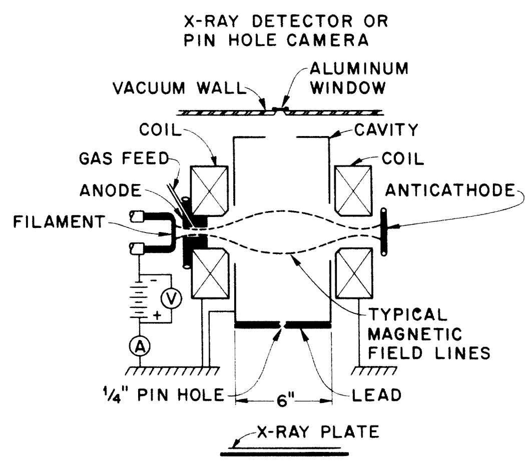

FIG. 1. A schematic diagram of the apparatus for producing the hot-electron plasma. Helium gas gave results similar to those described for deuterium. The cavity is cylindrical, 12 inches in diameter and 6 inches long; other dimensions may be scaled approximately.

A schematic diagram of the apparatus is shown in Fig. 1. Both the plasma and the electron beam were formed by apparatus similar to that used for the pressure gradient arc, sometimes called mode II.[3] The pressure gradient is produced by feeding gas into the aperture in the anode, which defines the electron beam, producing a local region with a pressure of about 10^{-3} Torr. The pressure between the mirrors is generally a factor of 100 lower. The electron beam (up to 500 mA) forms a dense plasma which may extend beyond the anode into the region between the magnetic mirrors. Previous experiments have shown that this plasma may be traversed by ionic sound waves[3] and emit ions random in energy and direction. A secondary plasma forms which has been shown to contain trapped deuterium ions of about 70 eV.[4]

X rays were first observed without the copper enclosure surrounding the plasma region. However, the addition of the enclosure enhanced the x-ray photon yield by at least a factor of 100. One possibility is that a resonance occurs between a cavity frequency and the electron cyclotron frequency. For example, the lowest frequency of a fundamental cavity mode was 0.76×10^9 cps, measured experimentally. Many other modes with higher frequencies were possible. The electron cyclotron frequency at the midplane of the apparatus was about 4×10^9 cps. As the frequencies of the cavity modes are increased by the presence of plasma, it is conceivable that even a fundamental mode of the cavity is in resonance with the electron cyclotron frequency. A resonance with the plasma frequency $(n_e e^2/\pi m_e)^{1/2} = 5 \times 10^9$ cps is also possible.

X-ray photographs of the plasma region were made using a pinhole camera technique. A camera used outside the vacuum chamber was a cube 25 cm on a side and was made of sheet lead. The "pinhole" was $\frac{1}{4}$ in. in diameter. Another x-ray photograph was made inside the vacuum chamber. Here, the "pinhole" was $\frac{1}{8}$ in. in diameter. Sheet lead covered the copper enclosure. The film was Eastman type AA x-ray film suitably shielded from heat and light.

Several x-ray exposures are shown in Fig. 2. A geometrical analysis of the x-ray photographs indicates that the central ellipsoidal area corresponds to x rays emitted from the volume of the plasma. The two x-ray sources at the ends of the plasma coincide with the entrances to the mirror coils. These x rays probably result from the bombardment of the metal walls by energetic electrons. The photographs showed that the x-ray flux from the plasma region was as dense as that from the walls.

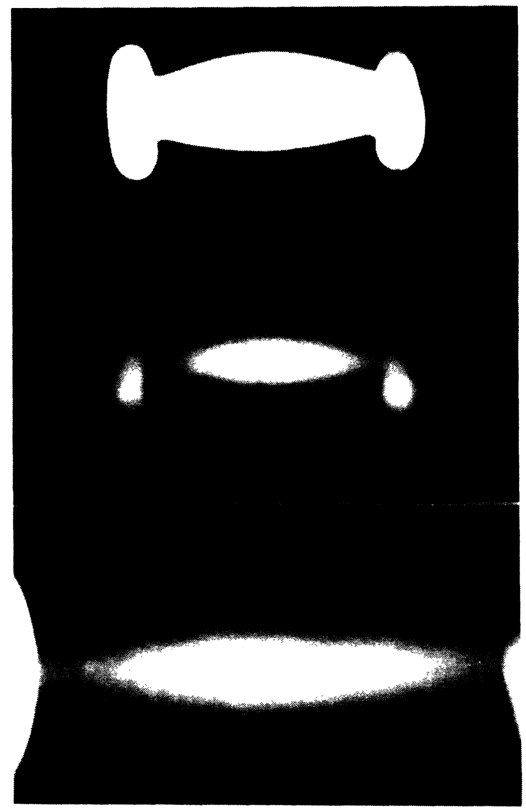

FIG. 2. X-ray photographs of the plasma. The upper two photographs were made with the "pinhole" camera outside the vacuum chamber and represent different exposure times. The lowest photograph was made with the "pinhole" and x-ray plate inside the vacuum chamber.

The energy spectrum and the total photon flux from the plasma region were measured by a sodium-iodide scintillation crystal, photomultiplier, and a 256-channel analyzer. A photon spectrum is shown in Fig. 3. The spectrometer was calibrated against a Cs^{137} source and had a channel width of 3 keV. The maximum photon energy was at least 250 keV. An additional check by an x-ray absorption technique supported the general features of the energy spectrum. The photon flux to the detector was measured through a total of one millimeter of aluminum. By assuming a point source and isotropic distribution, the total photon flux was conservatively estimated to be 10^{10} photons/sec.

Electron temperatures may be calculated from the x-ray spectrum using known formulas[5] if the plasma is fully ionized. The exponential section of the x-ray spectrum extends over a factor of 10^3 in intensity; if we associate its slope with

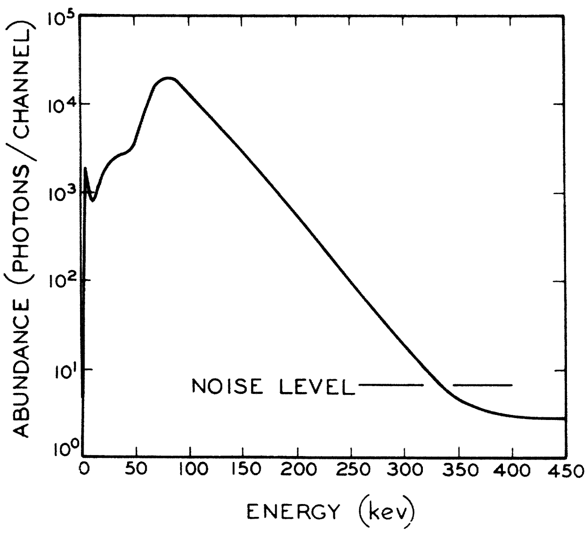

FIG. 3. X-ray energy distribution.

$-h\nu/kT_e$, we derive an electron temperature $T_e = 32$ keV.

The electron density can be inferred from the foregoing data[5] using the relation $\epsilon = 1.42\times 10^{-27} \times Z^3 n^2 T_e^{1/2}$, where the rate of energy emission ϵ takes the value 5.1 erg cm^{-3} sec^{-1} as indicated by the source strength and the energy spectrum. Inserting $Z = 1$ (deuterium) and a plasma volume of 100 cm^3 as derived from the pinhole image, together with $T_e = 32$ keV, we obtain a density value of $n = 4\times 10^{11}$ cm^{-3}. Contaminants with Z higher than 1 and lack of complete ionization may make this a slight overestimate.

Certainly, the plasma electrons have been heated beyond that expected from the applied voltage. Interplay of the natural plasma frequency with the electron cyclotron frequency and the frequency of resonant modes in the cavity may be a factor in the electron acceleration process, and is under investigation.

The following theory has been developed as a tentative explanation of these results. Let the vector potential of the cavity by expanded in a series of normal modes

$$\vec{A}(\vec{x}, t) = \sum_\lambda q_\lambda(t)\vec{A}_\lambda(\vec{x}). \tag{1}$$

The amplitudes, $q_\lambda(t)$, then satisfy

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = (1/c)\int \vec{J}\cdot\vec{A}_\lambda d^3x, \tag{2}$$

where ω_λ is the frequency of the λth mode and $\vec{J}$ is the current density. The electrons are assumed to be constrained by the magnetic field to move along the field. The current density is found from

$$\vec{J} = -e\int v f_1(v, z, t)dv, \tag{3}$$

where f_1 is the solution of the linearized one-dimensional Vlasov equation

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial z} = \frac{e}{m}\frac{\partial f_0}{\partial v}E_{\lambda z} = -\frac{e}{mc}\frac{\partial f_0}{\partial v}\dot{q}_\lambda A_{\lambda z}. \tag{4}$$

Only the modes customarily denoted[6] by TM_{mnp} with $m = 0$ have an electric field parallel to the magnetic field at the axis, so only these modes need be considered. Assuming that the time dependence of f_1 and q_λ is given by a factor $e^{i\omega t}$, Eq. (4) can be solved and the result used in Eq. (3). When this result is used in Eq. (1), one finds the dispersion relation

$$\omega^2 = \omega_\lambda^2 + \beta_\lambda \omega_p^2 \omega^2 \int f_0 \frac{\omega^2 + k_z^2 v^2}{(\omega^2 - k_z^2 v^2)^2} dv, \tag{5}$$

where

$$\beta_\lambda = \frac{\int_{V_p} |A_{\lambda z}|^2 d^3x}{\int_{V_c} |\vec{A}_\lambda|^2 d^3x}$$

and

$$k_z = p\pi/d, \quad p = 0, 1, 2, 3, \cdots;$$

ω_p is the plasma frequency, V_c is the volume of the cavity, and V_p is the volume occupied by the plasma; f_0 is normalized to unity. If $f_0 = \delta(v - V)$, then it can be shown that the system is unstable for $k_z V < \omega_\lambda$ which, in fact, is always true. The complex roots of Eq. (5) are given approximately by

$$\omega = \pm k_z V[1 \pm i(\tfrac{1}{2}\beta)^{1/2}\omega_p/\omega_\lambda]. \tag{6}$$

It seems likely that this instability produces large-amplitude oscillations in the cavity and that as a result, heating of the electrons by cyclotron resonance occurs.

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PHOTON CORRELATIONS*

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In an interesting recent note published in this journal, Glauber[1] refers to one of our papers[2] with a comment that a prediction made in that paper is misleading and follows from an inappropriate model of the maser beam. To the extent that we refer to the possible use of our results in connection with optical masers, our remarks were indeed misleading, for the results will certainly not apply to the output of a single, well-stabilized maser operating on one or a few modes. However, our paper was not at all concerned with constructing a model of a maser beam, but rather with showing how the state of coherence and polarization of light originating in thermal sources—and also the resolving time of the photoelectric detector—affects the kind of correlation first observed by Hanbury Brown and Twiss.[3]

The underlying assumption of our analysis (clearly stated on p. 1698 of reference 2) is that the probability distribution of the wave field is Gaussian, and this assumption is well-founded for light from thermal sources.[4] Glauber's observation that such an assumption is not appropriate for light from an ideal maser is not new.[5-7] It is likely, however, that this assumption is a reasonable first approximation if the light is obtained by superposing the outputs of a number of independent optical masers, as Glauber himself implies.

There is also an implication in reference 1 that the stochastic semiclassical description of light, initiated by Bothe[8] and Purcell[9] and developed further elsewhere,[10-12] ceases to be useful when the wave field cannot be represented classically by a Gaussian probability distribution. We do not believe this to be the case, and some support for our view, based on first-order perturbation theory, will be published elsewhere. For the analysis of many correlation and fluctuation experiments, the stochastic treatment has much to commend itself on account of relative simplicity and a strong physical appeal. There is actually a close connection between the description of light in terms of higher order correlation functions involving the classical field,[13,14] and the description involving ensembles of multiple photon states advocated by Glauber.[1,15] Moreover, on account of the very high value of the degeneracy parameter of maser light,[16,17] the stochastic semiclassical treatment may be expected to be even more appropriate than for light from thermal sources, for which the usefulness of the approach has already been amply demonstrated.

*Work supported by the Air Force Office of Scientific Research.

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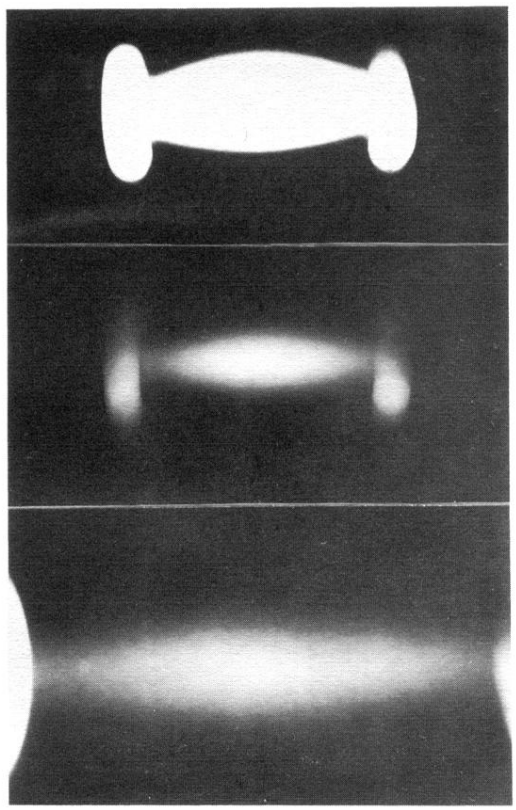

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