SOME EMPIRICAL SYSTEMATICS OF THE PION-NUCLEON SYSTEM*

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In the preceding Letter[1] experimental evidence for two bumps in the $\pi^- p$ total cross sections is given. The interpretation of these bumps as resonances due to isobars with masses 2.19 and 2.36 BeV and isotopic spins of $\frac{1}{2}$ and $\frac{3}{2}$, respectively, raises the number of observed nuclear excited states to seven. It is the purpose of this note to point out several empirical systematics of the quantum numbers and masses of these states. In what follows we label the observed pion-nucleon resonances in order of ascending mass by N_i, with the nucleon as N_0 and $i = 0, 1 \cdots, 7$.

Inspection of the quantum numbers of the nucleon and its three well-known isobars with masses 1238 MeV, 1512 MeV, and 1688 MeV suggests a possible selection rule. It relates I, the isotopic spin, J, the spin, and L, the orbital angular momentum, of each nuclear isobar in the following way:

$$J - L = I - 1. \qquad (1)$$

A study of the mass spectrum of the nucleon and three isobars mentioned above shows that the mass increases with increased L for fixed J, and with increased J for fixed L. One might then expect to find isobars with $I = \frac{1}{2}$, $p_{3/2}$, or $I = \frac{3}{2}$, $p_{1/2}$, or $I = \frac{1}{2}$, $d_{5/2}$, etc. None of these predicted isobars have been observed, in agreement with the empirical relationship (1). There are no known symmetry laws which would be violated by the existence of any of these isobars, but a possible explanation is that the spin-orbit potential between the pion and nucleon is strong and has opposite signs in the two isotopic spin states, i.e., could be written as $\propto (I-1)\vec{s}\cdot\vec{L}$.

It is possible to generate the quantum numbers for the eight isobars in the order of increasing mass by starting with $J = L - \frac{1}{2}$ and $J = L + \frac{1}{2}$, in that order, for the lowest L (1) and increasing L in unit steps to 4. The existence of an $L = 0$, $J = \frac{1}{2}$ isobar with isotopic spin $\frac{3}{2}$ is ruled out if the binding force between the pion and nucleon is solely due to a potential of the form suggested above, since it vanishes for $L = 0$. The results are shown in Fig. 1. Each circle represents an isobar; I, J, and L have their usual meaning, and M is the mass of the isobar expressed in MeV. To help in the identification of the resonances, we have included K, the corresponding approximate pion laboratory kinetic energy. It might be pointed out that half of the possible J, L, and I combinations are rejected by relation (1). All masses are taken directly from Rosenfeld's compilation[2] except for N_3, N_6, and N_7 which are not given. The N_3 mass is taken as 1.65 BeV and is seen as a shoulder in $\pi^+ - p$ total cross sections at 860 MeV. N_6 and N_7 masses are taken as 2.19 and 2.36 BeV.[1] All quoted masses are, of course, subject to experimental errors. These have been shown in Fig. 1 but, for the sake of brevity, have been omitted in the text.

Experimental results[3] strongly favor an assignment of either $f_{7/2}$ or $g_{7/2}$ for the resonance at 1.35 BeV. The scheme proposed here assigns $f_{7/2}$ to the state. It furthermore assigns the states $d_{5/2}$, $g_{7/2}$, and $g_{9/2}$ to N_3, N_6, and N_7, respectively. The selection of $d_{5/2}$ for N_3, together with that of $f_{5/2}$ for N_4, gives a possible explanation of the $\pi^- p$ elastic distribution around 900 MeV (see reference 3). It may be noted that in Fig. 1 each vertical pair of isobars differ only by two units of spin and can lie on the same Regge trajectory.

In addition to the systematics of the quantum numbers which we have already noted, several phenomena appear in the mass spectrum. If we write the mass difference between N_i and N_j

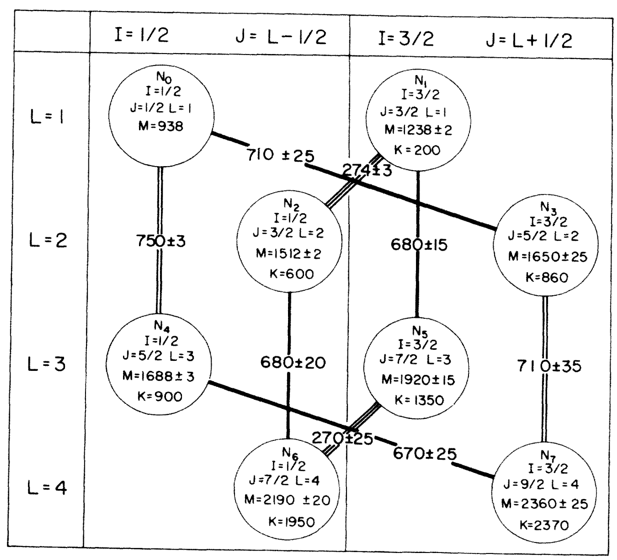

FIG. 1. Systematics of the pion-nucleon system.

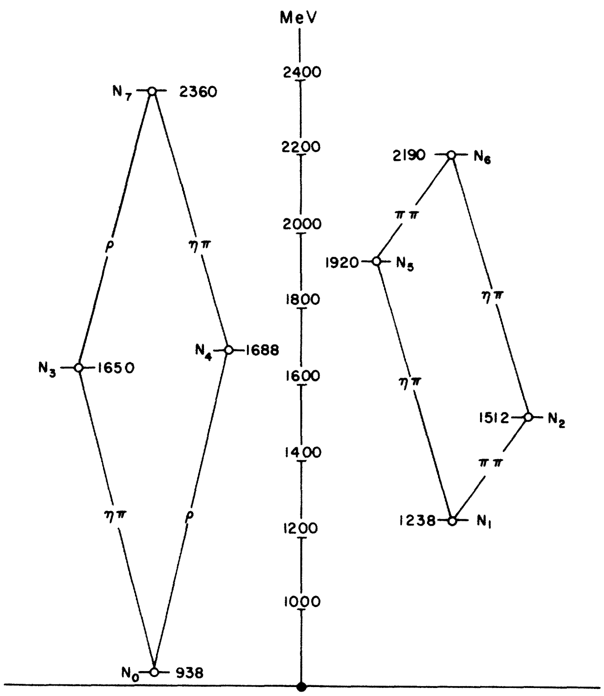

FIG. 2. The pion-nucleon "mass-level spectra."

as Δ_{ij}(MeV), we notice that

$$\Delta_{26} = 680 \pm 20, \quad \Delta_{15} = 680 \pm 15, \tag{2i}$$

$$\Delta_{12} = 274 \pm 3, \quad \Delta_{56} = 270 \pm 25, \tag{2ii}$$

$$\Delta_{04} = 750 \pm 3, \quad \Delta_{37} = 710 \pm 35, \tag{2iii}$$

$$\Delta_{03} = 710 \pm 25, \quad \Delta_{47} = 670 \pm 25, \tag{2iv}$$

i.e., that several pairs of mass differences are equal within the errors quoted. Further, these mass differences are, within errors, equal to the masses of three $I = 1$ systems: (2i) and (2iv), $\eta^0 + \pi^\pm$; (2ii), $\pi^0 + \pi^\pm$; (2iii), ρ.[4] Of course, the Δ_{ij} are not completely independent, and a change in any one mass affects two of the Δ_{ij}. When better mass values for the higher resonances are available, a more critical test of the above conclusions will be possible.

Using these observations, we can divide the eight isobars into two quadruplets and represent them as two "mass-level spectra." This is done in Fig. 2, where the symbols η, π, and ρ are to be understood as representing the masses of the corresponding particles. The selection rule for this scheme appears to be $\Delta J = 0, 2$.

It should be emphasized that the representation of the nucleon isobars presented in this note is based upon purely empirical rules, which do, however, make predictions about some as yet unknown spins and parities. It is presented in the hope that future confirmation or contradiction of the suggested systematics in both the quantum numbers and the mass spectrum will lead to a better understanding of the pion-nucleon interaction.

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[1]A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, preceding Letter [Phys. Rev. Letters 10, 262 (1963)].

[2]A. H. Rosenfeld (unpublished).

[3]J. A. Helland, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and C. D. Wood, Phys. Rev. Letters 10, 27 (1963).

[4]Because of the large errors in Δ_{37} and Δ_{03}, both $\eta + \pi$ and ρ are possible assignments for these two differences. The choice made in the text has been guided by relationships (2i) and (2ii).