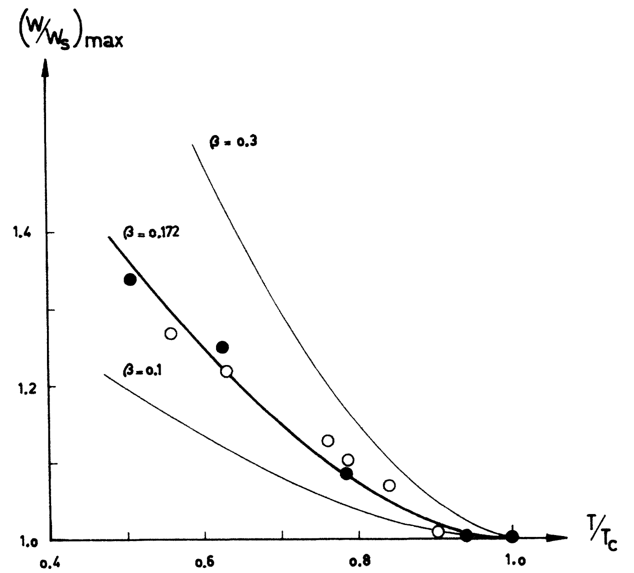

FIG. 2. The maximum of the thermal resistance in the intermediate state as a function of the reduced temperature with theoretical curves for various values of β. Closed circles, 0.186-mm specimen; open circles, 0.095-mm specimen.

variation with temperature of the energy gap $\epsilon_0(T)$ from the experimental data if β is known.

We are grateful to Professor G. Busch, the Director of the Laboratorium für Festkörperphysik, and to Professor P. Grassmann, the Director of the Institut für kalorische Apparate und Kältetechnik, for their encouragement. Our special thanks are due to Professor J. L. Olsen for suggesting this problem and for many helpful discussions and comments. We would also like to thank Professor A. Thellúng and Professor M. Fierz for for enlightening discussions. One of us (S.S.) is grateful to the Battelle Memorial Institute for financial support. The experimental work was supported by an Arbeitsbeschaffungs-Forschungskredit des Bundes.

---

*Now at the Department of Physics, University of California, Berkeley, California.

[1] J. K. Hulm, Phys. Rev. 90, 1116 (1953).

[2] N. V. Zavaritskii, Zh. Eksperim. i Teor. Fiz. 38, 1673 (1960) [translation: Soviet Phys.–JETP 11, 1207 (1960)].

[3] K. Mendelssohn and J. L. Olsen, Proc. Phys. Soc. (London) A63, 2 (1950); Phys. Rev. 80, 859 (1950).

[4] D. P. Detwiler and H. A. Fairbank, Phys. Rev. 88, 1049 (1952); J. L. Olsen and C. A. Renton, Phil. Mag. 43, 946 (1952); R. T. Webber and D. A. Spohr, Phys. Rev. 91, 414 (1953); C. A. Renton, Phil. Mag. 46, 47 (1955); K. Mendelssohn and C. A. Shiffman, Proc. Roy. Soc. (London) A255, 199 (1960).

[5] F. H. J. Cornish and J. L. Olsen, Helv. Phys. Acta 26, 369 (1953); S. J. Laredo and A. B. Pippard, Proc. Cambridge Phil. Soc. 51, 369 (1955).

[6] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[7] J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. 113, 982 (1959).

[8] P. Wyder (to be published).

[9] M. Tinkham, Low-Temperature Physics (Gordon and Breach Science Publishers, New York, 1962).

[10] We are grateful to Professor Fierz for pointing out that if the structure of the intermediate state were perfectly periodic, the additional reflection would not reduce the mean free path at all, but would only lead to a change in the dispersion relation.

[11] J. L. Olsen, Proc. Phys. Soc. (London) A65, 518 (1952).

[12] P. Rhodes, Proc. Roy. Soc. (London) A204, 396 (1950).

[13] J. L. Olsen and P. Wyder, Helv. Phys. Acta 32, 311 (1959).

[14] E. M. Lifshitz and Yu. V. Sharvin, Dokl. Akad. Nauk S.S.S.R. 79, 783 (1951).

---

de HAAS–van ALPHEN EFFECT AND INTERNAL FIELD IN IRON*

J. R. Anderson[†] and A. V. Gold
Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa
(Received 21 February 1963)

We wish to report the discovery of long-period de Haas–van Alphen oscillations in iron, to our knowledge the first time that the effect has been observed in a ferromagnetic metal; its discovery puts an end to the speculation that there might be some intrinsic reason whereby the existence of ferromagnetism would inhibit the effect in principle.[1] The present preliminary results shed but little light on the nature of the Fermi surface; indeed, major improvements in technique will be necessary in order to carry out a detailed study. However, the results obtained so far suggest that the usual theory of the de Haas – van Alphen effect[2] can be applied to conduction electrons in a ferromagnetic metal when the applied field H is replaced by the total field B.

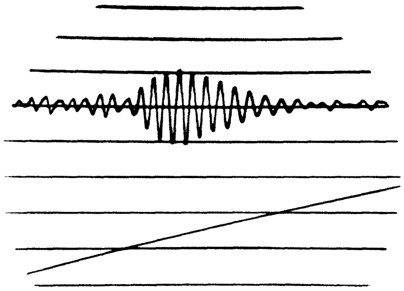

FIG. 1. Upper trace: de Haas–van Alphen oscillations in a [110] Fe whisker. Lower trace: magnetic field; the horizontal calibration lines are in steps of 11.41 kG, lowest line 22.82 kG. Peak field 115 kG, pulse duration 15 msec. Time increases from right to left; total sweep 2 msec.

The iron single crystals were "whiskers" grown by the Brenner method[3] by Dr. R. W. DeBlois of General Electric Research Laboratory, Schenectady, New York. The whiskers were of 8-mm length and about 0.1 mm thick, with their tips rounded by electropolishing,[4] and were found to have resistance ratios $\rho_{293°\text{K}}/\rho_{4.2°\text{K}}$ lying between 200 and 300. The de Haas–van Alphen oscillations were observed between 1 and 2°K, using the impulsive-field technique under resonance conditions,[5] with the whiskers oriented parallel to the field. Figure 1 shows the oscillations in a [110] whisker; they are of very low amplitude and, in fact, cannot be clearly observed without use of the resonance technique. The measurements reported here are for falling field only, since the rising-field oscillations are complicated by interference with the abnormally violent ringing of the pick-up circuit following the start of the pulse.

Figure 2 shows a plot vs integers n of the values of the reciprocal of the applied magnetic field H at the minima of the oscillations of Fig. 1. The points are always found to lie on smooth curves and not on straight lines, indicating that the oscillations are not strictly periodic in H^{-1}, as they would be for a nonferromagnetic metal. Instead, we assume that the oscillations have a constant period P_0 in $(H+H_0)^{-1}$, where H_0 is a constant internal field. If n is now regarded as a continuous variable, the field dependence of the slope $S=d(H^{-1})/dn$ of the integer plot (for either maxima or minima) should be given by

$$HS^{1/2}=P_0^{1/2}(H+H_0). \tag{1}$$

The oscillations may be analyzed most reliably by measuring the slope S_r at the maximum of the resonance where the signal-to-noise ratio is greatest; the position of the resonance maximum, H_r, can be made to occur between 40 kG and 70 kG by varying the resonant frequency of the pick-up circuit and the profile of the field pulse. In Fig. 3 we give a plot of $H_r S_r^{1/2}$ vss H_r for [100] and [110] whiskers; within experimental accuracy we find a linear relation in accordance with Eq. (1), and a least squares fit gives the following values of P_0 and H_0:

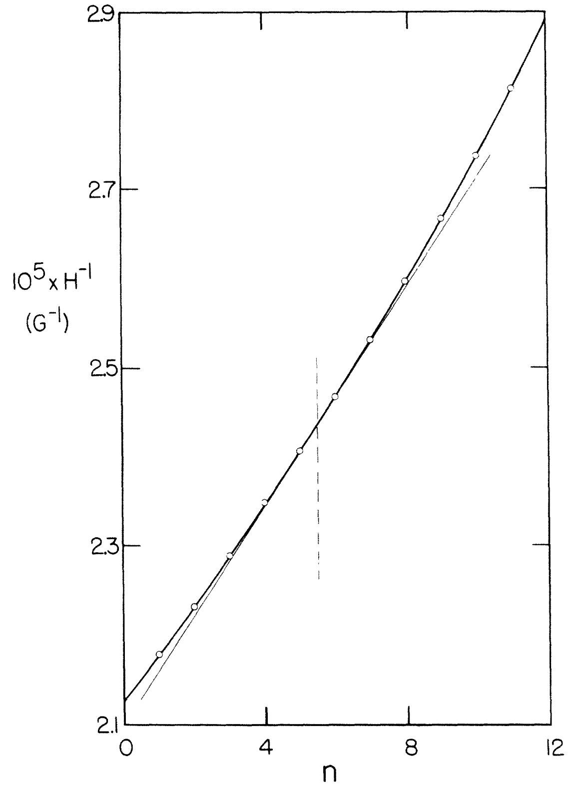

FIG. 2. Plot of H^{-1} at the minima of the oscillations in Fig. 1 vs integers n. S_r is the slope of the curve at the resonance maximum.

Orientation	$10^7 P_0$ (G^{-1})	H_0 (kG)
[100]	2.16 ± 0.15	21.4 ± 1.9
[110]	2.71 ± 0.16	21.9 ± 1.7

When the oscillations are analyzed in the above manner, the variation in phase across a resonance gives rise to systematic errors in P_0 and H_0. Rough estimates of this effect suggest that the periods may be too large by as much as 10 % and that the values of H_0 may be too small by a similar amount. The systematic errors arising from eddy currents[6] and demagnetizing ef-

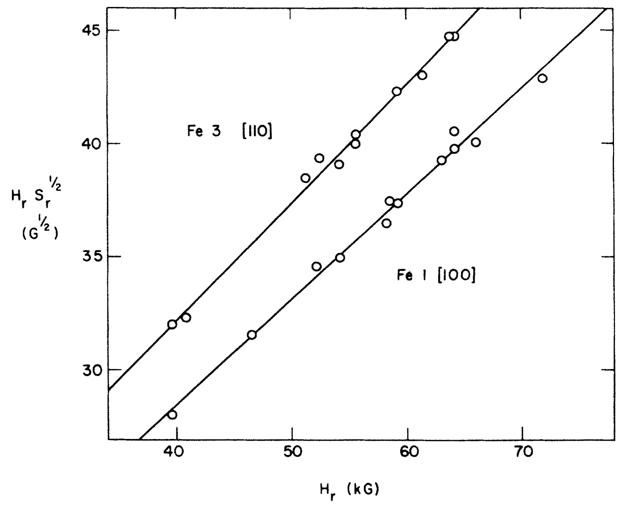

FIG. 3. Variation of $H_r S_r^{1/2}$ with H_r.

fects should be small.

The values of H_0 agree well with the saturation magnetization $4\pi M_S = 21.8$ kG,[7] and within experimental error the oscillations appear to be strictly periodic in $B^{-1} = (H + 4\pi M_S)^{-1}$. Our findings thus suggest that the usual theory of the de Haas–van Alphen effect[2] can be carried over to a ferromagnetic metal by simply replacing H by the total field B; this, in turn, would imply that the field quantity entering into the Lorentz force acting on a conduction electron in iron is $\vec{B}$ rather than $\frac{1}{2}(\vec{B}+\vec{H})$.[8] Periods of order 10^{-7} G^{-1} reflect relatively small parts of the Fermi surface, and H_0 is, in fact, the average internal field experienced by a conduction electron in a large real-space orbit with mean radius of roughly 500 lattice spacings at $H = 50$ kG. There also appears to be some poorly resolved oscillations of still longer period ($\sim 10^{-6}$ G^{-1}), but so far pulses up to 200 kG have revealed no clear evidence of faster oscillations associated with larger sections of the Fermi surface.

Walmsley[9] has measured a linear shift in the Fermi level of iron with applied magnetic field. A linear shift will give rise to a field dependence in the period of the oscillations given by

$$P(H) = P(0)/(1 + \alpha H),$$

where

$$\alpha = (dE_F/dH)(e\hbar/m_0 c)^{-1}(m^*/m_0)P(0).$$

The cyclotron mass m^* is found to be about $0.25 \times m_0$ from the temperature dependence of the amplitude of the oscillations. From this value and Walmsley's result, we estimate $\alpha H \cong 10^{-3}$ at $H = 50$ kG; the correction to the values of P_0 is thus negligibly small.

The stimulus to study the present iron whiskers came from a conversation with Dr. D. S. Rodbell. We are most grateful to Dr. DeBlois for giving us the high-quality whiskers, and to him and Dr. Rodbell for helpful discussions. We are also indebted to Dr. D. Shoenberg for pointing out an error in an earlier analysis, and wish to thank Mr. P. T. Panousis for help with the measurements.

---

*Contribution No. 1273. Work was performed in the Ames Laboratory of the U. S. Atomic Energy Commission.

†National Science Foundation Graduate Fellow. Present address: Royal Society Mond Laboratory, University of Cambridge, Cambridge, England.

[1] See review articles by D. Shoenberg and A. B. Pippard in The Fermi Surface (John Wiley & Sons, Inc., New York, 1960), pp. 82, 336, respectively. The de Haas–van Alphen effect has been previously observed in antiferromagnetic Cr [B. R. Watts (private communication)].

[2] L. Onsager, Phil. Mag. 43, 1006 (1952).

[3] S. S. Brenner, Acta Met. 4, 62 (1956).

[4] R. W. DeBlois, J. Appl. Phys. 32, 1561 (1961).

[5] D. Shoenberg, Progress in Low-Temperature Physics (North-Holland Publishing Co., Amsterdam, 1957), Vol. II, p. 226.

[6] B. Lüthi, Helv. Phys. Acta 33, 161 (1960), has observed nonsaturation of the transverse magnetoresistance in polycrystalline iron.

[7] R. M. Bozorth, Ferromagnetism (D. Van Nostrand Co., Inc., Princeton, New Jersey, 1951), p. 54.

[8] See D. L. Webster, Am. J. Phys. 14, 360 (1946); G. H. Wannier, Phys. Rev. 72, 304 (1947).

[9] R. H. Walmsley, Phys. Rev. Letters 8, 242 (1962).