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THERMAL CONDUCTIVITY AND ELECTRON SCATTERING AT INTERPHASE BOUNDARIES IN A SUPERCONDUCTOR

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It has long been realized that some scattering of electrons must occur at the normal-superconducting phase boundaries in the intermediate state. Such a scattering was suggested by Hulm[1] and in a modified form by Zavaritskii[2] to explain the maximum in the thermal resistivity in the intermediate state of superconductors discovered by Mendelssohn and Olsen[3] and since observed by several other workers.[4] A different mechanism has been shown to be responsible for the maximum where phonon conduction dominates in the superconducting state,[5] but such an additional scattering of electrons in the intermediate state is required to explain the maximum where electrons dominate the thermal conductivity in both the normal and the superconductive states. No calculation based on the Bardeen-Cooper-Schrieffer (BCS) theory[6] has, however, been reported so far for this case. We have now made such a calculation with the additional condition that the electronic mean free path in the normal state is longer than the scale, a, of the intermediate state structure. We have also made measurements on very pure indium specimens where these conditions hold, and find very good agreement with the predictions of our model.

Bardeen, Rickayzen, and Tewordt[7] in their calculation of the thermal conductivity of a superconductor make use of an expression which applies generally, and which is valid under the assumption that the electrons are scattered elastically. They state that an energy interval dE gives the following contribution to the thermal conductivity κ for a given energy E relative to the Fermi energy E_F:

$$d\kappa = -[2N(0)v_0/3T]l(E)E^2(\partial f/\partial E)dE.$$

Here, $N(0)$ is the density of states at the Fermi level, v_0 is the velocity of electrons at the Fermi surface in the normal state, T is the temperature, $l(E)$ is the mean free path as a function of the energy, and $f(E)$ is the Fermi distribution function. This expression is quite independent of the density-of-states function and only requires a knowledge of $l(E)$. We may therefore apply it to the intermediate state as a whole. The problem then arises of the value of l to be used. It has been shown by Bardeen, Rickayzen, and Tewordt[7] that under the condition of elastic scattering, $l(E)$ should be the same in the normal and in the superconducting states, and this has recently been proved directly by size-effect measurements by one of us.[8] In the intermediate state, however, we have to take into account the reduction of the mean free path caused by scattering at the phase boundaries.

To get an idea about this type of scattering, we compare a quasi-particle in the normal state with a BCS quasi-particle in the superconducting state.[9] The normal particle has only kinetic energy, Δ_n(KE), while the energy of the superconducting particle is composed of a kinetic energy, Δ_s(KE), and a potential energy, Δ_s(PE). For $\Delta_s(\mathrm{KE})=0$, Δ_s(PE) is just the ordinary BCS energy gap ϵ_0. For a transition without any phonon interaction of a particle through the phase boundary, the energy has to be conserved. If $\Delta_n(\mathrm{KE})<\epsilon_0$, the normal particle will be totally reflected at the boundary. If $\Delta_n(\mathrm{KE})>\epsilon_0$, a transition is possible, but then some kinetic energy has to be transformed to potential energy or vice versa. One must therefore also expect an additional scattering for particles with energies higher than ϵ_0.[10] One would expect that the energy region where the mean free path is strongly reduced by this mechanism would lie in the energy interval where the ratio of $\Delta_s(\mathrm{PE})/\Delta_s(\mathrm{KE})$ is large, $\epsilon_0<E\lesssim 2\epsilon_0$.

One has to distinguish between two cases: $a \gg l_n$, and $a \ll l_n$. For the first case, one can just add the thermal resistance of the normal and superconducting phases. In the intermediate state, the fraction of normal material varies linearly with magnetic field. One therefore expects a linear transition with magnetic field for the thermal resistance from the superconducting to the normal state as found in pure lead by Olsen.[11] In the present work we investigate just the opposite case where $a \ll l_n$. As a first approximation, we can then describe the mean free path as the following function of energy:

$$l(E)=0 \quad \text{for } E<\epsilon_0+\beta\epsilon_0,$$

$$l(E)=l \quad \text{for } E>\epsilon_0+\beta\epsilon_0,$$

where β is a temperature-independent parameter which lies between 0 and 1. An examination of other functions besides this step function shows that the results are not very sensitive to the special form of $l(E)$.

The maximum thermal resistivity of the intermediate state, W, divided by the thermal resistivity of the superconducting state, W_s, is now given by

$$(W/W_s)_{\max}=\int_{\epsilon_0}^{\infty}d\kappa\Big/\int_{\epsilon_0+\beta\epsilon_0}^{\infty}d\kappa=f(y)/f[(1+\beta)y],$$

where $y=\epsilon_0(T)/kT$ and $f(y)=2F_1(-y)+2y\ln(1+e^{-y})+y^2/(1+e^y)$. $F_1(-y)$ is given by

$$F_1(-y)=\int_0^{\infty}(1+e^{z+y})^{-1}dz,$$

which is tabulated by Rhodes.[12]

To test this prediction we have investigated two polycrystalline specimens of spectroscopically pure indium 40 mm long, and 0.186 mm and 0.095 mm in diameter. According to size effect data,[13] the mean free path, l_n, of the electrons was about 0.5 mm, and the period of the intermediate state structure, a, was calculated[14] to be about 3×10^{-2} mm and 2×10^{-2} mm, respectively. The thermal resistance divided by that in the superconducting state is given as a function of the reduced magnetic field $h=H/H_c$ for several temperatures in Fig. 1, and in Fig. 2 the maximum value of this ratio is plotted as a function of T/T_c.

In our calculation one undetermined parameter β is left. Comparing the experiments with this computation, we find a remarkable degree of agreement for both specimens throughout the temperature range investigated by using $\beta=0.172$. Inverting the procedure, one can also obtain the

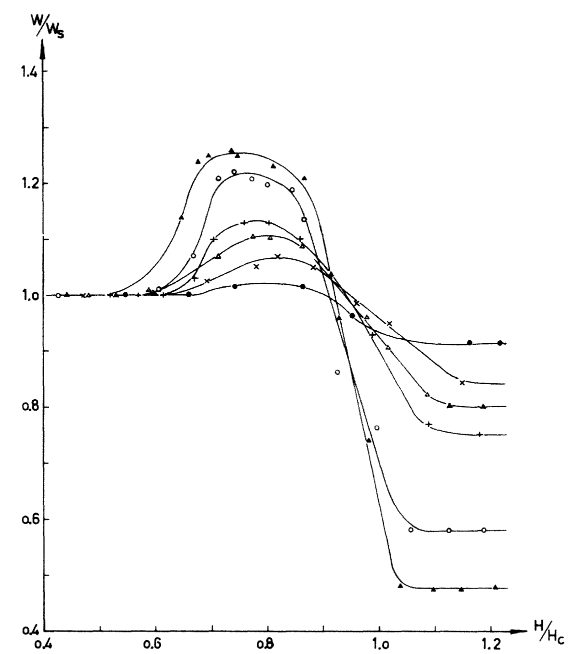

FIG. 1. The thermal resistance, W, in the intermediate state divided by its value in the superconducting state, W_s, as a function of the reduced magnetic field for different temperatures. Specimen diameter 0.095 mm. Closed circles, 3.04°K; exes, 2.82°K; open triangles, 2.68°K; crosses, 2.55°K; open circles, 2.12°K; closed triangles, 1.87°K.

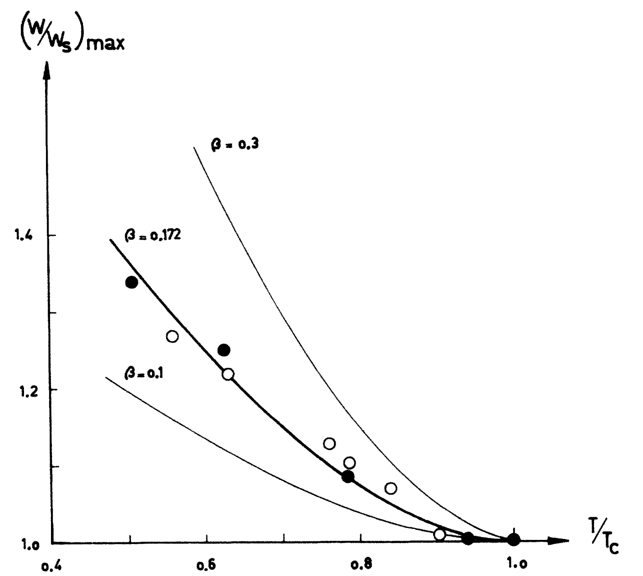

FIG. 2. The maximum of the thermal resistance in the intermediate state as a function of the reduced temperature with theoretical curves for various values of β. Closed circles, 0.186-mm specimen; open circles, 0.095-mm specimen.

variation with temperature of the energy gap $\epsilon_0(T)$ from the experimental data if β is known.

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de HAAS–van ALPHEN EFFECT AND INTERNAL FIELD IN IRON*

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We wish to report the discovery of long-period de Haas–van Alphen oscillations in iron, to our knowledge the first time that the effect has been observed in a ferromagnetic metal; its discovery puts an end to the speculation that there might be some intrinsic reason whereby the existence of ferromagnetism would inhibit the effect in principle.[1] The present preliminary results shed but little light on the nature of the Fermi surface; indeed, major improvements in technique will be necessary in order to carry out a detailed study. However, the results obtained so far suggest that the usual theory of the de Haas–van Alphen effect[2] can be applied to conduction electrons in a ferromagnetic metal when the applied field H is replaced by the total field B.