# INSTABILITY OF ELASTIC WAVES BY TIME-VARYING ELASTIC MODULUS IN FERROMAGNETS

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We report here the prediction and observation of a new type of instability of elastic waves in magnetic crystals due to an rf magnetic field applied parallel to the dc magnetic bias field. The purpose of this Letter is to describe the origin of the effect, to present a calculation of the instability threshold, and to give experimental confirmation of the prediction using europium iron garnet.

This instability has been observed for frequencies much less than the frequency at which the transverse elastic-wave spectrum intersects the spin-wave spectrum,[1-3] and thus the waves are predominantly elastic with a small magnetic (spin-wave) admixture. The effect of a spin wave with the same value of the wave number as a shear elastic wave propagating parallel or perpendicular to the magnetic bias field is to add a magnetic contribution to the elastic modulus ($c_{44}$ in isotropic media). This part of the elastic modulus depends on the dc bias field and can thus be modulated with a uniform rf magnetic field parallel to the dc field. This time-varying elastic modulus can then parametrically excite elastic waves propagating in opposite directions with a frequency equal to half the modulating frequency.

In the following we neglect exchange torque, which typically restricts the analysis to wavelengths greater than $10^{-4}$ cm. The equation of motion for the elastic displacement vector $R_z$, parallel to the magnetic field $H$, can be derived using the coupled equations for the magnetic-elastic system,[1-3] assuming shear strain with variations in the $x$ direction, with $\partial/\partial z = \partial/\partial y = R_x = R_y = 0$; and assuming $e^{j\omega t}$ time dependence, we find

$$-\omega^2 \rho R_z + j\omega\sigma R_z = c_{44}\partial^2 R_z/\partial x^2, \qquad (1)$$

where $\rho$ is the density, $\sigma$ is a viscous damping parameter, and $c_{44}$ is the elastic modulus, given by $c_{44}^0 + c_{44}^1(H)$. The unperturbed elastic modulus is $c_{44}^0$. In the case where the medium is isotropic or the [100] axis is along the magnetic field, the magnetic contribution $c_{44}^1(H)$ is given by $\gamma b_2^2/M_S(\omega - \gamma H)$, where $b_2$ is the magnetoelastic coupling constant, and $M_S$ is the saturation magnetization. If $H$ has a dc part $H_0$ and a small rf contribution $2h\cos\omega_p t$, we can expand $c_{44}^1(H)$ about $H_0$ (with $\omega \ll \gamma H_0$) and retain the first two terms. This transforms the wave equation into one with a time-varying coefficient. Plane-wave solutions can then be found which apply to the case of shear waves confined to an infinite plate (dc field in the plane) with appropriate boundary conditions. By constructing the solution from forward and backward traveling waves, $R_z = R_{+k}e^{-jkx} + R_{-k}e^{jkx}$, we find that the rf field $h$ parametrically couples $R_{+k}$ and $R_{-k}$. In the steady state the resulting secular equation applies only at the oscillation threshold, given by

$$h_{\text{crit}} = \frac{[(\omega_{\text{res}}^2 - \omega^2)^2 + (\omega^2/Q)^2]^{1/2}}{\omega_{\text{res}}^2 \partial c_{44}(H_0)/\partial H} c_{44}. \qquad (2)$$

The $Q$ and resonant frequency of the mode have elastic and magnetic contributions as discussed by LeCraw and Kasuya,[4] and $\partial c_{44}(H_0)/\partial H$ is given by $b_2^2/M_S H_0^2$ in isotropic media. Note that in a disc $c_{44}^{-1}\partial c_{44}/\partial H_0 = 2\nu_{\text{res}}^{-1}\partial\nu_{\text{res}}/\partial H_0$, where $\nu_{\text{res}}$ is the resonant frequency of a thickness shear mode. This expression is valid for any orientation of the bias field with respect to the crystal axes. In case $\omega$ equals $\omega_{\text{res}}$, the minimum threshold is given by $\nu_{\text{res}}[2Q\partial\nu_{\text{res}}/\partial H_0]^{-1}$.

To estimate the threshold, we use measurements of the resonant frequency and $Q$ vs $H_0$ on a single-crystal sphere of EuIG ($d = 0.065$ in.), with $H_0$ along [111] for the lowest-order oblate-prolate elastic mode, as shown in Fig. 1. The measurement technique is similar to that previously reported.[4,5] Assuming that the same frequency change and $Q$ would be observed in a plate at $H_0$ (operating), we calculate $h_{\text{crit}}(\text{min}) = 0.09$ Oe.

The EuIG sphere used for the data in Fig. 1 was placed in an evacuated tube and excited with a small coil with one component of its magnetic field along $H_0$, and the pump frequency was adjusted to approximately twice the resonant frequency of the oblate-prolate mode. At a well-defined threshold the mode was excited into oscillation as detected by the transverse component of magnetization at $\frac{1}{2}\omega_p$. The variation of the threshold with pump frequency ($\nu_p = 2\nu_{\text{res}} + \Delta\nu_p$) is shown in Fig. 2 and compared to the

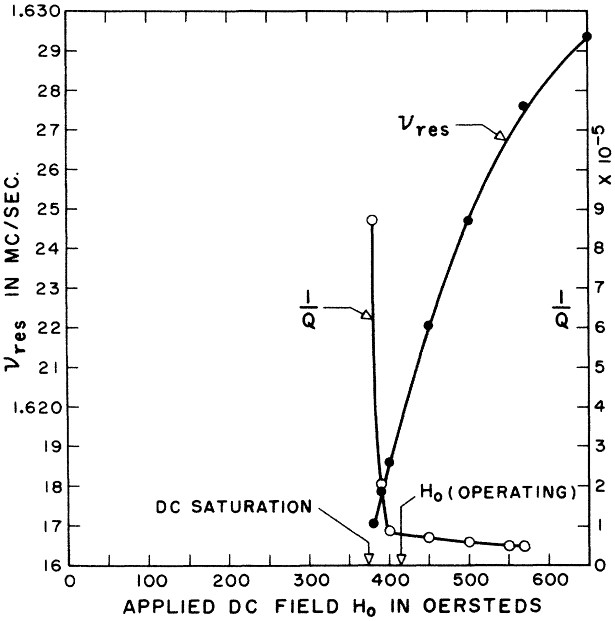


FIG. 1. Plots of $\nu_{res}$ and $Q^{-1}$ vs $H_0$ for a single-crystal EuIG sphere, with $H_0$ along the [111] axis for the oblate-prolate mode. The $H_0$ (operating) point was chosen by maximizing the product $Q\nu_{res}^{-1}\partial\nu_{res}/\partial H_0$.

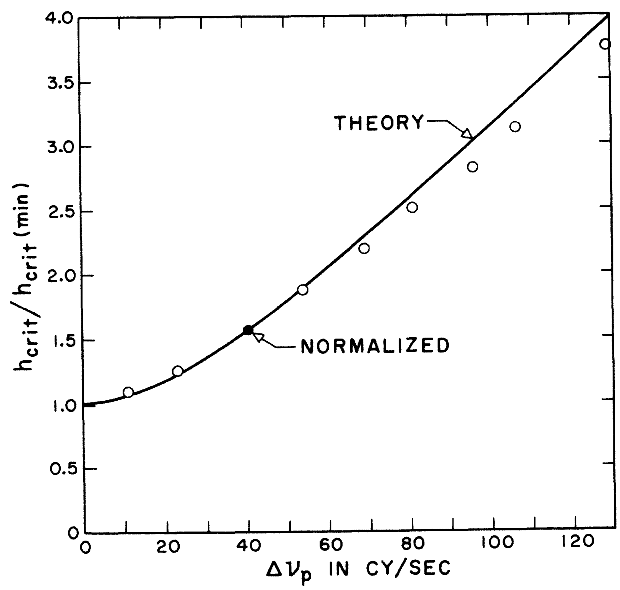


FIG. 2. Comparison of theory and experiment for $h_{crit}$ vs $\Delta\nu_p$.

theoretical curve [Eq. (2)]. The minimum threshold $h_{crit}$(min) was measured as approximately 0.20 Oe. It can be shown for the oblate-prolate mode vibrating with its axis displaced from the $z$ axis (which is the case in our experiments) that the displacement has shear components similar to those in the lowest-order thickness shear mode of a plate. Thus, we expect a comparable though higher threshold for this mode, as shown by the relatively good agreement between the theoretical and observed thresholds.

We expect that this effect can be observed at higher frequencies on either branch of the coupled-mode spectrum, and that also by mechanically loading the elastic mode, acoustic amplification should be possible.

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## ELECTRON PARAMAGNETIC RESONANCE INVESTIGATION OF THE VACANCY IN DIAMOND


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Griffiths, Owen, and Ward[1] reported that diamonds exposed to reactor neutrons developed an intense isotropic electron paramagnetic resonance (EPR) absorption line whose $g$ value was very close to that of the free electron. They found that a similar line was produced by 1-MeV electrons. The work herein reported has established that this absorption is composed of at least three superimposed lines. Following the precedent established for silicon,[2] they are called the diamond $A$, $B$, and $C$ centers. The widths and $g$ values of these lines are listed in Table I. The $A$- and $B$-center absorptions are observed with the reference phase of the lock-in detector in