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GROWING HELICAL DENSITY WAVES IN SEMICONDUCTOR PLASMAS

C. E. Hurwitz and A. L. McWhorter
Lincoln Laboratory* and Electrical Engineering Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts
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Spatially growing screw-shaped waves of electron-hole density have been excited in a semiconductor subjected to sufficiently large parallel electric and magnetic fields, in accordance with earlier theoretical predictions.[1] The phenomenon is closely related in physical mechanism to the oscillistor[2-5] (and also to a helical type of instability in gas discharges[6-9]), but differs from the oscillistor in that the observed effect is a unidirectional traveling-wave amplification, rather than an uncontrolled oscillation, and because no injected or light-generated plasma is involved. The growth rates, frequencies, and phase characteristics of the waves are in reasonable agreement with theoretical calculations.

A $1\times1\times25$-mm bar of 30 ohm-cm n-type germanium, with a bulk lifetime of 1400 μsec and a low-recombination surface, was supplied with five pairs of noninjecting n^+ probes, as shown in the inset of Fig. 1. The bar was placed in a longitudinal dc magnetic field H_0, and an electric field E_0, pulsed to avoid heating, was applied by means of Ohmic end contacts. The wave was excited with a signal generator using various probe combinations, and observed in magnitude and phase at the other probes as it traveled down the bar.

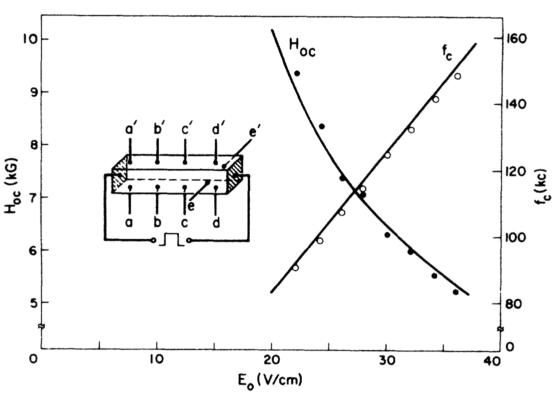

FIG. 1. Critical magnetic field H_{0c} and critical frequency f_c vs E_0. Points are experimental, solid curves are theoretical. Inset shows probe arrangement of sample.

The predicted helical wave, which would be of the form $\exp[i\omega t - i(k_r + ik_i)z \pm i\phi]$ for a cylindrical sample, travels at approximately the ambipolar drift velocity and is right- (left-) handed for H_0 parallel (antiparallel) to E_0. In confirmation of the helical nature, excitation at probes aa' produced a signal at dd' which led that at ee' by 90° for $E_0 \| H_0$, and which lagged by 90° when H_0 was reversed. Furthermore, if probes dd' and ee' were simultaneously excited 90° out of phase, only the 90° shift corresponding to the predicted screw sense produced a detectable traveling wave. Other measurements confirmed the expected phase velocity and, for not too strong excitation, the exponential variation in the longitudinal direction. The behavior of the growth constant k_i is rather complicated since it increases monotonically with both E_0 and H_0 for a fixed frequency f, while for fixed E_0 and H_0 it exhibits a maximum as a function of f, with the optimum frequency also increasing with E_0 and H_0. Hence, as H_0 is increased for a given E_0, there is a critical magnetic field H_{0c} and a critical frequency f_c at which growth is first observed. The experimental values of f_c and H_{0c} as a function of E_0 are shown in Fig. 1, while Fig. 2 gives k_i as a function of H_0 for one value of E_0 and $f = f_c$. It should be mentioned that k_i falls off very rapidly if H_0 deviates even a few degrees from the longitudinal axis and that, in agreement with the theory, no terminal VI oscillations are observed.

Before presenting the theoretical results, we give a physical picture of the growth mechanism based on Hoh and Lehnert's[7,8] discussion of the helical instability in gas discharges. A small

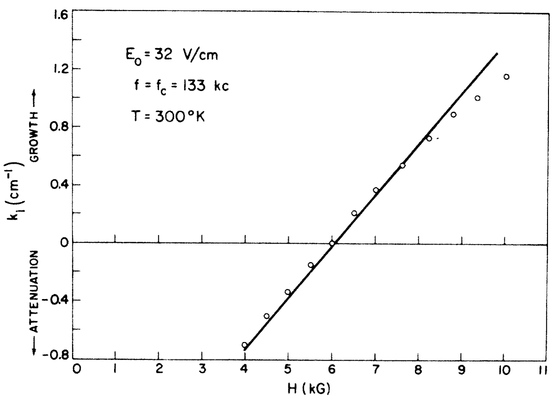

FIG. 2. Growth constant k_i vs H_0 for fixed E_0 and f. Points are experimental, solid curve is theoretical.

screw-shaped density perturbation of positive and negative carriers is assumed to be superimposed on the steady-state unperturbed distribution. The longitudinal field E_0 tends to separate the positive and negative screws axially, which is equivalent to a rotation of one screw relative to the other. The resulting charge separation creates an azimuthal electric field. In a linear approximation this field, together with H_0, acts on the unperturbed distribution and produces a radial flow of particles. If the unperturbed distribution has a radial gradient, as it does in a gas discharge or the usual form of the oscillistor, this flow is not divergenceless and hence can feed particles from the main distribution into the screw with the proper phase, thus producing a growth of the perturbation. It is worth pointing out that in a semiconductor a radial gradient of the unperturbed distribution can be produced in thermal equilibrium by proper doping, eliminating the need of creating a steady-state plasma. However, as indicated by the present experiment, an even simpler mode of operation requiring no radial gradient can exist in a semiconductor if the surface recombination is sufficiently low. In this mode, growth occurs because the radial flow piles up or depletes the carriers at the surface in the proper phase.

Since the analysis is similar to the previous instability treatments,[3,6] we merely sketch the method and give the results for the simplest situation. We assume a cylindrical sample of radius a, long in the z direction. The hole and electron continuity equations are linearized about equilibrium, quasi-neutrality is assumed, and the unknowns are Fourier analyzed in the form $f(r)\exp(i\omega t - ikz - im\phi)$. We further assume for both carriers that the Hall angle $\mu H/c \ll 1$, that the bulk lifetime $\tau_r \gg a^2/D$, and that the surface recombination velocity $s \ll D/a$. Then for the $m=1$ mode, which has the lowest threshold, we find that

$$\frac{H_{0c}}{c} = \frac{4\sqrt{3}}{a}\,\frac{kT}{e}\,\frac{(n_0+p_0)(n_0\mu_e+p_0\mu_h)}{n_0p_0(\mu_e+\mu_h)^2}\,\frac{1}{E_0} \qquad (1)$$

and

$$f_c = (1/\pi\sqrt{3}\,a)\mu_aE_0 - (20/9\pi a^2)(\mu_e-\mu_h)D_a(H_{0c}/c), \qquad (2)$$

where μ_a and D_a are the ambipolar mobility and diffusion constant, respectively. The motion of the helix is a superposition of a bodily translation with the ambipolar drift velocity, represented by the first term in (2), and a rotation due to carrier diffusion across the magnetic field, represented by the second term. The translational motion will dominate unless the ambipolar mobility is extremely small, i.e., for nearly intrinsic material.

Expressions for k have been obtained near threshold for the case when $[(n_0-p_0)/(n_0+p_0)]^2 \gg [(\mu_e+\mu_h)H_0/3c]^2$, which will be satisfied for all but nearly intrinsic material. For the forward traveling wave,

$$k = \frac{2\pi f}{\mu_aE_0} + i\frac{8}{3a^2}\left(\frac{n_0+p_0}{n_0-p_0}\right)\frac{kT}{eE_0}\left[2\frac{fH_0}{f_cH_{0c}} - \left(\frac{f}{f_c}\right)^2 - 1\right], \qquad (3)$$

while the reverse traveling wave is strongly attenuated and plays no role.

Calculated curves for H_{0c}, f_c, and k_i are plotted in Figs. 1 and 2. Mobilities $\mu = 3900$ cm^2/V-sec and $\mu_h = 1900$ cm^2/V-sec were used.[10] The densities n_0 and p_0 were obtained from the reported value[10] of the np product, 5.76×10^{26} cm^{-6} at the operating temperature, and the measured sample resistivity. The resulting value of n_0 agreed within experimental error with that obtained by Hall measurements at 77°K. An effective radius $a = 0.56$ mm (the radius of a cylinder with the same cross-sectional area as the sample) was arbitrarily chosen. Agreement with the theory is quite satisfactory. The deviation between the theoretical and experimental values of the growth rate at fields above 7.5 kG is due to the breakdown of the assumption $\mu H/c \ll 1$. A calculation without this assumption provides good agreement over the whole range of magnetic field.

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ELECTRON-HYDROGEN PHASE SHIFTS JUST BELOW THE INELASTIC THRESHOLD

A. Temkin and R. Pohle*
Theoretical Division, Goddard Space Flight Center,
National Aeronautics and Space Administration, Greenbelt, Maryland
(Received 17 September 1962)

Burke and Schey[1] have recently calculated a resonance in the elastic scattering of electrons by atomic hydrogen just below the threshold (10.203 eV) for inelastic scattering. The basis of Burke and Schey's calculation was the close-coupling approximation with $1s$ - $2s$ - $2p$ states. The nomenclature refers to hydrogenic states, each of which is multiplied by an initially undetermined function. For S-wave[2] scattering their wave function can be written

$$r_1 r_2 \Psi_{cc} = [u(r_1)R_{1s}(r_2) + v(r_1)R_{2s}(r_2) + (1 \rightleftarrows 2)]P_0(\cos\theta_{12}) + \sqrt{3}\,[w(r_1)R_{2p}(r_2) + (1 \rightleftarrows 2)]P_1(\cos\theta_{12}), \quad (1)$$

where $R_{nl}(r)$ is r times the (nl) radial wave function of hydrogen. It is clear from (1) that this function is approximate in two ways. First, it contains only two of an infinite number of relative angular momenta $P_l(\cos\theta_{12})$. Second, the "coefficients" of the included P_l have a comparatively restricted form (which, however, is manifestly symmetric with respect to interchange of 1 and 2 corresponding to the necessary symmetry of singlet scattering, with which we shall be concerned here).

Clearly, the most general function containing P_0 and P_1 can be written

$$r_1 r_2 \Psi = \Phi_0(r_1 r_2)P_0(\cos\theta_{12}) + \sqrt{3}\,\Phi_1(r_1 r_2)P_1(\cos\theta_{12}), \quad (2)$$

where the two-dimensional functions Φ_0 and Φ_1 are required to have the correct symmetry. Substitution of (2) into the variational principle $\delta\int\Psi^*(H - E)\Psi d\tau = 0$ yields the following coupled set of partial differential equations[3] in the region $r_1 \geq r_2$ with appropriate boundary conditions[4,5]:

$$(\Delta_{12} + 2/r_2 + E)\Phi_0 = 2(3)^{-1/2} r_2 r_1^{-2}\Phi_1, \quad (3a)$$

$$[\Delta_{12} - 2(r_1^{-2} + r_2^{-2}) + 2r_2^{-1} - 0.8\, r_2^2 r_1^{-3} + E]\Phi_1 = 2(3)^{-1/2} r_2 r_1^{-2}\Phi_0. \quad (3b)$$

It has been the object of our nonadiabatic theory[4,5] to identify and attack directly these partial differential equations, and thus avoid the second category of approximation implicit in the close-coupling method. The purpose of this note is to report on results of calculations on Eq. (3) just below threshold, where Burke and Schey have found that the Ansatz (1) has led to a resonance.

The method of calculation presupposes an accurate solution of a zeroth order problem,

$$(\Delta_{12} + E + 2r_2^{-1})\Phi_0^{(0)} = 0, \quad (4)$$