out the whole region will provide a final answer to the spin question and information on the parity. In particular, the energy dependence of the $\Sigma^-\pi^+$ differential cross sections must be established carefully, because we cannot rule out the possibility that the large value of $A_2(\Sigma^-\pi^+)$ at 760 MeV/$c$ is due mainly to nonresonant terms. If this is the case, the value of $x_K$ would be much smaller, and this, in turn, would decrease our confidence in the spin assignment.

Finally, let us say a few words about the three-body final states. If $\Lambda\pi^+\pi^-$ is formed predominantly through the chain $K^-p \to \Sigma(1660) \to \Sigma(1385) + \pi \to \Lambda\pi^+\pi^-$, one could hope to say something about the spin and parity of $\Sigma(1660)$, taking $\Sigma(1385)$ to be $\frac{3}{2}^+$.[6-8] However, the observed cross section for $K^-p \to \Lambda\pi^+\pi^-$ at 760 MeV/$c$ is 4.3 mb, and $\Sigma(1660)$ can contribute only ~0.4 mb to that value since the normalized channel width for $\Sigma(1660) \to \Lambda\pi^+\pi^-$ is approximately 0.10. Therefore, $Y_1^*(1660)$ cannot affect the $\Lambda\pi^+\pi^-$ distributions significantly. Similar negative conclusions are drawn about the $\Sigma\pi\pi$ channels.

We wish to thank Professor Luis W. Alvarez, Donald H. Miller, Arthur H. Rosenfeld, Robert D. Tripp, and Dr. Massimiliano Ferro-Luzzi and Dr. Joseph J. Murray for advice and encouragement.

†Work done under the auspices of the U. S. Atomic Energy Commission.

[1]L. W. Alvarez, M. Alston, J. B. Shafer, M. Ferro-Luzzi, D. O. Huwe, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and S. G. Wojcicki, preceding Letter [Phys. Rev. Letters 10, 184 (1963)].

[2]P. L. Bastien, O. I. Dahl, J. J. Murray, M. B. Watson, R. Ammar, and P. Schlein, in Proceedings of the International Conference on Instrumentation for High-Energy Physics, Berkeley, California, September 1960 (Interscience Publishers, Inc., New York, 1961), p. 299.

[3]These cross sections have appeared in preliminary form in P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, J. Kirz, D. H. Miller, J. J. Murray, A. H. Rosenfeld, R. D. Tripp, and M B. Watson, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 373.

[4]M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters 8, 28 (1962).

[5]Both the square of the resonant amplitude and interference terms between the resonant amplitude and the nonresonant background appear in the coefficients of the even powers of $\cos\theta$. With the observed normalized partial widths for $\Sigma(1660)$, the former are of the order of 0.02 for all channels in our dimensionless units, whereas the latter can be one or two orders of magnitude larger (see reference 4).

[6]J. P. Berge, P. Bastien, O. Dahl, M. Ferro-Luzzi, J. Kirz, D. H. Miller, J. J. Murray, A. H. Rosenfeld, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters 6, 557 (1961).

[7]R. Dalitz and D. H. Miller, Phys. Rev. Letters 6, 562 (1961).

[8]Robert P. Ely, Sun-Yiu Fung, George Gidal, Yu-Li Pan, Wilson H. Powell, and Howard S. White, Phys. Rev. Letters 7, 461 (1961).

# EIGHTFOLD-WAY ASSIGNMENTS FOR $Y_1^*(1660)$ AND OTHER BARYONS†

Sheldon L. Glashow* and Arthur H. Rosenfeld

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California
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In the preceding papers establishing the existence of a new $T=1$, $Y=0$ hyperon resonance at 1660 MeV, $Y_1^*(1660)$,[1] it is noted that the baryons and the many low-lying baryon-meson resonances may be included within just four unitary-symmetry multiplets[2]: the $j=\frac{1}{2}^+$ $\alpha$ octet, the $j=\frac{1}{2}^-$ $\beta$ singlet, the $j=\frac{3}{2}^-$ $\gamma$ octet, and the $j=\frac{3}{2}^+$ $\delta$ decuplet.[3] All members of these unitary multiplets are seen except $\Omega_\delta^-$ (sometimes called $Z^-$) and $\Xi_\gamma$, whose discoveries would give crucial tests of the eightfold way. Each of the four unitary multiplets may be viewed as the "ground" (i.e., lowest angular momentum) state of a different Regge trajectory, and each such trajectory may have several manifestations with different physical values of angular momentum. These "Regge recurrences" (should they occur at all) must characterize all the members of a given unitary multiplet, and must be spaced by two units of angular momentum. Identifying the $N_{3/2}^*(1920)$ resonance with $\Delta_\delta$ ($j=\frac{7}{2}^+$), the $Y_0^*(1815)$ with $\Lambda_\alpha$ ($j=\frac{5}{2}^+$), and the $N_{1/2}^*(1688)$ resonance with $N_\alpha$ ($j=\frac{5}{2}^+$), we find that all presently known baryon states are described with only four unitary-multiplet Regge trajectories. Because the first Regge recurrences of some

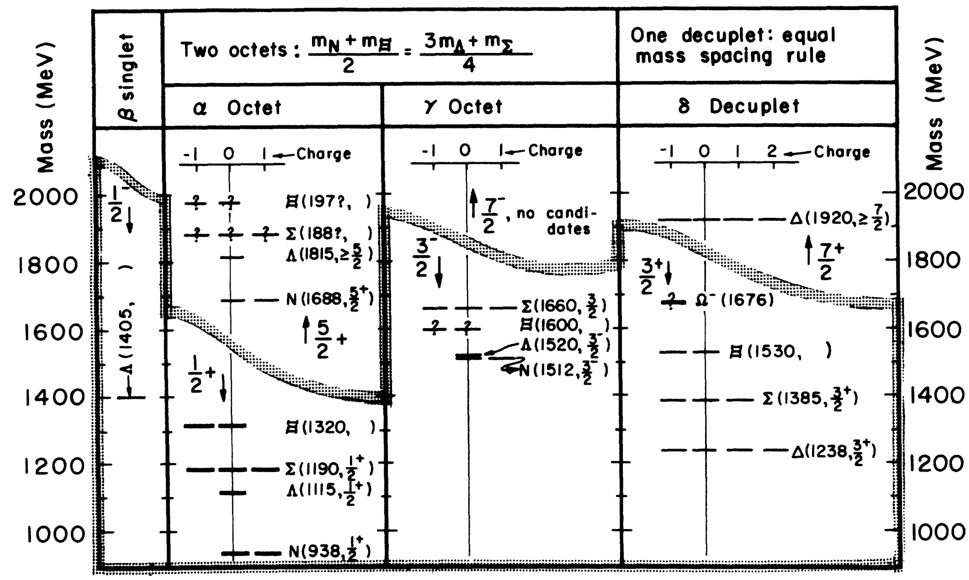


FIG. 1. Baryons: the four unitary multiplets and their Regge recurrences. Spin and parity assignments $J^P$ are written beside each particle if they are supported by any experimental evidence; if not, $J^P$ have been conjectured by assigning one known resonance to each set of quantum numbers. The notation was introduced in the Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), pp. 783 and 325. Observe that the families so defined coincide with the unitary multiplets of the eightfold way. Heavy bars show stable or metastable particles; light lines show resonances. States predicted by the eightfold way but not yet seen are indicated by question marks. The masses of $\Xi_\gamma$ and $\Omega_\delta^-$ follow from the mass formulas alone; those of the $\frac{5}{2}^+$ $\Sigma_\alpha$ and $\Xi_\alpha$ also require the assumption of nearly parallel Regge trajectories.

members of the $\alpha$ and $\delta$ unitary multiplets have appeared, one predicts the existence of $\Sigma_\alpha$ and $\Xi_\alpha$ states of $j = \frac{5}{2}^+$ and of the remaining members of the $j = \frac{7}{2}^+$ $\delta$ decuplets. These four unitary multiplets, and their first recurrences, are displayed in Fig. 1. The analogous meson display appears in Fig. 2.

Beyond the mere classification of particles and resonances, there must be more quantitative, dynamical verifications of the "broken eightfold way."[4] In this note, we show that the partial widths for the various two-body decay modes of the $\gamma$ octet and of the $\delta$ decuplet are compatible with unitary symmetry of strong interactions. In Table I the experimental partial widths for decay into meson plus baryon are summarized. Two of these are used as input variables determining the eightfold-way $D$ and $F$ decay-coupling constants for the $\gamma$ octet; the remaining five partial widths are calculated after adjustment of a radius of interaction.[5] With the same form factor, the calculation is repeated for the $\delta$ decuplet, for which there is a unique invariant coupling to meson plus baryon. In each case, agreement with experiment is excellent, in general, to within the accuracy with which the partial widths are known.

Some of the agreement may be due merely to our use of a reasonable form factor. On the other hand, the theoretical result for $\Gamma(\Sigma_\gamma \to \bar{K}N):\Gamma(\Sigma_\gamma \to \pi\Lambda):\Gamma(N_\gamma \to \pi N)$ of about 1:4:22 (compared with the experimental values of about 1:4:27) refers to modes of roughly the same momenta. Hence we see that the eightfold way supplies nontrivial coefficients. Moreover, the $\delta$ coupling strengths are consistently greater by an order of magnitude than the $\gamma$ coupling strengths [e.g., $\Gamma(\Sigma_\delta \to \pi\Lambda) \approx 4\Gamma(\Sigma_\gamma \to \pi\Lambda)$, even though the $\gamma$ mode has three times as much available energy as the $\delta$ mode].[6] These considerations preclude any purely kinematical explanation for the agreement between theory and experiment.[7]

Three-body decay modes of baryon resonances are less of a test of the eightfold way than two-body modes, because there are many possible

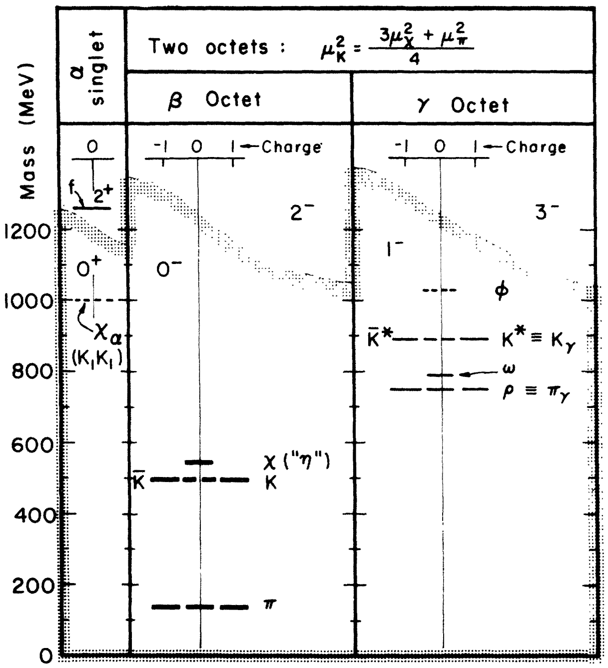


FIG. 2. Mesons: the meson unitary multiplets include a $\beta$ (pseudoscalar) octet and a $\gamma$ (vector) octet. There are two observed $\chi_\gamma$ states (i.e., vector isotopic singlets) called $\omega$ and $\phi$; one linear combination of these is presumably the eighth member of the $\gamma$ octet, and the orthogonal linear combination is assigned to a unitary singlet. $\phi$ is seen as a $K_1K_2$ enhancement at 1030 MeV [see L. Bertanza et al., Phys. Rev. Letters 9, 180 (1962), and J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962)]. In addition there appear to be two $\alpha$ singlets: a $J^{PG}=2^{++}$ pion-pion resonance at 1250 MeV called $f$, and a $0^{++}$ $K_1K_1$ interaction near $\bar{K}K$ threshold. For more complete references, see W. H. Barkas and A. H. Rosenfeld, Lawrence Radiation Laboratory Report UCRL-8030, February 1963 (unpublished). All the meson states have charge-conjugation properties such that they may couple to baryon-antibaryon states, e.g., $\pi^0$ and $\chi$ have $C=+1$ (and decay into two photons), while $\rho^0$ and $\omega$ have $C=-1$ (and couple to a single photon).

Table I. Two-body partial widths for the $\gamma$ octet and $\delta$ decuplet.

| Resonance and total width Γ | Decay mode | Momentum (MeV/$c$) | Width, Γ (MeV) Experimental[a] | Calculated[b] |
|---|---|---|---|---|
| | | $\gamma$ octet | | |
| $\Xi(1600?)$ | $\Xi\pi$ | 220 | ? | 0.6[c] |
| $\Sigma(1600)$ | $\bar{K}N$ | 406 | 3[d] | 3 |
| $\Gamma=40$ MeV | $\Lambda\pi$ | 441 | 11 | Input = 11 |
| | $\Sigma\pi$ | 386 | 13 | Input = 13 |
| $\Lambda(1520)$ | $\bar{K}N$ | 244 | 5 | 6 |
| $\Gamma=16$ MeV | $\Sigma\pi$ | 267 | 9 | Input = 8 |
| $N(1512)$ | $N\pi$ | 450 | 80 | 67 |
| $\Gamma=100$ MeV | | | | |
| | | $\delta$ decuplet | | |
| $\Omega^-(1676?)$ | | Decays weakly into $\Xi\pi$, $\Lambda\bar{K}$, or leptonically | | |
| $\Xi(1530)$ | $\Xi\pi$ | 148 | <7 | 12 |
| $\Gamma<7$ MeV | | | | |
| $\Sigma(1385)$ | $\Lambda\pi$ | 210 | 50 | 35 |
| $\Gamma=50$ MeV | $\Sigma\pi$ | 119 | $\lesssim 4$ | 5 |
| $\Delta(1238)$ | $N\pi$ | 233 | 100 | Input = 100 |
| $\Gamma=100$ MeV | | | | |

[a]For references to the data, see W. H. Barkas and A. H. Rosenfeld, Lawrence Radiation Laboratory Report UCRL-8030, February 1963 (unpublished).

[b]The $D-F$ mixing ratio giving best fit between experiment and theory is $\alpha=0.655$ (in Gell-Mann's notation) or $\theta=35°$ [in the notation of R. Cutkosky, Carnegie Institute of Technology (unpublished)]. This value is in good agreement with dynamical considerations given in preprints by Capps, by Cutkosky, and by Martin and Wali. Moreover, Cutkosky shows that a value of $\theta$ near 33° is probably demanded for the bootstrap appearance of the $\gamma$ octet and the $\delta$ decuplet. We thank Professor Cutkosky for telling us of his result after our calculation was completed.

[c]The very low predicted partial width for $\Xi_\gamma \to \Xi\pi$ suggests that an alternative decay mode may predominate (such as $\Xi+\gamma$, $\Xi\pi\pi$, or $\Lambda K$, if $\Xi\gamma$ is sufficiently heavy).

[d]There is some discrepancy in the measurement of $\Gamma(\Sigma_\gamma \to \bar{K}N)$. According to Alvarez et al., it is $\lesssim 2$ Mev; according to Bastien and Berge, it is $\gtrsim 4$ MeV.

Table II. Two-body decays of the $\alpha$ recurrences into stable baryons + mesons.

| Resonance, total width, and threshold | Decay mode | Momentum (MeV/$c$) | Width, Γ (MeV) Experimental | Calculated[a] |
|---|---|---|---|---|
| $N_\alpha(1688)$ | $N\pi$ | 572 | 80 | Input = 80 |
| $\Gamma = 100$ MeV | $N\chi(``\eta")$ | 387 | <20 | 0.5 |
| $P_\pi = 1.00$ GeV/$c$ | $\Lambda K$ | 235 | < 2 | 1 |
| $\Lambda_\alpha(1815)$ | $N\bar{K}$ | 538 | 70 | 41 |
| $\Gamma = 120$ MeV | $\Sigma\pi$ | 504 | <40 | 29 |
| $P_{K^-} = 1.05$ GeV/$c$ | $\Lambda\chi(``\eta")$ | 345 | < 1.3 | 3 |
| $\Sigma_\alpha(1875)$ | $N\bar{K}$ | 586 | | 5 |
| | $\Lambda\pi$ | 595 | | 15 |
| $P_{K^-} = 1.20$ GeV/$c$ | $\Sigma\pi$ | 548 | | 20 |
| | $\Xi K$ | 208 | | 1 |
| | $\Sigma\chi(``\eta")$ | 322 | | 2 |
| $\Xi_\alpha(1972)$ | $\Xi\pi$ | 531 | | 5 |
| | $\Xi\chi(``\eta")$ | 290 | | 2 |
| $P_{K^-} = 1.40$ GeV/$c$ | $\Lambda\bar{K}$ | 540 | | 1 |
| | $\Sigma\bar{K}$ | 479 | | 41 |

[a]There are again two invariant coupling constants. We use the same $D-F$ coupling constant ratio as we have determined from the decays of the $\gamma$ octet, and we adjust the strength to fit the $N\pi$ decay mode of $N_\alpha(1688)$ (theoretical arguments of Cutkosky suggest that the same "self-consistent" value of $\theta$ should apply to all couplings of mesons to baryon octets). The same form factor is used as earlier (reference 6).

forms of symmetrical interaction involving two final-state mesons. However, it is instructive to consider the possible two-body decays of one baryon resonance into another baryon resonance plus a meson (giving, eventually, a three-body mode), e.g., $N_\gamma \to N_\delta\pi$, $\Sigma_\gamma \to \Sigma_\delta\pi$, $\Sigma_\gamma \to \Lambda_\beta\pi$.

Only inequalities for these partial widths have thus far been experimentally determined:

$$\Gamma(N_\gamma \to N_\delta\pi) \lesssim \Gamma(N_\gamma \to N\pi\pi) = 30 \text{ MeV}, \quad (1)$$

$$\Gamma(\Sigma_\gamma \to \Sigma_\delta\pi) \lesssim \Gamma(\Sigma_\gamma \to \Lambda\pi\pi) = 8 \text{ MeV}, \quad (2)$$

$$\Gamma(\Sigma_\gamma \to \Lambda_\beta\pi) \lesssim \Gamma(\Sigma_\gamma \to \Sigma\pi\pi) = 8 \text{ MeV}. \quad (3)$$

There is again only a single invariant coupling strength for modes (1) and (2) so that their ratio is determined in the eightfold way:

$$\Gamma(N_\gamma \to N_\delta\pi)/\Gamma(\Sigma_\gamma \to \Sigma_\delta\pi) = 6.5.$$

The existence of the mode (3), $\Sigma_\gamma \to \Lambda_\beta\pi$, is a test of the correctness of the assignments of resonances to unitary multiplets: If $\Lambda_\beta$ is a unitary singlet, then it can be shown that unitary symmetry allows this mode if $\Sigma_\gamma$ is a member of an octet, but forbids it if it is a member of either the 10- or 27-plet.

Finally, we consider decays of the two higher energy resonances supposed to be first Regge recurrences of the $\alpha$ octet. The experimental situation is summarized in Table II: $N_\alpha(1688)$ and $\Lambda_\alpha(1815)$ are known to be mainly "elastic" resonances; i.e., for $N_\alpha(1688)$ we know $\Gamma(N\pi)/\Gamma \approx 80\,\%$,[8] for $\Lambda_\alpha(1815)$, $\Gamma(N\bar{K})/\Gamma \approx 60\,\%$[9]; perhaps the reason that $\Sigma_\alpha(187?)$ has not yet been found is because its coupling to $N\bar{K}$ is small; in any case it is not strongly formed by 1.2-GeV/$c$ $K^-$ on protons. Also in Table II are given the calculated partial widths for all decays of the Regge recurrences of the $\alpha$ octet into two-body states of meson plus stable baryon, according to the eightfold way. With satisfaction and relief we find that the calculated results are completely compatible with experiment.

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†Work done under the auspices of the U. S. Atomic Energy Commission.

*Alfred P. Sloan Foundation Fellow.

[1]L. W. Alvarez et al., second preceding Letter [Phys. Rev. Letters 10, 184 (1963)]; J. P. Berge, preceding Letter [Phys. Rev. Letters 10, 188 (1963)].

[2]Reference 1 contains the appropriate citations; for further detail, see J. J. Sakurai, in Proceedings of the International Summer School at Varenna, 1962 (to be

published); S. L. Glashow, in Proceedings of the International Summer School in Theoretical Physics, Istanbul, 1962 (Gordon and Breach, London, to be published).

[3]Words signifying sets of similar particles find their origins in musical terminology. Thus, a trio, quartet, ···, octet, nonet, decimet, ··· is a composition for 3, 4, ···, 8, 9, 10, ··· voices or instruments; but a triplet, quadruplet, ···, octuplet, nonuplet, decuplet, ··· refers to 3, 4, ···, 8, 9, 10, ··· notes played in one beat. After triplet (of pions) and quadruplet (of Δ isobars), we use "decuplet" for the 10. Because of an unfortunate earlier misuse, "octet" has become commonplace for the 8, rather than the more appropriate "octuplet." [See P. Scholes, Oxford Companion to Music (Oxford University Press, London, 1955), 9th ed.]

[4]The success of Gell-Mann's mass formula and its generalizations is certainly one such verification. More important is to understand why an octet resonates at $J=\frac{3}{2}^-$ and a decuplet resonates at $J=\frac{3}{2}^+$, and why there seem to be no resonances in the 27-plet and other decuplet channels. Some progress in this direction has been reported by R. Cutkosky, J. Kalcar, and P. Tarjanne, Phys. Letters 1, 93 (1962), and by R. H. Capps (to be published).

[5]The momentum dependence used included a form factor and $p/M$, i.e.,

$$\Gamma \propto \left|\frac{p^2}{p^2+X^2}\right|^l \frac{p}{M},$$

where $p$ = momentum of decay products of a resonance of mass $M$, and $X$ is related to the size of the interaction. The two coupling constants $D$ and $F$ were adjusted along with $X^2$ to fit the three input data shown in Table I. We found $X = 350$ MeV.

[6]In this connection, there also seems to be evidence that the $\delta$ resonances are produced more copiously than the $\gamma$ resonances.

[7]Also relevant are the two-body decay modes of the vector particles. Using coupling constants of the eightfold way, and with the same form factor as above, we find $\Gamma(\rho\to 2\pi)/\Gamma(K^*\to K\pi) = 2.4$, in rough agreement with experiment.

[8]From summary prepared by R. Omnés and G. Valladas, at the Aix-en-Provence Conference on Elementary Particles, 1961 (C. E. N. Saclay, France, 1961), p. 472.

[9]W. F. Beall et al., in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 368.

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# QUANTIZATION OF ELEMENTARY PARTICLE MASSES

Ernest E. H. Shin
National Magnet Laboratory,* Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 13 December 1962; revised manuscript received 12 February 1963)

Yukawa's[1] original theory of bilocal fields has been extended and applied to obtain the quantization of elementary particle masses. Born's[2] principle of reciprocity is used to develop the theory in the form of an eigenvalue equation whose solution leads to a mass formula which accounts for the mass values of the elementary particles known to date, and, in addition, predicts the existence and mass values of many more particles which have not yet been observed.

The present theory differs from that of Born's,[3,4] in that we attribute the quantization of elementary particles to the dynamics described by the internal space-time variables $r_\mu$ and their corresponding 4-momenta $q^\mu = -i\partial/\partial r_\mu$. A mass operator $\bar{M}(r,q)$ is defined which meets all of the requirements of the principle of reciprocity and relativistic covariance; the center-of-mass (c.m.) kinematics is given in the usual Klein-Gordon form. In its crudest form, the mass operator $\bar{M}(r,q)$ may be expressed as $r_\mu r^\mu + q_\mu q^\mu$, aside from multiplicative and additive (i.e., commutative) constants. The mass eigenvalue equation

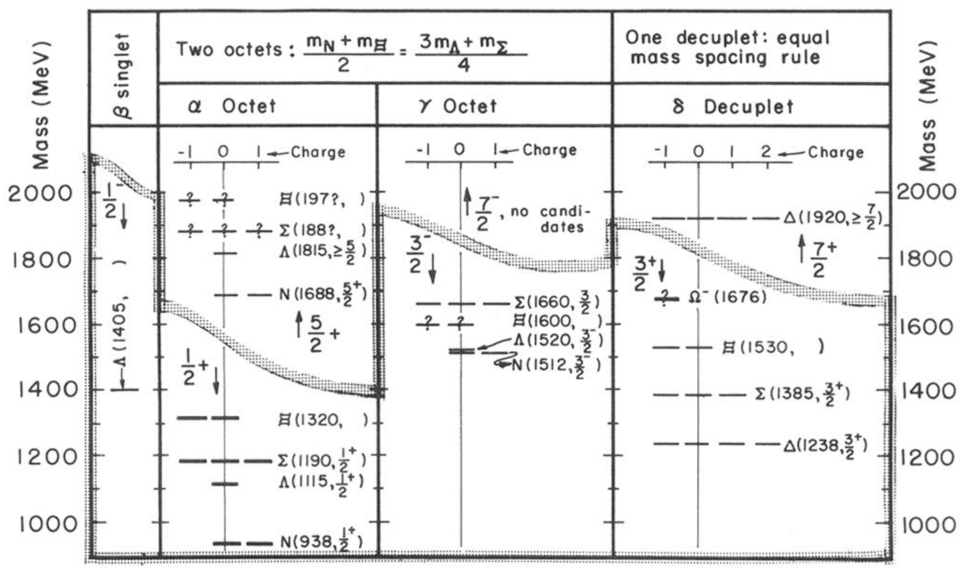
$$\bar{M} f_n(r) = M_n f_n(r) \tag{1}$$

is solved where $f_n(r)$ is the eigenfunction for the state $n$ and $M_n$ is the mass value. The solution of Eq. (1) leads to a quantized mass formula ($\hbar = c = 1$)

$$M_n = \lambda_0^{-1}(n+\tfrac{1}{2}) \tag{2}$$

($n = 1, 2, 3\cdots$), where $\lambda_0$ is a fundamental length (independent of $n$) introduced by the theory.

The development of the theory reveals some of the basic properties of a bilocal field associated with the elementary particles. A bilocal field $\Phi(X, r)$ represents an extended packet of matter, whose c.m. motion is described by $X_\mu$ and $P_\mu$ and whose internal dynamics is described by the variables $r_\mu$ and $q_\mu$. The packet is confined preferentially within a characteristic space-time volume of the order $\lambda_0^4/c$; the normalization condition requires that $f_n(r)\to 0$ as $r^2\to\infty$, and it may be shown that $f_n(r)$ vanishes like

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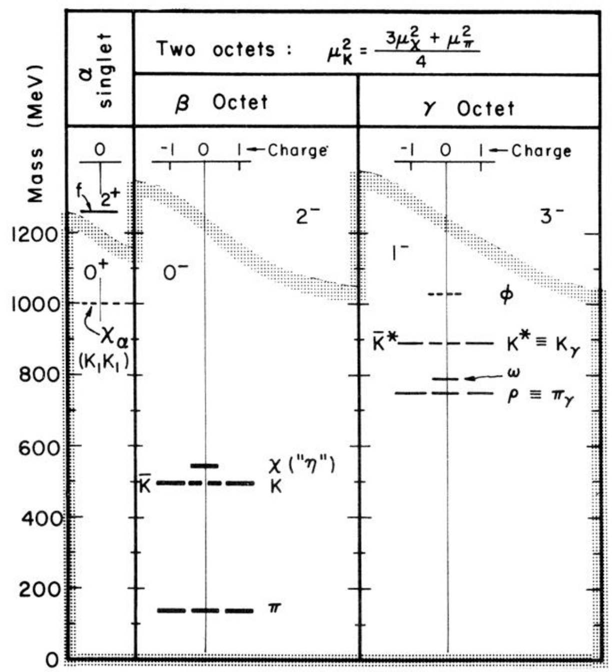


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