SOLUTIONS TO DISPERSION EQUATIONS FOR NUCLEON-NUCLEON SCATTERING*

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Since the existence of the multipion resonances ρ, ω, and η has now been firmly established,[1] it leads naturally to the question of what implication such resonances have on the nucleon-nucleon interaction. In this paper we use relativistic dispersion relations to calculate elastic nucleon-nucleon scattering phase shifts from zero to 400 MeV, including the exchange of such resonances.[2] In addition to the single-pion exchange and the exchange of the above mentioned resonances, we also include the exchange of an S-wave pion-pion pair in the form of an effective scalar particle.

Our procedure is as follows: First, we calculate the pole terms (renormalized Born approximation) corresponding to the exchange of an $I=1$ pseudoscalar (π), an $I=0$ scalar (S-wave π - π pair) an $I=0$ pseudoscalar (η), an $I=1$ vector (ρ), and an $I=0$ vector (ω). Then we solve for the partial-wave amplitudes using the N/D method which is essentially a procedure of imposing the unitarity condition keeping singularities from the pole terms unchanged. If the ρ and ω were treated as elementary vector particles, we would encounter the familiar divergence difficulties which necessitate a cutoff. However, if the ρ and the ω are treated as Regge poles rather than elementary vector particles, a cutoff is inherently contained in the Regge pole description.[3] In this paper, we use a crude version of ρ and ω Regge poles which amounts to simply multiplying the Born term for the ρ exchange by an exponential factor $\exp[\alpha_\rho' \times(t-m_\rho^2)\log(s/2m^2-1)]$ and that for the ω exchange by $\exp[\alpha_\omega'(t-m_\omega^2)\log(s/2m^2-1)]$. Here s is the center-of-mass energy squared, t is minus the momentum transfer squared, m, m_ρ, and m_ω are the nucleon mass, the ρ mass, and the ω mass, respectively. The constants α_ρ' and α_ω' are positive definite parameters introduced to account for the asymptotic Regge pole behavior. Note that these exponential factors are near unity in the low-energy region so that the Born terms are substantially modified only at high energies. Other parameters of the problem are m_π^2, g_π^2, m_S^2, g_S^2, m_η^2, g_η^2, m_ρ^2, $g_{\rho 1}^2$, $g_{\rho 2}^2$, m_ω^2, and g_ω^2, where the g's are the coupling constants. We have two coupling constants $g_{\rho 1}^2$ and $g_{\rho 2}^2$ for the ρ meson coupled, respectively, to the charge and the anomalous magnetic moment of the nucleon. For the ω meson, only the charge coupling g_ω^2 is included. The isoscalar anomalous magnetic moment is known to be small. In addition, we also introduce the singlet and triplet scattering lengths a_s and a_t as subtraction constants on account of the fact that S-wave scattering amplitudes could be sensitive to extreme short-range forces not included in our calculation.

Some of the parameters we mentioned have predetermined values:

$$m_\pi = 140\text{ MeV},\quad m_\eta = 500\text{ MeV},\quad m_\omega = 780\text{ MeV},$$

$$g^2 = 14.4,\quad a_s = 5.4\times 10^{-13}\text{ cm},\quad a_t = -7.7\times 10^{-13}\text{ cm}.$$

Although the ρ resonance observed in production processes also has a well-determined peak (~760 MeV), the width of the resonance is sufficiently broad so that the effective mass which enters the nucleon-nucleon scattering problem may be shifted substantially from the observed peak in production. Therefore, we include m_ρ among the remaining nine adjustable parameters: m_S, g_S^2, m_ρ, $g_{\rho 1}^2$, $g_{\rho 2}^2$, g_ω^2, α_ρ', α_ω', and g_η^2.

We define partial-wave amplitudes in terms of nuclear bar phase shifts as follows:

$$\begin{aligned}
h_J &= (E/2imp)[\exp(2i\delta_J)-1],\\
h_{JJ} &= (E/2imp)[\exp(2i\delta_{JJ})-1],\\
h_{J-1,J} &= (E/2imp)[(\cos 2\epsilon_J)\exp(2i\delta_{J-1,J})-1],\\
h_{J+1,J} &= (E/2imp)[(\cos 2\epsilon_J)\exp(2i\delta_{J+1,J})-1],\\
h^J &= (E/2mp)(\sin 2\epsilon_J)\exp[i(\delta_{J-1,J}+\delta_{J+1,J})],
\end{aligned}\tag{1}$$

where E is the center-of-mass energy; $E=\frac{1}{2}(s)^{1/2}$, and p is the center-of-mass momentum. We denote that part of the h's which is obtained by taking the partial-wave projection of the pole terms by $B_J(s)$, $B_{JJ}(s)$, $B_{J-1,J}(s)$, $B_{J+1,J}(s)$, and $B^J(s)$. The B's, of course, contain only singularities in the unphysical region $s<4m^2$ (left-hand cuts). However, to satisfy unitarity the h's must have branch cuts starting at the physical threshold ($s=4m^2$) with discontinuities given by

$$\begin{aligned}
\mathrm{Im}h_J &= (mp/E)\,|h_J|^2,\\
\mathrm{Im}h_{JJ} &= (mp/E)\,|h_{JJ}|^2,\\
\mathrm{Im}h_{J-1,J} &= (mp/E)(|h_{J-1,J}|^2+|h^J|^2),\\
\mathrm{Im}h_{J+1,J} &= (mp/E)(|h_{J+1,J}|^2+|h^J|^2),\\
\mathrm{Im}h^J &= (mp/E)(h^*_{J-1,J}h^J+h^{*J}h_{J+1,J});\\
& s\geq 4m^2.
\end{aligned}\tag{2}$$

In this paper, we make some approximation to the above unitarity condition. For the S and P waves, we ignore the mixing with higher partial waves, i.e., we use the uncoupled unitarity condition $\mathrm{Im}h=(mp/E)\,|h|^2$. For higher partial waves ($l\geq 2$), the phase shifts are rather small so that the branch cut due to the unitarity condition is not very important. For the imaginary part of all these waves except $h^{J=2}$, we keep only the one-pion exchange terms which enter quadratically on the right-hand side of (2). The Omnès method will be used for the calculation of $h^{J=2}$ which is coupled to the triplet P wave by unitarity. We can now formulate dispersion relations for the h's containing the B's and the right-hand cuts as well as satisfying the boundedness condition at infinity and the p^{2l+1} behavior at threshold. We find it convenient to use the following dispersion relation which satisfies all the above requirements:

$$h(s)=B(s)+\left(\frac{s-4m^2}{s-s_1}\right)^{l-1}\frac{(s-4m^2)}{\pi}\times\int_{4m^2}^{\infty}ds'\left(\frac{s'-s_1}{s'-4m^2}\right)^{l-1}\frac{\mathrm{Im}h(s')}{(s'-4m^2)(s'-s)},\tag{3}$$

where the J indices are now suppressed but the orbital angular momentum is denoted by l. Here, an $(l-1)$th order pole ($l\geq 2$) at $s=s_1$ is added so that both the threshold behavior and the unitarity bound at infinity are maintained. Since the second term on the right side of Eq. (3) is, in general, quite small for $l\geq 2$, the final result is not sensitive to s_1. The only exceptions are the $h_{J+1,J}$ amplitudes (for example, $^3F_2$ and $^3H_4$ in p-p scattering) where the integral term may be comparable to B. This is due to the fact that $h_{J+1,J}$ couples to an amplitude two units lower in orbital angular momentum; hence the unitarity integral becomes rather important. We choose $s_1=60\,m_\pi^2$ corresponding to additional left-hand singularities slightly beyond the ω threshold.

For the P-wave amplitudes, the dispersion relation given by Eq. (3) is, of course, independent of s_1. For the S-wave amplitudes, Eq. (3) becomes a dispersion relation with a subtraction at s_1. We readjust s_1 for each of the S-wave amplitudes to obtain the correct scattering length a_s or a_t.

The dispersion Eq. (3) is solved by the N/D method for S and P waves. Solutions of the higher partial-wave amplitudes are obtained by simple integration using the approximation for the unitarity integral mentioned above. The nine adjustable parameters are used to fit our solutions to values given by direct phase-shift analysis.[4,5] Resulting p-p phase shifts are shown in Figs. 1 and 2 comparing to one of the energy dependent phase-shift analysis solutions of the Livermore group (MIDPOP), all of which fit the p-p observables. The n-p $^3S_1$ phase parameter is shown comparing to the analysis of the Yale group.[5] Other n-p phase shifts are also calculated, but the comparison with direct phase-shift analysis is less meaningful since the latter has a rather wide range of variation. The adjustable parameters corresponding to our results in Figs. 1 and 2 take on the values $m_S=420$ MeV, $m_\rho=700$ MeV, $\alpha_\omega'=0.0135\lambda_\pi^2$, $\alpha_\rho'=0.010\lambda_\pi^2$, $g_S^2=5.61$, $g_\eta^2=9.2$, $g_\omega^2=16.7$, $g_{\rho 1}^2=5.06$, and $g_{\rho 2}^2=49.0$. To avoid possible ambiguties in the definition of coupling constants, we write out the $B_2(I=1)$ amplitude near the threshold explicitly in terms of the masses and coupling constants according to our normalization. We find

$$\begin{aligned}
B_2(s)=\frac{2}{15}\Bigg[&\left(\frac{g_\pi^2}{m}\right)\left(\frac{p^2}{m_\pi^2}\right)^2+\left(\frac{g_S^2}{m}\right)\left(\frac{2m^2}{m_S^2}+\frac{1}{2}\right)\left(\frac{p^2}{m_S^2}\right)^2+\left(\frac{g_\eta^2}{m}\right)\left(\frac{p^2}{m_\eta^2}\right)^2-\left(\frac{g_\omega^2}{m}\right)\left(\frac{m^2}{m_\omega^2}\right)\left(\frac{p^2}{m_\omega^2}\right)^2-\left(\frac{g_{\rho 1}^2}{m}\right)\left(\frac{m^2}{m_\rho^2}\right)\left(\frac{p^2}{m_\rho^2}\right)^2\\
&+\left(\frac{g_{\rho 2}^2}{m}\right)\left(\frac{m_\rho^2}{m^2}\right)\left(\frac{m^2}{2m_\rho^2}-\frac{1}{16}\right)\left(\frac{p^2}{m_\rho^2}\right)^2+\frac{1}{2}\left(\frac{g_{\rho 1}g_{\rho 2}}{m}\right)\left(\frac{p^2}{m_\rho^2}\right)^2\Bigg]+O(p^6).
\end{aligned}\tag{4}$$

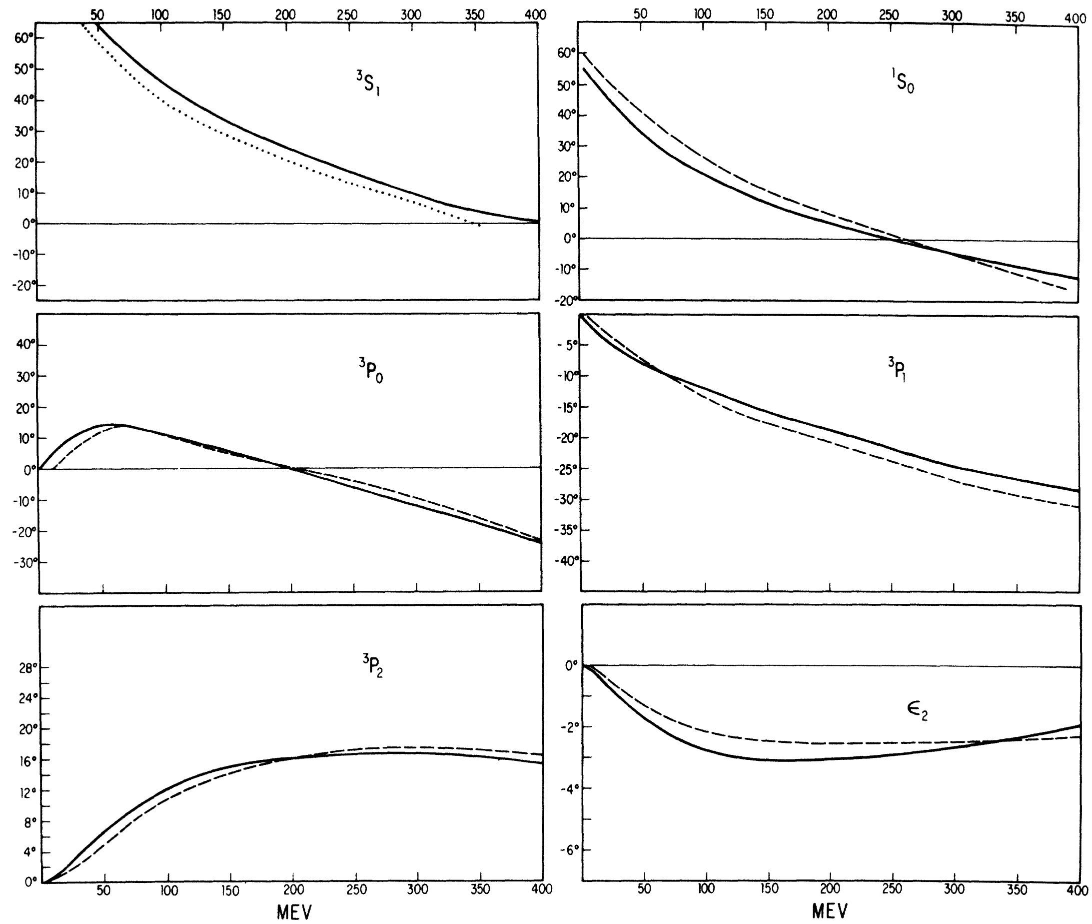

FIG. 1. Energy dependence of the $^3S_1$ n-p phase-shift and low partial-wave p-p phase shifts. The dotted line is the result of the Yale group, dashed lines are Livermore results, and solid lines are our calculated curves.

Our $^1S_0$ phase shift lies below the Livermore analysis by a few degrees in the low-energy region. A careful treatment of the static Coulomb correction could bring our calculation into better agreement with the Livermore result. As we mentioned before, the $^3F_2$ and the $^3H_4$ phase shifts have stronger dependence on the unitarity integral than that of the other high partial waves. In fact, for these two amplitudes, there is a significant cancellation between the unitarity integral and the pole terms so that the result is somewhat uncertain. However, both of these two phase shifts are very small. Therefore, results of the direct phase-shift analysis are also quite uncertain. For the same reason, different sets of the Livermore solutions differ widely for $^3F_3$ as well as $^3F_4$. For the picture as a whole, we find it rather convincing that the simple exchange mechanism described above with the aid of the relativistic dispersion relations does produce a coherent description of nucleon-nucleon scattering. The number of parameters required in the present work is substantially smaller than the familiar approach using phenomenological potentials. Moreover, all of our parameters except the scattering lengths are quantities which, in principle, can be related to other physical processes such as pion-pion scattering, pion-nucleon scattering, and the electromagnetic form factors of the nucleon. For example, the effective S-wave pion-pion pole and the ρ-meson poles can be treated by a more sophisticated method if the $N\bar{N} \to \pi\pi$ amplitudes are known. Some works in this direction are in progress. Incidentally, the value of m_ρ in our analysis is substantially below the ρ-meson peak observed in production processes. The same shifting of the effective ρ mass is also found in the analysis of the nucleon form factors.[6] Theoretical discussions on this point have been given by Ball and one of us (DYW).[7] Although our deter-

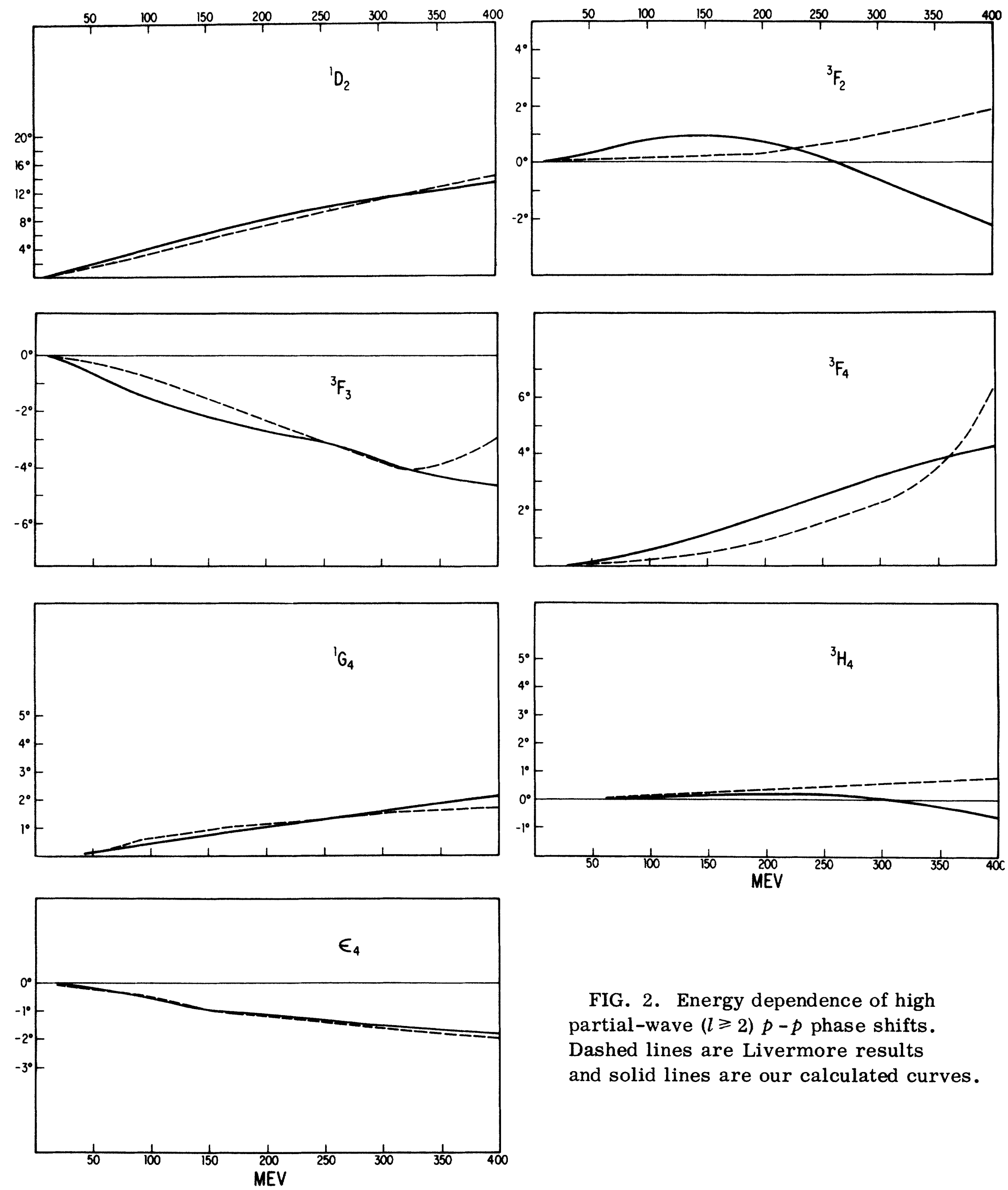

FIG. 2. Energy dependence of high partial-wave ($l \geq 2$) $p-p$ phase shifts. Dashed lines are Livermore results and solid lines are our calculated curves.

mination of various coupling constants and Regge slopes are tentative results which may change somewhat in a more complete treatment of the problem, it is clear that the experimental information on the quantum numbers of the resonances has played an important role in simplifying the theory of nucleon-nucleon interaction. Finally, we mention that results of our calculation can also be used to fit the experimental data directly rather than fitting results of phase-shift analysis. The latter is chosen for convenience in the present work.

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EVIDENCE FOR A PRIMARY COSMIC-RAY PARTICLE WITH ENERGY 10^{20} eV†

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Analysis of a cosmic-ray air shower recorded at the MIT Volcano Ranch station in February 1962 indicates that the total number of particles in the shower (Serial No. 2-4834) was 5×10^{10}. The total energy of the primary particle which produced the shower was 1.0×10^{20} eV. The shower was about twice the size of the largest we had reported previously (No. 1-15832, recorded in March 1961).[1]

The existence of cosmic-ray particles having such a great energy is of importance to astrophysics because such particles (believed to be atomic nuclei) have very great magnetic rigidity. It is believed that the region in which such a particle originates must be large enough and possess a strong enough magnetic field so that $RH \gg (1/300) \times (E/Z)$, where R is the radius of the region (cm) and H is the intensity of the magnetic field (gauss). E is the total energy of the particle (eV) and Z is its charge. Recent evidence favors the choice $Z=1$ (proton primaries) for the region of highest cosmic-ray energies.[2] For the present event one obtains the condition $RH \gg 3\times10^{17}$. This condition is not satisfied by our galaxy (for which $RH\approx5\times10^{17}$, halo included) or known objects within it, such as supernovae.

The technique we use has been described elsewhere.[1] An array of scintillation detectors is used to find the direction (from pulse times) and size (from pulse amplitudes) of shower events which satisfy a triggering requirement. In the present case, the direction of the shower was nearly vertical (zenith angle $10\pm5°$). The values of shower density registered at the various points of the array are shown in Fig. 1. It can be verified by close inspection of the figure that the core of the shower must have struck near the point marked "A," assuming only (1) that shower particles are distributed symmetrically about an axis (the "core"), and (2) that the density of particles decreases monotonically with increasing distance from the axis. The observed densities

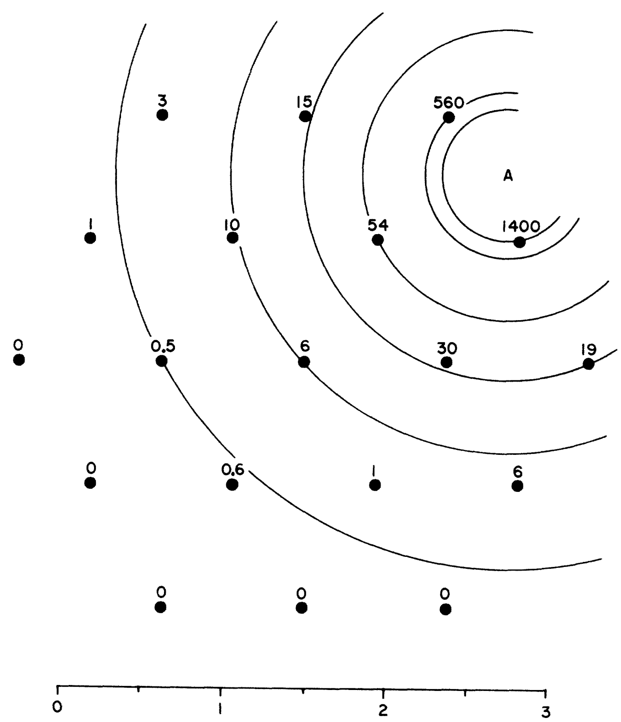

FIG. 1. Plan of the Volcano Ranch array in February 1962. The circles represent 3.3-m^2 scintillation detectors. The numbers near the circles are the shower densities (particles/m^2) registered in this event, No. 2-4834. Point "A" is the estimated location of the shower core. The circular contours about that point aid in verifying the core location by inspection.