We thank Dr. H. J. G. Meyer who suggested this mechanism from our experimental data. We are also indebted to Dr. I. Imai for his discussion and cooperation in this work.

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"BOTTLENECK" IN THE ATTENUATION OF HYPERSONIC WAVES IN QUARTZ AT LOW TEMPERATURES*

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In the investigation of the attenuation of 1000-Mc/sec hypersonic waves in single-crystal quartz at liquid helium temperatures, an unexpected decrease in the attenuation with decreasing temperature was observed below about 2°K for both compressional and shear waves. It is suggested that this decrease is due to a phonon "bottleneck." Energy scattered elastically into modes of the same frequency as the primary beam must subsequently be brought into the thermal modes; at sufficiently low temperatures this second step limits the attenuation of the primary beam. A qualitative and roughly quantitative agreement with theory is obtained.

The experimental arrangement, utilizing the surface excitation technique of generating and detecting hypersonic waves,[1] has been described in detail by de Klerk and Bolef.[2] A high-power pulsed transmitter was used to generate microsecond hypersonic pulses in single-crystal quartz. The exponential decay pattern of the pulses was detected by means of a sensitive superheterodyne receiver and displayed on an oscilloscope. The attenuation measurements were made using an exponential generator, calibrated by means of a precision step attenuator. Changes in attenuation of 0.001 dB/cm could be measured in the temperature range of interest.

Curves of attenuation as a function of temperature for an AC- (shear-)cut quartz rod (diameter 0.25 inches, length 1 inch) are given in Fig. 1 at frequencies of 1000 Mc/sec and 500 Mc/sec, and in each case for pulse repetition rates of 25 pulses per second (pps) and 75 pps. In each case the decrease of attenuation at lowest temperatures is observed, but the onset of the effect varies with frequency and pulse repetition rate. Fig. 1 also shows curves (at these two frequencies) for the case when the receiver end of the rod has been coated with Canada balsam, resulting in the suppression of the effect.

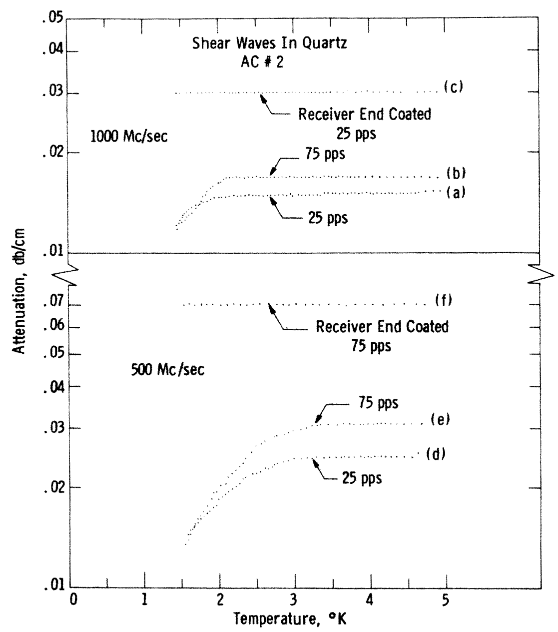

FIG. 1. Attenuation of hypersonic shear waves in AC-cut quartz as a function of temperature. 1000 Mc/sec: (a) pulse repetition rate of 25 pulses per second; (b) 75 pps; (c) 25 pps, but receiver end coated with Canada balsam. 500 Mc/sec: (d) 25 pps; (e) 75 pps; (f) 75pps, but receiver end coated with Canada balsam.

The following explanation is offered. In agreement with the work of Bömmel and Dransfeld,[1] one can neglect the direct effect of anharmonic processes on the attenuation below about 10°K, and the acoustic attenuation α tends to a constant value α_0. This attenuation arises from the elastic scattering of the sound beam (the primary mode) by defects in the body of the crystal or by surface irregularities into a large number of secondary modes of the same frequency ν. The number of secondary modes is the number of modes of the crystal in the frequency interval ν, $\Delta\nu$, where, roughly, $\Delta\nu \simeq \alpha_0 v$, v being the acoustic velocity. From some of these secondary modes (the transverse ones) energy is transferred to the high-frequency thermal modes by means of anharmonic three-phonon processes. Since the number of secondary modes is very large, the second step does not limit the over-all effectiveness of the attenuation process, unless the relaxation time of each secondary wave for three-phonon interaction (l_2/v) exceeds a characteristic time $\tau_0 = 1/\alpha_0 v$ by a comparable factor. Since l_2 increases rapidly with decreasing temperature, this condition will be satisfied at some low temperature. Below that temperature, energy will pile up in the secondary modes, and this will cause the over-all attenuation of the primary beam to decrease. The transfer of energy from the secondary modes to the thermal modes thus forms the same bottleneck which was discussed by Van Vleck[3] in connection with electron-spin resonance.

One can consider the energy balance of a typical secondary mode which receives energy from a pulsed primary beam, decaying with time τ_0, and has a repetition period t. One can easily show that the effective attenuation of the primary beam is

$$\alpha = \alpha_0[1 + l_2(\tau_0/t)v^2/4\pi V\nu^2]^{-1}, \qquad (1)$$

where V is the volume of the crystal and l_2 the mean free path for anharmonic processes of the transverse modes; these interact most strongly with the thermal modes. One can use the theory of Landau and Rumer[4] in its modified form[5] to calculate l_2. It is important to note that $l_2 \propto \nu^{-1}T^{-4}$, so that (1) becomes

$$\alpha = \alpha_0[1 + (\tau_0/t)(\lambda^3/V)A/T^4]^{-1}, \qquad (2)$$

where λ is the wavelength of the primary beam and the parameter A can be estimated roughly in terms of the Grüneisen constant and the elastic constants.

For $\nu = 1000$ Mc/sec and $t = 40$ msec, taking the observed value of $\tau_0 \simeq 2$ msec, we estimate the second term in the square bracket of (2) to be $0.08/T^4$, where T is expressed in degrees absolute. The behavior of curve (a) of Fig. 1 would require this to be $1.0/T^4$. Thus the theory underestimates the bottleneck correction by a factor $\simeq 12$. This numerical discrepancy may arise partly from uncertainties in our theoretical expressions for $\Delta\nu$ and l_2, and partly from the likelihood that the primary beam is not a single mode. The latter would decrease the ratio of secondary to primary modes and increase the bottleneck correction.

In other respects, Eq. (2) seems to describe the observed decrease of α with decreasing T. By doubling the wavelength, the correction term should be increased, and the "knee" shifted to higher temperatures by a factor $2^{3/4} = 1.68$. The knee is indeed shifted from 1.9°K to 3.0°K [see curve (d)]. Again, increasing the pulse repetition rate by a factor 3 should shift the temperature of the knee by a factor $3^{1/4} = 1.32$. Such a shift, but less than predicted, is observed both at 1000 Mc/sec [compare curves (a) and (b)] and at 500 Mc/sec [curves (d) and (e)].

The appearance of the bottleneck effect requires that the secondary waves suffer no appreciable inelastic scattering, except for the anharmonic processes considered, in a time l_2/v. During that time a given phonon will suffer numerous elastic scattering processes at the boundaries (~10^5 to 10^6). Thus even a small escape probability at the boundaries will enable the phonons to bypass the bottleneck. This is why attaching a good bonding material such as Canada balsam to even a small part of the surface eliminates the low-temperature decrease in α [curves (c) and (f)].

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