(600 seconds) suggests the utilization of techniques involving substantial compression of the polarized He^3 gas.

The possibility of using the polarized neutral He^3 gas as a source of polarized He^{3+} or He^{3++} ion beams, for use in particle accelerators, has been considered and is thought to be feasible.[5] The ions would be extracted from the aforementioned weak discharge which is employed in polarizing the neutral atoms. As a result of the He^{3+} hyperfine interaction, the theoretical upper limit for the ion beam nuclear polarization will be one half the polarization of the neutral He^3 gas from which the ions are derived.

The authors wish to acknowledge valuable discussions with P. A. Franken, M. deWit, and G. C. Phillips. The technical assistance of F. D. Sinclair is appreciated. The lamps described are similar in design to those used by C. Cohen-Tannoudji.[6]

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EXPERIMENTAL DETERMINATION OF THE BRANCHING RATIO $\Gamma(\eta\to\pi^+\pi^-\gamma)/\Gamma(\eta\to\pi^+\pi^-\pi^0)$†

Earle C. Fowler
Duke University, Durham, North Carolina

and

Frank S. Crawford, Jr., L. J. Lloyd, Ronald A. Grossman, and LeRoy Price
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 14 January 1963)

The spin (J), parity (P), isotopic spin (I), and G-parity (G) of the η meson have now been fairly well established,[1] mostly through studies of the Dalitz-Fabri plot for the most readily observable decay mode

$$\eta\to\pi^+ + \pi^- + \pi^0. \tag{1}$$

The generally accepted result is

$$J^P I^G = 0^- 0^+. \tag{2}$$

Direct observation,[2] in a heavy-liquid bubble chamber, of the mode

$$\eta\to\gamma+\gamma \tag{3}$$

eliminates the possibility $J=1$, and thus strengthens the assignments of Eq. (2). The expected decay mode

$$\eta\to\pi^0+\pi^0+\pi^0 \tag{4}$$

is difficult to observe and has not yet been established.[2] However, the ratio

$$\Gamma(\eta\to\text{“all neutrals”})/\Gamma(\eta\to\pi^+\pi^-\pi^0)=3.0\pm 0.5 \tag{5}$$

has been found in several experiments.[1] The "neutrals" are expected to correspond to Reactions (3) and (4), if Eq. (2) is correct. One also then expects the η to decay via the electromagnetic mode

$$\eta\to\pi^+ + \pi^- + \gamma. \tag{6}$$

Previous to the present experiment, Reaction (6) had not been observed. The branching ratio

$$R(\pi^+\pi^-\gamma/\pi^+\pi^-\pi^0)\equiv\Gamma(\eta\to\pi^+\pi^-\gamma)/\Gamma(\eta\to\pi^+\pi^-\pi^0)$$

was found to be consistent with zero[1,3,4] (with upper limit about 0.2). Thus, there have been reservations as to the correctness of Eq. (2).

We have determined the value of $R(\pi^+\pi^-\gamma/\pi^+\pi^-\pi^0)$, using η's produced in the Alvarez 72-inch hydrogen bubble chamber via the reaction

$$\pi^+ + p\to\pi^+ + p+\eta, \tag{7}$$

where the η subsequently decays via

$$\eta\to\pi^+ + \pi^- + X^0, \tag{8}$$

and where X^0 is an unknown neutral (or neutrals).

Our experimental result is that X^0 is always a single γ ray or a π^0, and we find the ratio to be

$$R(\pi^+\pi^-\gamma/\pi^+\pi^-\pi^0) = 0.26 \pm 0.08. \qquad (9)$$

The experimental procedures are given in the following nine numbered and lettered paragraphs; then the above result is compared with theoretical predictions.

The momentum of the incident π^+ is 1170 MeV/c at the center of the chamber, with a spread of ± 6 MeV/c.[5] The fact that the beam momentum is well defined is important in separating $X^0 = \gamma$, $X^0 = \pi^0$, and $X^0 =$ nothing.

The following numbered paragraphs correspond to successive procedures and cutoffs applied to a sample consisting initially of 4500 four-pronged events,

$$\pi_1^+, p \to p_2, +_3, +_4, -_5, X^0, \qquad (10)$$

where X^0 is an unknown missing neutral and the subscripts are track numbers. The track p_2 has been identified as a proton by a trained observer who compares the bubble density of the track with that predicted from its measured momentum. (We believe that the proton has been correctly identified in nearly 100% of the cases.)

(1) Dalitz pair cutoff. We assign the electron mass to tracks 3, 4, and 5. Using the PANG momentum and angles for each track, we calculate the invariant masses $M(e_3^+, e_5^-)$ and $M(e_4^+, e_5^-)$. If either of these is less than 100 MeV/c^2, we reject the event (280 events rejected). It is expected that 99.8% of the e^+e^- pairs arising from $\pi^0 \to e^+e^-\gamma$ are eliminated in this way.[6] In all that follows, we assume that tracks 3, 4, and 5 are π^+, π^+, and π^-. Therefore we replaced Eq. (10) by

$$\pi_1^+, p \to p_2, \pi_3^+, \pi_4^+, \pi_5^-, X^0. \qquad (11)$$

(2) "X^0 = nothing" cutoff. We assume that X^0 is absent in Reaction (11), and make a four-constraint (4C) GUTS fit; for $\chi^2 < 30$, we reject the event (3700 events rejected). This cutoff removes most of the "X^0 = nothing" events; namely, the most common event, $\pi^+ p\pi^+\pi^-$. However, we find that each of the charged tracks (2, 3, 4, and 5) has roughly one chance in 100 to undergo a scattering in the hydrogen sufficiently large to spoil the 4C fit, but small enough to escape detection by the scanner or measurer. The next cutoff, No. 3, removes all those cases in which only one track scatters.

(3) "X^0 = nothing: track n (where $n = 2, 3, 4,$ or 5) suffers an unnoticed scattering or decay" cutoff. We delete track n, fit the remaining tracks of Reaction (11) to the hypothesis "X^0 = nothing" with the one-constraint (1C) GUTS fit. For $\chi^2 < 6$, we reject the event (200 events rejected).

The kinematic fitting program GUTS is not used in the remainder of this procedure; momenta and angles from the spatial reconstruction program PANG are used exclusively.

(4) Error cutoff. We calculate the squares of the missing energy $E^2(X^0)$, momentum $p^2(X^0)$, and mass $M^2(X^0) = E^2(X^0) - p^2(X^0)$, and their errors. For $\delta[M^2(X^0)] > (63\text{ MeV}/c^2)^2 = 4 \times 10^3\ (\text{MeV}/c^2)^2$, we reject the event (230 events rejected). This cutoff is necessary for a clean separation between $X^0 = \gamma$ and $X^0 = \pi^0$ (see Fig. 3).

(5) Low-energy γ cutoff. We designate as γ-ray events those having $M^2(X^0) < +(95\text{ MeV}/c^2)^2 = +9 \times 10^3\ (\text{MeV}/c^2)^2$ (see Fig. 3). It is clear that one cannot distinguish a γ-ray event in which the γ ray has zero energy (lab) from an "X^0 = nothing" event, so that a cutoff is needed. The choice of the cutoff P_γ(lab) is dictated by the choice of the error cutoff of Paragraph (4). For γ-ray events having $p^2(X^0) < (94.5\text{ MeV}/c)^2 = 9 \times 10^3\ (\text{MeV}/c)^2$, we reject the event (14 events rejected). This $p^2(X^0)$ cutoff value is $\frac{9}{4}$ times the maximum allowed error in $M^2(X^0)$. No similar cutoff is needed for the π^0 events, since they do not everlap the "X^0 = nothing" category.

No more cutoffs are applied. We now examine the remaining 76 events.

(A) In Fig. 1 we plot $M(\pi_3^+, \pi_5^-, X^0)$ versus

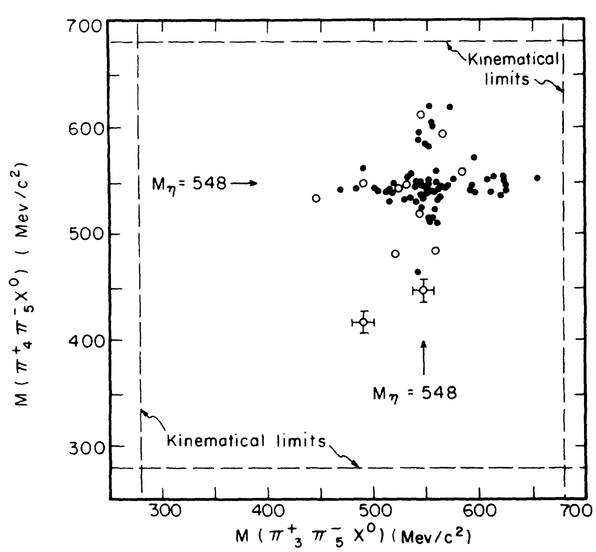

FIG. 1. Effective-mass scatter diagram. Open circles = γ events; closed circles = π^0 events. Typical errors are shown on two of the γ events [see text, Paragraph (A)].

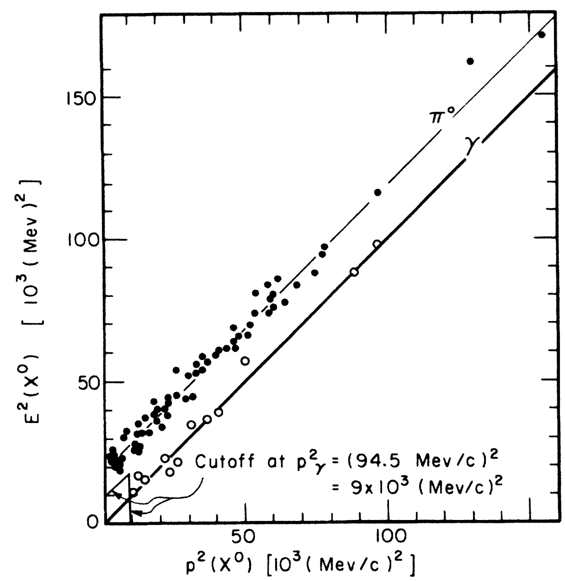

FIG. 2. Plot of $E^2(X^0)$ versus $p^2(X^0)$. Open circles $=\gamma$ events; closed circles $=\pi^0$ events [see text, Paragraph (B)].

$M(\pi_4{}^+, \pi_5{}^-, X^0)$ for each event. The choice between which π^+ is track 3 and which is track 4 is random. In every case at least one of the two values of M has (within the errors) the value of 548 MeV/c^2. We conclude that our sample of 76 events consists entirely of η's, produced via Reaction (7), and decaying via Reaction (8). The η mass distribution has a central value of 547.5 $\pm$ 1 MeV/c^2, and the width of the distribution is consistent with that calculated from the PANG errors (about ± 9 MeV/c^2).

(B) In Fig. 2 we plot $E^2(X^0)$ versus $p^2(X^0)$ for each event. The two 45° lines labeled γ and π^0 correspond to $M^2(X^0)=0$ and $(135.0\text{ MeV}/c^2)^2$, respectively. The events cluster near γ and π^0 and nowhere else.

(C) In Fig. 3(b) we plot a histogram in $M^2(X^0)$. Figure 3 demonstrates that the ratio $R(\pi^+\pi^-\gamma/\pi^+\pi^-\pi^0)$ is not sensitive to the choice of the value of $\delta[M^2(X^0)]$ at which the error cutoff of Paragraph 4 is made. (This is to be expected so long as the laboratory-system momentum spectra of the π^+ and π^- from $\eta\to\pi^+\pi^-\gamma$ are not very different from those from $\eta\to\pi^+\pi^-\pi^0$.) Having chosen the error cutoff corresponding to Fig. 3(b), we see that the γ and π^0 peaks are clearly separated, yielding 12 good γ events and 64 good π^0 events.[7] We thus find

$$R(\pi^+\pi^-\gamma/\pi^+\pi^-\pi^0)=\tfrac{12}{64}=0.19\pm 0.06 \text{ (uncorrected).}$$

(D) Figure 4 shows a histogram of the momentum (lab) of the 12 observed γ rays. The two theoretical curves correspond to (a) Lorentz-invariant phase space and (b) a calculation by Huff[8] using the "two intermediate ρ's" model of

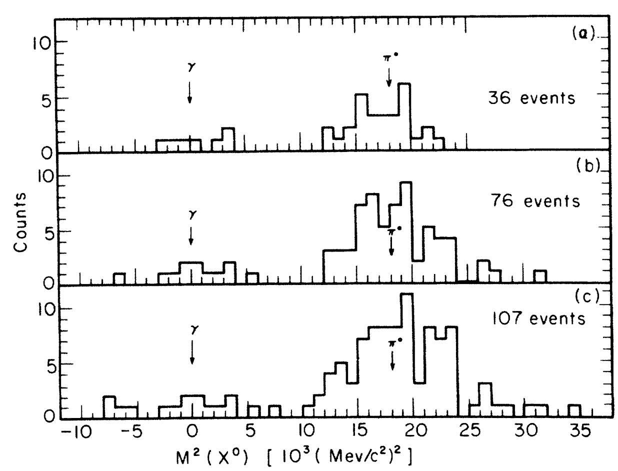

FIG. 3. Histograms in $M^2(X^0)$ for three choices of cutoff on the error, $\delta M^2(X^0)$. Events with $M^2(X^0)$ $<+9\times10^3$ are called γ's; the rest are called π^0's. (a) $\delta M^2(X^0)<3\times10^3$ (MeV/c^2)^2. $\gamma/\pi^0=\frac{7}{29}=0.24\pm0.11$. (b) $\delta M^2(X^0)<4\times10^3$. $\gamma/\pi^0=\frac{12}{64}=0.19\pm0.06$. (c) $\delta M^2(X^0)$ $<6\times10^3$. $\gamma/\pi^0=\frac{16}{91}=0.18\pm0.05$ [see text, Paragraph (C)].

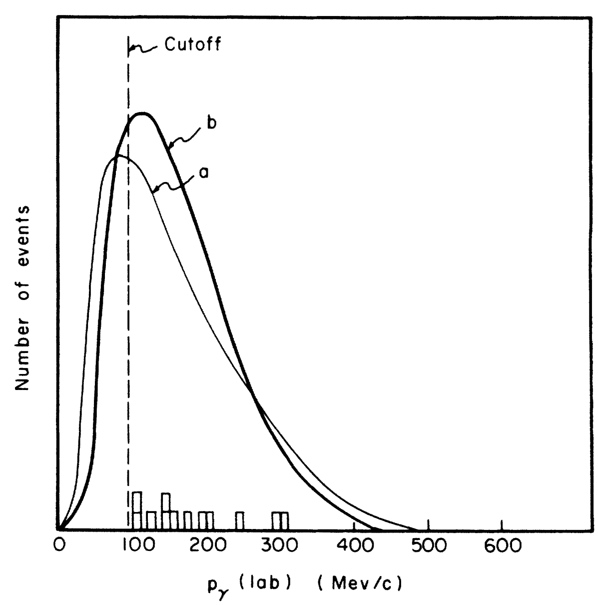

FIG. 4. Histogram of p_γ(lab) for γ events. Curve a: Lorentz-invariant phase space. Curve b: Calculation by Huff [see text, Paragraph (D)].

Gell-Mann, Sharp, and Wagner[9]:

$$\eta \to \rho_1{}^0 + \rho_2{}^0,$$
$$\rho_1{}^0 \to \gamma,$$
$$\rho_2{}^0 \to \pi^+ + \pi^-. \qquad (12)$$

The curves have been transformed from the η rest frame to the laboratory system by using the observed distribution of η momentum (lab). It is seen that the distribution of the events agrees within the limitations of statistics with either of the calculated distributions. This agreement serves as a further check on the validity of the 12 γ-ray events–i.e., their distribution is successfully predicted. From the curves of Fig. 4 we calculate that our cutoff at $p_\gamma = 95$ MeV/c rejects 28% of the total for the phase-space model and 22% for the "two intermediate ρ's" model. We strike a compromise and assume a correction corresponding to 25% loss–i.e., four events.[10]

None of the other cutoffs leads to a correction factor for the ratio.[11] We obtain, as our final result,

$$\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \pi^+\pi^-\pi^0) = (1.00/0.75)\tfrac{12}{64}$$
$$= 0.26 \pm 0.08. \qquad (9)$$

We now compare our result (9) with some theoretical predictions.

The model of Eq. (12) allows an estimate of the ratio

$$R(\pi^+\pi^-\gamma/\gamma\gamma) = \Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma).$$

The prediction[9] is

$$R(\pi^+\pi^-\gamma/\gamma\gamma) \approx \tfrac{1}{4}. \qquad (13)$$

We cannot compare our experimental result (9) directly with this prediction. However, Wali has shown that, provided Eq. (2) holds, the 3π decay modes of η are closely related to the 3π decay modes of K^+ and $K_2{}^0$.[12] Wali predicts a dependence of

$$R(3\pi^0/\pi^+\pi^-\pi^0) \equiv \Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$$

on the shape of the π^0 energy spectrum in the decay $\eta \to \pi^+\pi^-\pi^0$. Using a recent compilation[4] of the Dalitz-Fabri plot for this decay, one obtains the prediction

$$R(3\pi^0/\pi^+\pi^-\pi^0) = 1.68 \pm 0.05. \qquad (14)$$

(This prediction has not yet been experimentally confirmed.) Combining Eqs. (14), (13), and (5), and assuming that "all neutrals" means $\gamma\gamma$ and $3\pi^0$ only, we obtain

$$R(\pi^+\pi^-\gamma/\pi^+\pi^-\pi^0) = \tfrac{1}{4}(3.01 - 1.68) = 0.33. \qquad (15)$$

The prediction (15) is in excellent agreement with our experimental result (9). Our result therefore substantiates the assignments of Eq. (2) and also the prediction (13) of the "two intermediate ρ's" model.

It is a pleasure to acknowledge the advice and encouragement of Luis W. Alvarez. We are grateful to Arthur H. Rosenfeld, Frank T. Solmitz, and Donald Stern for useful discussions, and to our scanners for their excellent work. One of us (E.C.F.) wishes to express his gratitude for the hospitality shown to him by the Alvarez bubble chamber group, and to acknowledge financial support during part of the experiment from the Lawrence Radiation Laboratory and from Yale University.

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[7]From Fig. 1 we see that when the "correct" π^+ is chosen, corresponding to $\eta \to \pi^+\pi^-X^0$, the γ events and π^0 events have the same distribution of $M(\pi^+\pi^-X^0)$ near the η mass. For the "wrong" π^+, $M(\pi^+\pi^-X^0)$ is not peaked at the η mass, and has a somewhat wider spread in values for $X^0 = \gamma$ than for $X^0 = \pi^0$. This is consistent with the larger effective Q value for the decay $\eta \to \pi^+\pi^-\gamma$ than for $\eta \to \pi^+\pi^-\pi^0$.

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[10]If we examine the γ events below our cutoff at $p_\gamma = 94.5$ MeV/c we find 14 events. Four of these actually appear to be η's giving $\pi^+\pi^-\gamma$. The remainder do not appear to be η's. It is likely that they are "X^0 = nothing" events in which two of the tracks suffered small scatterings. The number observed is consistent with such an interpretation.

[11]The Dalitz pair cutoff of Paragraph (1) removes about 5 % of each mode, and does not affect the ratio. The "X^0 = nothing" cutoffs of Paragraphs (2) and (3) reject a negligible number of $\pi^+\pi^-\pi^0$, and because of the cutoff on p_γ(lab), they also reject a negligible number of $\pi^+\pi^-\gamma$.

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COMPARISON OF η, τ, AND τ' DECAYS*

D. Berley,† D. Colley,‡ and J. Schultz
Columbia University, New York, New York
(Received 9 November 1962)

In this note, pion spectra from η, τ, and τ' decays are compared. The theoretical comparison between the decays depends on the assumption that the three final pions have isotopic spin 1. This requirement follows from the $\Delta I = \frac{1}{2}$ rule for τ and τ', and from the assignment of quantum numbers $I = 0$, $G = +1$ for the η.[1] The comparison between η and τ' assumes further that the decay processes proceed through a one-pion intermediate state, i.e., that the diagrams corresponding to the decays are those shown in Fig. 1. The number of observed τ' decays is small, while the τ data are extensive. Consequently, the relation between τ and τ' provides a convenient means of comparing the τ' and η decays.

The decay matrix elements can be described with the three variables s_1, s_2, s_3, where

$$s_j = (p_0 - p_j)^2 = (m - \mu)^2 - 2mT_j.$$

Here p_0, p_j are the 4-momenta of the decaying particle (η or K) and the jth pion, m is the mass of the decaying particle, μ is the pion mass, and T_j is the kinetic energy of the jth pion.

There are only two independent invariants in the decay, which may be chosen to be

$$s_3 = (m - \mu)^2 - 2mT_3$$

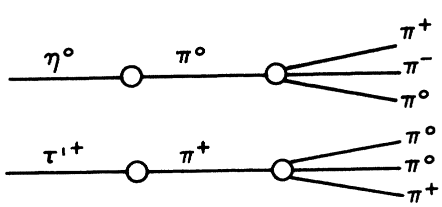

FIG. 1. Diagrams for η and τ' decays assuming one-pion intermediate states.

and

$$x = s_1 - s_2 = 2m(T_2 - T_1).$$

Particle 3 is the unlike pion in τ and τ' decays, and the neutral pion in η decay.

The final pion vertex is the same for η and τ' decays from which it follows that the matrix elements are proportional[2]:

$$M_{\tau'}(s_3, x) = \text{const}\cdot M_\eta(s_3, x). \quad (1)$$

The comparison of the τ and τ' spectra follows from the results of a calculation by Weinberg[3] which imply the relations

$$\left.\frac{\partial M_\tau}{\partial s_3}\right|_{s_3 = s_0, x=0} = \left.\frac{\partial M_{\tau'}}{\partial s_3}\right|_{s_3 = s_0, x=0} \quad (2)$$

and

$$M_\tau(s_0, 0) = -2M_{\tau'}(s_0, 0), \quad (3)$$

where

$$s_0 = \tfrac{1}{3}(s_1 + s_2 + s_3) = \tfrac{1}{3}(m_K^{\ 2} + 3\mu^2).$$

Note that the matrix elements must be even under interchange of pions 1 and 2, so that for the kth decay, $M_k(s_3, x)$ contains only even powers of x.

The first-order expansion of the matrix element about the point $(s_0, 0)$ has the form

$$M_k = C_k[1 + \alpha_k(s_3 - s_0)]. \quad (4)$$

In Fig. 2, the quantity

$$[N_k(s_{3j})/\rho_k(s_{3j})\Delta s_{3j}]^{1/2}, \quad (5)$$

which is proportional to $|M_k|/\omega_k^{1/2}$, is plotted for the three decays. Here $N_k(s_{3j})$ is the number