

³C. Dean, Phys. Rev. **96**, 1053 (1954).

⁴At the T_1 minima, both nuclear species will contribute to the resonance observed. In principle, different T_1 's should be resolvable for the two species; however, the proton signal will usually be much stronger than that of the other species.

⁵The spin exchange probabilities will also depend upon the relative signs of γ_1 , γ_2 , q and Q .

⁶Buchta, Gutowsky, and Woessner, Rev. Sci. Instr. **29**, 44 (1958).

⁷Anderson, Garwin, Hahn, Horton, Tucker, and Walker, J. Appl. Phys. **26**, 1324 (1955); **27**, 196 (EV) (1956).

⁸R. Braunstein, Phys. Rev. **107**, 1195 (1957).

⁹J. Itoh, J. Phys. Soc. Japan **12**, 1053 (1957).

INSTABILITY, TURBULENCE, AND CONDUCTIVITY IN CURRENT-CARRYING PLASMA

O. Buneman

Stanford Electronics Laboratories,
Stanford University, Stanford, California
(Received June 2, 1958)

In fully ionized plasma, close (large-angle) collisions between particles in appreciable relative motion are rare. However, collective Coulomb interactions (i.e., small-angle collisions) cause instabilities which grow so rapidly that relative motions of ions and electrons, i.e., currents, are continually damped down by con-

version of directed energy into (random) fluctuation energy.

The growth mechanism is the familiar¹⁻⁴ two-stream amplification. For initially stationary ions of uniform charge density ρ [plasma frequency $\omega_{pi} = (e\rho/M\epsilon_0)^{1/2}$] traversed by electrons [charge density $-\rho$, plasma frequency $\omega_{pe} = (e\rho/m\epsilon_0)^{1/2}$] with velocity \underline{u} , the (real) wavelength $2\pi/\beta$ of fluctuations and their (possibly complex) frequency ω obey the dispersion formula

$$\omega_{pi}^2/\omega^2 + \omega_{pe}/(\beta u - \omega^2) = 1.$$

[A fluctuation Coulomb field \vec{E} creates ion velocities $e\vec{E}/i\omega M$ and, by virtue of continuity, ion charge density fluctuations $-(\rho/i\omega)\vec{\nabla}\cdot e\vec{E}/i\omega M = \epsilon_0\vec{\nabla}\cdot\vec{E}\omega_{pi}^2/\omega^2$. Similarly the electrons contribute charge density fluctuations $\epsilon_0\vec{\nabla}\cdot\vec{E}\omega_{pe}^2/(\beta u - \omega^2)$, encountering the fluctuations at the Doppler-shifted frequency $\beta u - \omega$. Poisson's equation furnishes the dispersion law.] $2\pi/\beta$ is measured along the electron flow; transverse wavelengths are irrelevant.

Exploring the likely range $\omega_{pi} \ll |\omega| \ll \omega_{pe}$ for possible complex $\omega = |\omega| \exp(i\theta)$, one equates first the imaginary parts in the approximation

$$\beta u/\omega_{pe} - \omega/\omega_{pe} = (1 - \omega_{pi}^2/\omega^2)^{-1/2} \approx 1 + \omega_{pi}^2/2\omega^2,$$

giving $|\omega|^3 = \omega_{pi}^2\omega_{pe} \cos \theta$. Then, from the range $0 < \theta < \pi/2$ and the real part one deduces the following table.

$\beta u - \omega_{pe}$	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
Re ω	0.05	0.09	0.15	0.25	0.40	0.58	0.78	1.00
Im ω	0.48	0.54	0.60	0.66	0.69	0.66	0.51	0

All quantities in the table are in units of $\omega_{pi}^{2/3} \times \omega_{pe}^{1/3}$. The growth rate, $\text{Im}\omega = (\omega_{pi}^2\omega_{pe}\cos\theta)^{1/3} \times \sin\theta$, maximizes at $\theta = \pi/3$, where $\beta u = \omega_{pe}$.

For hydrogen, $\omega_{pe} = 43\omega_{pi}$ and the shortest e-folding times of fluctuation amplitudes are $18\omega_{pe}$. (Multiply by $A^{1/3}$ for ions of mass number A .) The wavelength is then the distance traveled by the electrons during one electron-plasma period. But appreciable growth occurs over a band of wavelengths 20% longer to 10% shorter (for hydrogen). Exponential rise is accompanied by very little oscillation (small Re ω). Energy doubles in just about one electron plasma period. The growth rate is independent of \underline{u} , but

our arguments imply that \underline{u} exceeds random electron velocities.

If a field is briefly applied to a plasma in thermal equilibrium, accelerating electrons to energy eV within an interval shorter than a plasma period, initial thermal energy eV_0 of plasma oscillation within the relevant wavelength band will amplify sufficiently to destroy the directed electron motion in about $\log_2(V/V_0)$ electron-plasma periods (probably 10-20 for typical cases).

When a quasi-steady current is maintained by a lasting applied field \vec{E} , fluctuations will never be allowed to drop appreciably below the level

of the mean dc electron velocity: they are restored with a relaxation time of a few plasma periods. The plasma is turbulent and far from neutral, electric fields other than \bar{E} are continually excited. Electron motion is damped because electrons and ions collide in bunches rather than individually. The collision frequency is the inverse of the relaxation time, say ω_{pe}/K where K is perhaps 10.

With the appropriate collision term in the equation of motion of the electrons, their mean velocity \bar{u} becomes steady when

$$m\omega_{pe}\bar{u}/K = e\bar{E},$$

giving a current density $\rho\bar{u} = K e \rho \bar{E} / m \omega_{pe} = K \epsilon_0 \omega_{pe} \bar{E}$ and a real conductivity $K \epsilon \omega_{pe}$. This is of the order of mhos per meter for a density 10^{12} per cc, much smaller than typical conductivities calculated from individual collisions (see Spitzer⁵). It applies at low frequencies (less than ω_{pe}/K) only.

Radiation from bunches of charge accelerated by the two-stream mechanism is more intense than Bremsstrahlung (owing to coherence). Boundary conditions permitting, it may account for noise received from ionized media. Hoyle⁶ describes qualitatively a mechanism for noise production by interstreaming charges.

The mechanism may wreck some ambitious schemes for channeling electrons through ions or vice versa^{7, 8} and it is of value in thermonuclear work where rapid heating is both desired and observed.⁹ It may be the answer to the Langmuir paradox (see Gabor *et al.*¹⁰), and the rapid growth of collective instabilities, compared with the poor interaction through close collisions, would suggest that coarse-grained turbulence (rather than micro-randomness of particle motion) is the closest one can get to a "thermo"-nuclear regime.

¹J. R. Pierce and W. B. Hebenstreit, *Bell System Tech. J.* **28**, 33 (1949).

²A. V. Haeff, *Proc. Inst. Radio Engrs.* **37**, 4 (1949).

³L. S. Nergaard, *RCA Rev.* **9**, 585 (1948).

⁴J. R. Pierce, *J. Appl. Phys.* **19**, 231 (1949).

⁵L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, New York, 1956).

⁶F. Hoyle, *Nature* **172**, 296 (1953).

⁷G. J. Budker, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organ-

ization of Nuclear Research, Geneva, 1956), Vol. 1.

⁸W. H. Bennett, *Phys. Rev.* **98**, 1584 (1955).

⁹Release in *Nature* **181**, 217 (1958).

¹⁰D. Gabor, *Nature* **176**, 916 (1955).

ENERGY vs MOMENTUM RELATION FOR THE EXCITATIONS IN LIQUID HELIUM*

J. L. Yarnell, G. P. Arnold, P. J. Bendt,
and E. C. Kerr

Los Alamos Scientific Laboratory,
University of California,
Los Alamos, New Mexico
(Received March 20, 1958)

Cohen and Feynman¹ have pointed out that the elementary excitations in liquid helium may be studied by scattering monoenergetic cold neutrons from the liquid. At temperatures below 2°K, the most important scattering process is that in which the neutron creates a single excitation in the liquid. In this case, the energy and momentum of the excitation are given by the change of energy and momentum suffered by the neutron in the scattering.

Thermal neutrons which have been passed through a polycrystalline filter may be used instead of monoenergetic neutrons in this experiment. In this case, the sharp cutoff in the neutron spectrum transmitted by the filter serves as an energy marker. If the predominant scattering process is the creation of a single excitation, the energy spectrum of the scattered neutrons should also possess a sharp cutoff, but at a lower energy.

Palevsky, Otnes, Larsson, Pauli, and Stedman² have done an experiment of this type. They find that at a helium temperature of 1.4°K, the shape of the cutoff is preserved in the scattered beam. At 4.2°K, no indication of a sharp cutoff is seen in the scattered beam. This indicates that the scattering process proposed by Cohen and Feynman is essentially correct. These authors have obtained a number of points on the energy-momentum curve for the excitations.

This note is to report preliminary results of a similar experiment in progress at Los Alamos. A beam of thermal neutrons from the Los Alamos Omega West Reactor was filtered through 16 in. of beryllium, and allowed to strike a target of liquid helium 10 in. in diameter and 3 in. high. The energy spectrum of neutrons scattered through an angle θ was measured with a