SEMIPHENOMENOLOGICAL THEORY OF Σ^+ HYPERON-PROTON SCATTERING UP TO 150 Mev *

R. A. Bryan, J. J. de Swart, and R. E. Marshak
University of Rochester, Rochester, New York

and
P. S. Signell
Bucknell University, Lewisburg, Pennsylvania

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Two of the authors[1] have found a semiphenomenological potential which gives an excellent fit of all the proton-proton scattering data up to 150 Mev, including the recent triple scattering experiment at 150 Mev.[2] The semiphenomenological SM potential is compounded out of the "pion-theoretic" Gartenhaus potential and a phenomenological short-range, attractive, spin-orbit potential whose meson origin is still obscure.[3] One interesting feature of the SM potential is that substantial modifications can be made in the cores for the various states (except the S state) without hurting significantly the fit with the scattering data. From this observation, we draw two conclusions: (1) the SM potential cannot make reliable predictions concerning bound states, and (2) the properties of the SM potential which determine the scattering predictions in the energy region under consideration are to be ascribed to the pion field and will not be much affected by the details of the K-meson field.

If we accept conclusion (2) above and further assume that the coupling of the pion to the Σ hyperon has a strength equal to the pion-nucleon coupling,[4] it follows that the SM potential can be taken over directly for the Σ^+-p system. Indeed, it can be shown quite generally that the existence of a universal pion-baryon coupling (and neglect of the K-meson coupling) implies that the Σ-nucleon interaction in the $T = 3/2$ (T is the isotopic spin) state is identical with the nucleon-nucleon interaction in the $T = 1$ state.[5] One point to be kept in mind is that the Pauli exclusion principle does not apply to the Σ-nucleon system so that the T=1 nucleon-nucleon potential is to be used for both even and odd parity states of the T=3/2 Σ-nucleon system. In order to compare directly with experiment, we confine our attention to Σ^+-p scattering which necessitates the inclusion of the Coulomb field (without antisymmetrization as for p-p scattering).

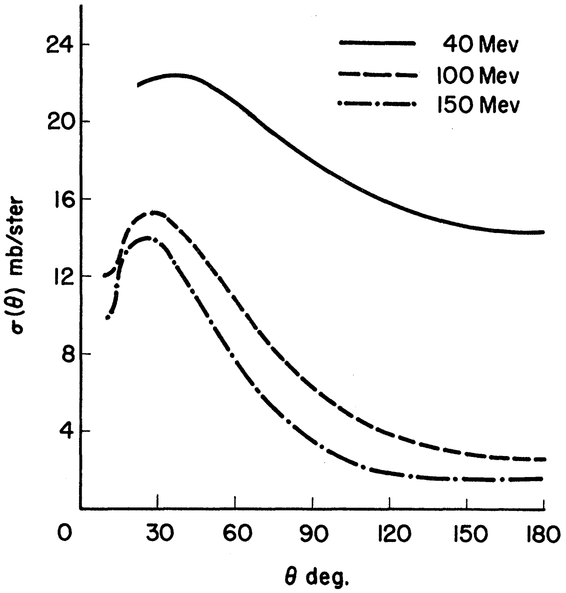

FIG. 1. Differential cross sections for Σ^+-p scattering at 40, 100, and 150 Mev in the laboratory system.

We have computed the differential cross-section $\sigma(\theta)$ and the polarization $P(\theta)$ for Σ^+-p scattering on the basis of the SM potential at three energies for Σ^+ in the laboratory system—40, 100, and 150 Mev. In carrying out these computations (with the SM IBM-650 program), it has been necessary to cut off the deep triplet central potential[6] in the even parity states. Without cutoff, the (${}^3S_1 + {}^3D_1$) state of the Σ^+-p system is strongly bound (by more than 200 Mev). There is no evidence for such a strongly bound state and indeed no real evidence that there is a bound state at all for the Σ^+-p system.[7] Under these circumstances, we have determined the (zero) cutoff for the triplet even central potential so that the binding energy of the ${}^3S_1$ state is exactly zero[8]; we find $\mu r_c = 0.335$. We have then applied this cutoff to the total SM potential in the triplet even states. The results for the Σ^+-p cross sections are shown in Fig. 1. The Σ^+ polarizations are shown in Fig. 2. The phase shifts are listed in Table I. In order to assess the sensitivity of $\sigma(\theta)$ and $P(\theta)$ to the binding energy of the (${}^3S_1 + {}^3D_1$) state, we have redone the calculations for μr_c=0.285 (corresponding to approximately 10-Mev binding) and for μr_c=0.385 (corresponding to a "virtual" state at about 10 Mev). We find that within these plausible limits on the binding energy, the $\sigma(\theta)$ and $P(\theta)$ are changed in magnitude by a maximum of about

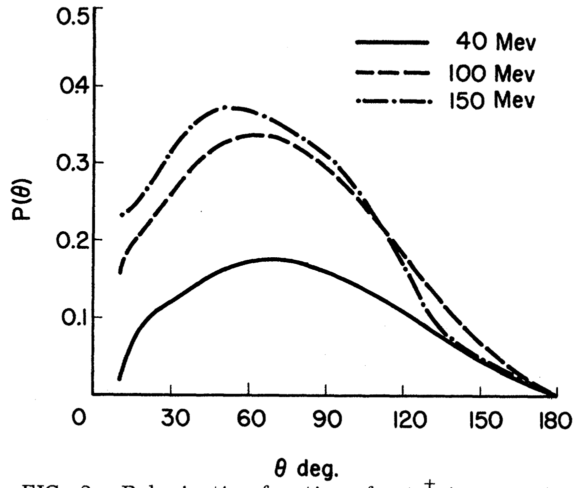

FIG. 2. Polarization functions for Σ^+-p scattering at 40, 100, and 150 Mev in the laboratory system.

30% at the three energies and in shape by lesser amounts. We have also examined the dependence of the $^3S_1$ phase shifts at the three energies on the type of cutoff (it is not expected that the higher triplet even phase shifts would depend on the cutoff and they do not); we find that a repulsive rather than a zero cutoff does not significantly alter the $^3S_1$ phase shifts as long as the ("positive" or "negative") binding energy of the ($^3S_1 + {}^3D_1$) state stays below 10 Mev. If it actually turns out that the ($^3S_1 + {}^3D_1$) state of the Σ^+-p system is strongly bound or "unbound", the large $^3S_1$ phase shifts will change appreciably and so will $\sigma(\theta)$ and $P(\theta)$. In order to allow for this possibility we have included Table I wherein essentially all phase shifts except the $^3S_1$ are fixed uniquely by the p-p potential and "global symmetry".

The peaking of $\sigma(\theta)$ in the forward direction and to a lesser extent its magnitude should be capable of experimental check. It will be more difficult to measure $P(\theta)$, which can in principle be determined by measuring the up-down asymmetry (with respect to the normal to the Σ^+-p scattering plane) of the π^+ resulting from the decay of the Σ^+ and knowing α (the intrinsic asymmetry factor characterizing the decay). If α is small— as seems indicated by recent experiments[9] with Σ^-—$P(\theta)$ will be hard to measure. As a minimum, it would be worthwhile to test our predictions concerning $\sigma(\theta)$ (or more precisely all except the $^3S_1$ phase shifts) since it is not at all evident that the pion-baryon coupling is universal.[10] If this "global symmetry" were actually established by some other means, comparsion between theory and experiment for Σ^+-p scattering below 150 Mev should throw light on the binding of the ($^3S_1 + {}^3D_1$) state of the Σ^+-p system and consequently on the K-meson contribution to the core of the Σ^+-p potential in triplet even states.

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[1] P. S. Signell and R. E. Marshak, Phys. Rev. 106, 832 (1957) and 109, 1229 (1958); this potential has been referred to as the SM potential, and we shall continue this practice

[2] A. E. Taylor and co-workers at Harwell have measured the D function at 150 Mev (private communication); the SM potential predicts the correct D function whereas the phenomenological potential of J. L. Gammel and R. M. Thaler [Phys. Rev. 107, 291 (1957)] does not.

[3] See S. Okubo and R. E. Marshak, Ann. Phys. (to be published). R. T. Sharp has recently derived a spin-orbit potential from pion theory which bears a strong resemblance to the SM spin-orbit potential (private communication).

[4] See E. P. Wigner, Proc. Natl. Acad. Sci. U. S. 38, 449 (1952); J. Schwinger, Ann. Phys. 2, 407 (1957); and M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

[5] See D. B. Lichtenberg and M. H. Ross, Phys. Rev. 103, 1131 (1956) and 107, 714 (1957); also N. Dallaporta and F. Ferrari, Nuovo Cimento 5, 111 (1957). The

TABLE I. Phase shifts: Σ^+-p scattering (zero cutoff for triplet even potential to $\mu r_c = 0.335$).

E	$^1S_0$	$^1P_1$	$^1D_2$	$^1F_3$	$^1G_4$	$^1H_5$	$^3P_0$	$^3P_1$	$^3D_2$	$^3F_3$	$^3G_4$	$^3H_5$
40	49.5	10.5	0.9	0.1	0.0	0.0	9.4	-6.9	-0.8	-0.1	0.0	0.0
100	32.2	22.1	3.9	0.8	0.2	0.0	17.9	-15.0	-4.0	-0.8	-0.1	0.0
150	22.6	26.6	6.4	1.7	0.5	0.1	17.5	-18.4	-6.7	-1.8	-0.4	-0.1

E	$^3S_1$	$^3D_1$	ϵ_1	$^3P_2$	$^3F_2$	ϵ_2	$^3D_3$	$^3G_3$	ϵ_3	$^3F_4$	$^3H_4$	ϵ_4	$^3G_5$	$^3K_5$	ϵ_5
40	74.8	-0.7	-2.7	6.9	-0.2	-5.4	1.0	0.0	-2.2	0.1	0.0	-0.8	+0.0	0.0	-0.2
100	60.1	-1.8	-1.6	16.0	-1.4	-10.7	5.4	-0.3	-5.9	1.2	-0.1	-2.7	0.3	0.0	-0.8
150	52.7	-1.8	0.9	19.5	-2.7	-12.3	9.9	-1.0	-8.1	2.8	-0.3	-4.9	0.9	-0.1	-1.8

mass difference between the hyperon and the nucleon is disregarded in deriving the potential but not in the kinematics.

[6] See Fig. 5 of the second SM paper, reference 1.

[7] See R. H. Dalitz, Reports on Progress in Physics (The Physical Society, London, 1957), Vol. 20, p. 163.

[8] Taking account of the admixture of $^3D_1$ with $^3S_1$ should not appreciably modify the cutoff, since the tensor force is small compared to the central part of the SM potential.

[9] See M. Gell-Mann and A. Rosenfeld, Annual Review of Nuclear Science (Annual Reviews, Inc., Stanford, 1957), Vol 7, p. 407.

[10] Some preliminary evidence comes from a study of the reaction $K^- + p$ [see A. Fujii and R. Marshak, Nuovo cimento (to be published)]; other arguments are given in M. Gell-Mann and A. Rosenfeld, reference 9, and A. Pais, Phys. Rev. 110, 574 (1958).

OPTICAL MODEL POTENTIAL AT THE NUCLEAR SURFACE FOR THE ELASTIC SCATTERING OF ALPHA PARTICLES *

George Igo

Los Alamos Scientific Laboratory,
Los Alamos, New Mexico, and
Stanford University, Stanford, California

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The elastic scattering of 18-Mev alpha particles from argon,[1] 40-Mev alpha particles from copper,[2] and 48-Mev alpha particles from lead[3] has been analyzed in terms of the optical model. A radial nuclear potential[4] $(V + iW)/\{1 + \exp(r - r_0/d)\}$ and a radial charge distribution $\rho(r)$ of the form[5]

$$\rho(r) \sim 1 - \tfrac{1}{2}\exp[n(r/r_0 - 1)], \quad r \leq r_0$$
$$\sim \tfrac{1}{2}\exp[n(1 - r/r_0)], \quad r \leq r_0$$

have been employed. This method of analysis is by now very familiar and will not be described here.[6]

In previous work[6] we have mainly tested the family of nuclear potentials generated by changing V and W while keeping r_0 and d equal to $1.37A^{1/3} + 1.30$ and 0.5 fermis, respectively (1 fermi $= 10^{-13}$cm). We had found best average values of V and W to be about -45 Mev and -10 Mev for all elements considered at bombarding energies of 22[7] and 40 Mev.[2, 8] Cheston and Glassgold[9] have pointed out that is is probably possible to fit the data with potentials resulting from other values of r_0 and d.

The family of nuclear potentials which can be generated by changing all four parameters was employed in the present analysis. Since the average time to calculate an angular distribution for one set of parameters is about 12 minutes, it was necessary to restrict the number of angular distributions to be analyzed. Three angular distributions were chosen to represent the main characteristics of the elastic scattering of alpha particles from nuclei; the monotonic decrease with increasing angle found in the heavy-element distributions, the diffraction structure in the light-element distributions, and the variations associated with changing the kinetic energy of the alpha particle. Approximately 100 different sets of the four parameters were tested for each of the three angular distributions.

Only very broad restrictions can be placed on each parameter. However, it is found that the parameters are important only so far as they combine together to determine the surface of the potential, i.e., where the potential is $\gtrsim$ -10 Mev. Table I lists the parameters for the best potentials obtained. Figure 1 shows a plot of the magnitude of the real part of the best nuclear poten-

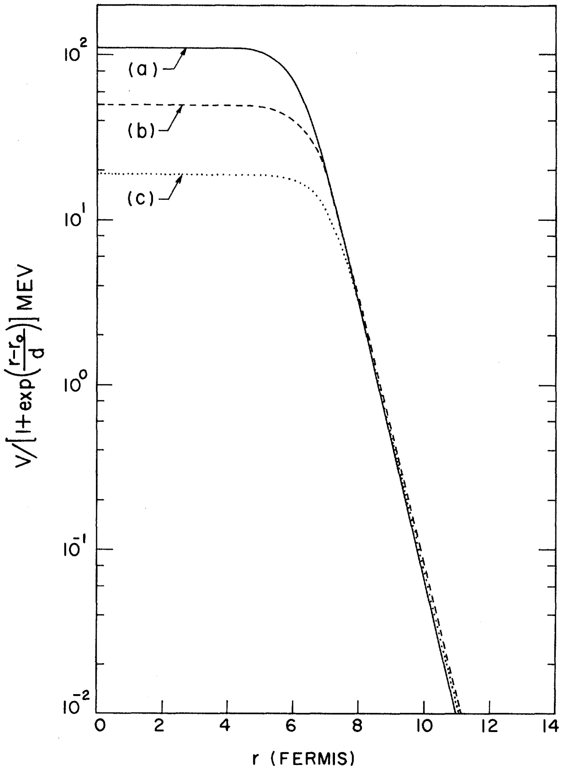

FIG. 1. Comparison of best real potentials for the elastic scattering of 40-Mev alpha particle from copper (V and W are in Mev; r_0 and d in fermis). (a) $(V, W, r_0, d) = (-110, -20, 6.30, 0.5)$; (b) (-49.3, -11, 6.78, 0.5); (c) (-19, -13, 7.22, 0.5).