RELATIVISTIC CORRECTIONS TO THE FERMI MATRIX ELEMENT*

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The ft values for $J = 0^+ \rightarrow J = 0^+$ positron decays of O^{14}, Al^{26}, and Cl^{34} have been used to determine the magnitude of the Fierz interference term.[1] The analysis is subject, however, to an uncertainty in the variations of the ft values which could be produced by the Coulomb interaction or by the energy-dependent relativistic correction terms. The Coulomb correction for Cl^{34} has been calculated to be ~1.5%.[2] We now consider the effect of relativistic terms.

There are no relativistic correction terms to the vector nuclear matrix element for a transition between two states of zero angular momentum. The scalar nuclear matrix element would have a relativistic correction term, however. The question one must answer is whether the energy dependence of the correction term would be sufficient to mask the presence of a Fierz interference term. It is important to realize that only a change in the ratio of the ft values will affect the limit on the magnitude of the Fierz term.

The scalar matrix element is

$$\int \beta \equiv \int \Psi_f^* \sum_k (t_1^{(k)} + it_2^{(k)}) \beta^{(k)} \Psi_i, \qquad (1)$$

where Ψ_i, Ψ_f are the initial and final nuclear states and β is a Dirac matrix. The matrix element of $\beta^{(k)}$ is equal to the matrix element of $(1 - v_k^2/2c^2)$ to this order in v/c. The calculation of this matrix element will be made first in the shell model with harmonic oscillator wave functions. The kinetic energy of the proton in an (nl) orbital is

$$T = \tfrac{1}{2}\hbar\omega(2n + l - \tfrac{1}{2}), \quad n = 1, 2, \ldots, \quad l = 0, 1, \ldots. \qquad (2)$$

Only the kinetic energy of the last particle appears in the correction term. Using oscillator parameters determined from Coulomb energy differences we find

$$\text{for } O^{14}: \quad T = 19.1 \text{ Mev}, \quad 1p \text{ proton};$$
$$\text{for } Cl^{34}: \quad T = 21.4 \text{ Mev}, \quad 1d \text{ proton}. \qquad (3)$$

The relativistic correction terms decrease the scalar nuclear matrix elements for the $Cl^{34} \rightarrow S^{34}$ and $O^{14} \rightarrow N^{14*}$ (2.31 Mev) positron decay by the amounts 2.3% and 2.0% respectively. The ratio of the ft values for the O^{14} and Cl^{34} decays would deviate from unity:

$$ft(Cl^{34})/ft(O^{14}) = 1.003. \qquad (4)$$

This ratio is calculated assuming equal amounts of the scalar and vector interaction.

The interesting aspect of this result is that the relativistic corrections are individually large, yet the effect on the ratio of the ft values is very small.

To check whether the result is sensitive to the shape of the potential, the calculation was done by using the Dirac wave function for a particle in a finite square well. The radial functions are simply spherical Bessel and Hankel functions. The complete solution can be determined by inserting the nuclear binding energy of the last proton and the radius into a characteristic equation obtained from the requirement of continuity of the current across the circumference of the well.

The nuclear binding energy used must not include the Coulomb energy. The Coulomb energy to be subtracted was found from the differences in binding of mirror nuclei by making a correction for the changes in radius produced by the addition or subtraction of a neutron. The binding by the nuclear potential of the $1p$ proton in O^{14} was found to be 8.15 Mev; and of the $1d$ proton of Cl^{34}, 11.48 Mev. The radii used were the equivalent uniform-model radii deduced from high-energy electron scattering experiments. These radii are 3.24×10^{-13} cm for O^{14} and 4.19×10^{-13} for Cl^{34}.

Using these radii and nuclear binding energies of the last proton, we find the relativistic corrections to produce a decrease in the scalar nuclear matrix elements of 2.3% for $Cl^{34} \rightarrow S^{34}$ and 1.8% for $O^{14} \rightarrow N^{14*}$ (2.31 Mev). The ratio of the ft values is then calculated to be

$$ft(Cl^{34})/ft(O^{14}) = 1.005. \qquad (5)$$

The comparison of Eqs. (4), (5) shows that while the effect of the relativistic corrections to the ratio of the ft values is uncertain in the exact value, the magnitude is a fraction of one percent. These correction terms are insufficient, therefore, to mask the presence of a Fierz term larger than the limit set by Gerhart.

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[1]J. B. Gerhart and R. Sherr, Bull. Am. Phys. Soc. Ser. II, 1, 195 (1956); J. B. Gerhart, Phys. Rev. 109, 897 (1958).

[2]W. M. MacDonald, Phys. Rev. 110, 1420 (1958).

ELECTROMAGNETIC MASS CORRECTION FOR SPIN 0 PARTICLES

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Because of the requirement of gauge invariance, the electromagnetic coupling of a charged spin-zero field consists of two terms,

$$-ie\left(\varphi^*(x)\cdot\partial_\mu\varphi(x) - \partial_\mu\varphi^*(x)\cdot\varphi(x)\right)A_\mu(x) \qquad \text{(1a)}$$

$$-e^2\varphi^*(x)\varphi(x)A_\mu(x)A_\mu(x). \qquad \text{(1b)}$$

In lowest-order perturbation theory (ignoring for the moment all strong interactions) the two self-energy terms are represented by the diagrams in Fig. 1.

To separate the contributions of the two terms (each of which is gauge dependent), it is useful to choose the gauge in which the photon Green's function in momentum space takes the form

$$D_{F\mu\nu}(K) = \frac{1}{K^2}\left(\delta_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}\right). \qquad (2)$$

With this choice, the contribution of the terms (1a) to the self-energy are seen to be logarithmically divergent, since

$$\int d^4K\,(q-\tfrac{1}{2}K)_\mu \frac{1}{(q-K)^2+\mu^2}(q-\tfrac{1}{2}K)_\nu \frac{1}{K^2}\left(\delta_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}\right)$$

$$= \int d^4K \frac{1}{K^2}\frac{1}{(q-K)^2+\mu^2}\left(q^2 - \frac{(q\cdot K)^2}{K^2}\right). \qquad (3)$$

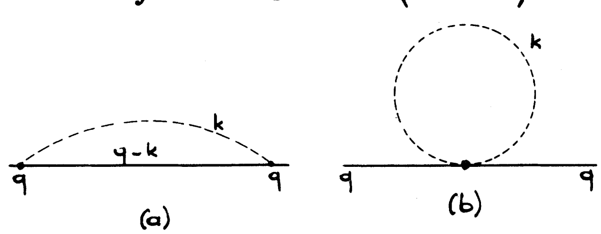

FIG. 1. Lowest order electromagnetic effect.

The contribution of the term (1b) is quadratically divergent. It is proportional to

$$3e^2\int\frac{d^4K}{K^2}, \qquad (4)$$

which behaves as Λ^2, if Λ is a cutoff momentum for the photon. With this choice of gauge, the familiar result that the spin-zero boson electromagnetic self-energy is quadratically divergent[1] is made very transparent. As would be expected on dimensional grounds, it is also clear that the dominant term [(1b) in this gauge], which is the quadratically divergent one, is independent of the mass of the boson. On these grounds, if it is assumed that the mass difference between charged and neutral members of the same multiplet is purely electromagnetic, one might expect the result

$$\frac{m_{K^+} - m_{K^0}}{m_{\pi^+} - m_{\pi^0}} = \frac{m_\pi}{m_K} \qquad (5)$$

to hold to good accuracy.

It is of course necessary to enquire how this result is affected by the inclusion of strong interactions. The terms of the type shown in Fig. 2 (a) correspond to taking into account, for example, the fact that in the strong interaction "box," which changes the boson mass from the bare to the physical mass, the masses of intermediate charged and neutral members of a multiplet are their physical (slightly unequal) masses. We expect the contribution of these terms to the electromagnetic mass difference to be rather small. There are also terms of a type exhibited in Fig. 2 (b). We do not, of course, know how to calculate these terms but, if we assume that the vertex operator behaves in a way similar to that indicated by perturbation

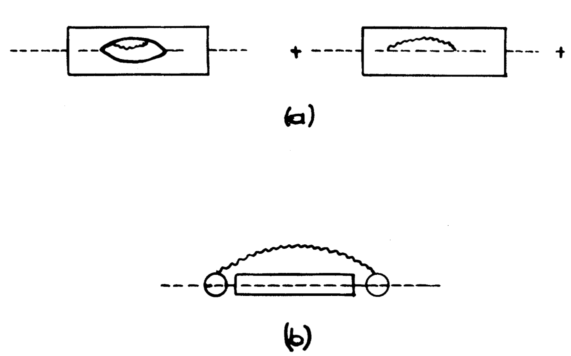

FIG. 2. Strong interactions corrections.