[5]Dresselhaus, Kip, and Kittel, Phys. Rev. 98, 368 (1955).

[6]Dexter, Zeiger, and Lax, Phys. Rev. 104, 637 (1956). See the review article by B. Lax, Revs. Modern Phys. 30, 122 (1958) for a comprehensive list of references.

[7]Rauch, Behrndt, and Zeiger, Lincoln Laboratory Quarterly Report, Solid State Research, February 1, 1958 (unpublished).

[8]Fletcher, Yager, and Merritt, Phys. Rev. 100, 747 (1955).

[9]Indications of distortion on the hole peak have been obtained by the Lincoln Laboratory group in work with plane waves [Zeiger, Behrndt, and Rauch, Proceedings of the Rochester Conference on Semiconductors, J. Phys. Chem. Solids (to be published)]. Structure on the heavy-hole peak on the $+H$ side had been reported by Dresselhaus, Kip, and Kittel [Phys. Rev. 95, 568 (1954)]. The high-resolution spectrum of Fletcher, Yager, and Merritt[8] (circular polarization, $+H$ side) shows no structure on the heavy-hole line at 4° K.

[10]J. M. Luttinger and W. Kohn, Phys. Rev. 97, 869 (1955).

[11]A distortion has been found with plane waves on the low-field side of the heavy-hole line. The absolute value of the mass is about $0.26m_0$, which is within the range where the emission-type effect occurs with circular polarization.

THEORY OF THE MEISSNER EFFECT

D. Pines* and J. R. Schrieffer†
Ecole d'Eté de Physique Théorique, Les Houches (Haute Savoie), France
(Received November 7, 1958)

In order to obtain an explicitly gauge-invariant description of the Meissner effect in superconductors, Wentzel[1] has recently suggested an approach which differs in several essential respects from that developed by Bardeen, Cooper, and Schrieffer.[2] Wentzel finds in the long-wavelength limit a relation between the current density and the magnetic vector potential which differs from the London value given by the BCS theory. We wish to point out that this discrepancy is due to Wentzel's assumption that corrections to his approach possess a convergent expansion in powers of the phonon-electron coupling constant, g. It is our belief that this expansion does not exist.

Wentzel considers the Hamiltonian

$$H = H_0 + H_g + H_A \equiv H' + H_A, \qquad (1)$$

where the three terms describe the free electrons and phonons, the electron-phonon interaction of strength g, and the coupling of the electrons to the vector potential, A. Coulomb interactions between electrons are neglected. By carrying out a perturbation theoretic canonical transformation based on the unperturbed Hamiltonian, H_0, Wentzel eliminates the coupling of the system to the vector potential through order g^2. Thus he finds

$$\tilde{H} = e^{-K} H e^{K} = H_0 + H_g + O(g^3 A), \qquad (2)$$

where

$$K = K_A + K_{gA} + K_{ggA}, \qquad (3)$$

and the generators are defined by

$$[H_0, K_A] = -H_A, \qquad (4)$$

$$[H_0, K_{gA}] = -[H_g, K_A], \qquad (5)$$

$$[H_0, K_{ggA}] = -[H_g, K_{gA}]. \qquad (6)$$

To the same order in g, the transformed current density operator is given by

$$\tilde{j}_\mu(\vec{q}) = j_\mu(\vec{q}) + [j_\mu(\vec{q}), K], \qquad (7)$$

where $j_\mu(\vec{q})$ is the current density operator in the presence of the vector potential in the original representation. Wentzel then calculates the current density by taking the expectation value of (7) with respect to the Bogoliubov[3]-Valatin[4] representation of the BCS ground state appropriate to $H' \cong \tilde{H}$. Since H_g is not treated as a perturbation in this last step, a Meissner effect is obtained and it is found that

$$\lim_{q^2 \to 0} J_\perp(\vec{q}) = -\frac{\rho}{2} \frac{ne^2}{m} A_\perp(\vec{q}), \qquad (8)$$

where $\rho = g^2 N(0) \simeq \frac{1}{4}$, $N(0)$ being the density of states in energy of electrons of one spin at the Fermi surface and $\perp$ denotes the transverse components.[5]

To understand the difficulties with this approach, we note that the exact expression for the current density including terms linear in A may be written as[6]

$$J_\mu(\vec{q}) = (\Psi_0 | j_\mu(\vec{q}) | \Psi_0) + (\Psi_0 | [j_\mu(\vec{q}), S_A] | \Psi_0) \equiv \sum_\nu K_{\mu\nu}(\vec{q}) A_\nu(\vec{q}), \qquad (9)$$

where

$$[H', S_A] = -H_A, \qquad (10)$$

and Ψ_0 is the ground-state eigenfunction of H':

$$H' \Psi_\alpha = W_\alpha \Psi_\alpha. \qquad (11)$$

If S_A were expanded in powers of g, one would obtain Wentzel's expression for the current density. The exact expression for S_A is

$$(\Psi_\beta | S_A | \Psi_\alpha) = -\frac{(\Psi_\beta | H_A | \Psi_\alpha)}{W_\beta - W_\alpha} \quad (W_\beta \neq W_\alpha). \quad (12)$$

As Schafroth[7] has shown, the Meissner effect is established by considering the form of $K_{\mu\nu}(\vec{q})$ in the limit $q^2 \to 0$. We argue that for a superconductor, the excitation energies that appear in (12) have a radically different behavior from the free-electron excitation energies in this limit. Thus, the free-electron energies, $\epsilon_{k+q} - \epsilon_k = \vec{q}\cdot(2\vec{k}+\vec{q})/2m$, vanish in the limit of small q^2 while the superconductor possesses a finite energy gap, $2\epsilon_0$, for the single-particle-like excitations involved.

Wentzel's procedure is equivalent to assuming that to lowest order the excitation energies which appear in (12) may be replaced by their free-electron counterparts and that higher-order corrections may be included by a systematic expansion in powers of g. It is clear, however, that such an expansion will not converge in the limit $q^2 \to 0$, since the expansion parameter is of the order of $(m\epsilon_0/\vec{q}\cdot\vec{k})$. The BCS approach corresponds to replacing H' by the reduced or quasi-particle Hamiltonian. Indeed, it is the fact that $\vec{q}\cdot\vec{k}/m\epsilon_0 \to 0$ in this limit which is responsible for the London value of $K_{\mu\nu}(\vec{q})$ given by the BCS theory. In a more complete treatment, the BCS kernel would be corrected for the change in density of states due to the phonon-electron interaction, as shown by Rickayzen.[8] This correction should vanish for weak coupling, leading back to the London value, while Wentzel's approach would give a kernel which becomes vanishingly small in this limit.

The BCS approach will lead to gauge-invariant results only if the approximate eigenstates of H' are sufficiently accurate to satisfy the longitudinal f-sum rule. At present the BCS-Bogoliubov-Valatin states are not that accurate. This apparent lack of gauge invariance is resolved when the Coulomb interactions between electrons are included, since the plasmons play a dominant role in satisfying the sum rule in the long-wavelength limit. The authors have given elsewhere[9] an extension of the BCS theory of the Meissner effect in an arbitrary gauge and have shown the BCS result to be essentially unaltered.

It is a pleasure to thank Dr. Cecile DeWitt and Dr. Philippe Nozieres for the gracious hospitality extended to us by the Ecole d'Eté de Physique Théorique. We also thank Dr. G. Wentzel for a preprint of his paper and for valuable discussions.

*National Science Foundation Senior Post-Doctoral Fellow, on leave of absence from Princeton University, 1957-58; present address: The Institute for Advanced Study, Princeton, New Jersey.

†National Science Foundation Post-Doctoral Fellow, on leave of absence from the Department of Physics and The Institute for the Study of Metals, University of Chicago, Chicago, Illinois, 1957-58.

[1]G. Wentzel, Phys. Rev. 111, 1488 (1958).

[2]Bardeen, Cooper, and Schrieffer, Phys. Rev. 108, 1175 (1957), hereafter referred to as BCS.

[3]N. N. Bogoliubov, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 66 (1958).

[4]J. Valatin, Nuovo cimento 7, 843 (1958).

[5]We choose units $\hbar = c = 1$.

[6]Note that $K_{\mu\nu}(\vec{q})$ should not be confused with K defined by (2).

[7]M. R. Schafroth, Helv. Phys. Acta 24, 645 (1951).

[8]G. Rickayzen, Phys. Rev. 111, 817 (1958).

[9]D. Pines and J. R. Schrieffer (to be published).

CYCLOTRON RESONANCE IN FLAMES*

Jürgen Schneider and F. W. Hofmann
Department of Physics, Duke University,
Durham, North Carolina
(Received November 10, 1958)

Cyclotron resonance of electrons in a gas discharge has been detected by Ingram and Tapley.[1] We have observed this effect in low-pressure flames at a frequency near 24 kMc/sec. Starting from the classical equation of motion of an electron moving in a homogenous, constant magnetic field H and an alternating electric field one obtains, for the complex electric conductivity σ perpendicular to H,

$$\sigma = \frac{Ne^2}{m} \frac{\nu + i\omega}{(\nu + i\omega)^2 + \omega_c^2}, \quad (1)$$

where e and m are the electronic charge and mass, N is the number of free electrons per cc, ω the frequency of the applied electric field, ν the collision frequency of electrons with neutral molecules, and $\omega_c = eH/mc$ the cyclotron frequency. The same equation can be deduced from the Boltzmann transfer equation assuming ν is independent of the electron velocity. Cyclotron resonance will occur near $\omega_c = \omega$ if the electron is able to make at least one revolution in the magnetic field before colliding with another particle, i.e., for $\omega/\nu > 1$. In an acetylene-air flame we found at K-band frequencies and at atmospheric pressure no indication of a cyclotron resonance, whereas at 100 mm Hg a broad absorption appeared which became considerably sharper as the pressure was decreased. Figure 1 shows