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TWO-FLUID MODEL OF SUPERCONDUCTIVITY

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Analogies between superconductivity and superfluidity in liquid helium have been noted by many people.[1,2] We wish to point out here that Landau's interpretation[3] of the two-fluid model of liquid helium in terms of motion of elementary excitations can be extended to provide a basis for a two-fluid model for superconductivity. The quasi-particle excitations of the Bardeen, Cooper, Schrieffer[4] (BCS) theory of superconductivity obey Fermi rather than Bose statistics, but the phenomenological equations of the two-fluid model are formally identical. The main assumption is that interactions involve only relative momenta, and so are unchanged by a uniform displacement of the whole system in momentum space. While this is strictly true for helium, it is only an approximation for metals because of the periodic field of the lattice. The physical content is, of course, quite different for the two cases. The connecting link is the common momentum in the ground state of the pairs in the BCS theory and of particles in the Einstein-Bose condensation of helium.

In superconductors there is a coherence distance of the order of 10^{-4} cm, and it is only when changes in motion occur slowly over distances of this order that point relations can be used. We shall be concerned here only with this limit, where the Pippard nonlocal relation between current and field goes over into the usual London equations[1]:

$$-c \operatorname{curl} \Lambda_T \vec{j}_S = \vec{H}; \ \partial(\Lambda_T \vec{j}_S)/\partial t = \vec{E}. \qquad (1)$$

Here Λ_T is the London parameter, which in a free-electron approximation is equal to m/ne^2 at $T=0$ and increases to ∞ as $T \rightarrow T_C$. One may write $j_S = -e\rho_S v_S/m$, where ρ_S is the density and v_S the velocity of the superfluid component. At T=0, $\rho_S = \rho = nm$, and at some finite temperature,

$$\rho_S/\rho = 1 - \rho_n/\rho = \Lambda/\Lambda_T. \qquad (2)$$

The acceleration equation then becomes $\dot{\vec{v}}_S = -e\vec{E}/m$ at any temperature. An expression for (2) was derived by BCS by treating the external field as a perturbation.

An identical expression for ρ_S may be obtained by following Landau's derivation of the two-fluid model. We shall use a free-electron model, although the analysis can be extended readily to a more general Bloch model for the normal metal. The quasi-particle excitations have an energy $E(p) = (\epsilon^2 + \epsilon_0^2)^{1/2}$, where $\epsilon = (p^2/2m) - E_F$ is the Bloch energy measured relative to the Fermi surface and ϵ_0 is the temperature-dependent energy gap parameter. First consider the superfluid at rest (corresponding to a zero-momentum pairing condition) and suppose that the excitations (normal component) have a net momentum $\vec{J}_n$. Their distribution function, $f(\vec{p})$, is chosen to minimize the free energy, F, subject to the subsidiary condition

$$\vec{J}_n = \sum \vec{p} f(\vec{p}), \qquad (3)$$

Note that only "single" excitations in the BCS sense contribute to (3). If $\vec{v}$ is the Lagrange multiplier for $\vec{J}_n$,

$$\delta F - \vec{v} \cdot \delta \vec{J}_n = 0, \qquad (4)$$

giving, with a slight generalization of the procedure of BCS,

$$f(\vec{p}) = \frac{1}{1 + \exp\{[E(p) - \vec{v}\cdot\vec{p}]/kT\}}. \qquad (5)$$

For small $\vec{v}$, $\vec{J}_n$ is porportional to $\vec{v}$, and the coefficient of proportionality is defined as the normal density, ρ_n. This gives

$$\rho_n = \lim_{\vec{v}\to 0} (\vec{J}_n/\vec{v}) = -\frac{4\pi}{3h^3} \int_0^\infty p^4 \frac{d}{dE}\left(\frac{1}{1+e^{E(p)/kT}}\right) dp, \quad (6)$$

which is identical with Landau's expression, except for use of the Fermi in place of the Einstein-Bose distribution function. It also agrees with that derived from (2) with the BCS value for Λ/Λ_T.

If the whole system is displaced in momentum space and moves with a velocity $\vec{v}_s$, the pairs have a common velocity $\vec{v}_s$. Defining $\vec{v}_n = \vec{v} + \vec{v}_s$ and $\rho_s = \rho - \rho_n$, it follows quite generally that for small v,

$$\vec{J} = \rho\vec{v}_s + \rho_n\vec{v} = \rho_s\vec{v}_s + \rho_n\vec{v}_n, \quad (7)$$

and the associated increase in free energy is

$$\Delta F = \tfrac{1}{2}\rho_n \vec{v}_n^{\,2} + \tfrac{1}{2}\rho_s \vec{v}_s^{\,2}. \quad (8)$$

The other equations of the two-fluid model of liquid He also follow. In particular, it is possible to construct states, in the way Feynman[5] has done for He, for which $\vec{v}_s$ is a slowly varying function of position. As for He, it is required that curl $\vec{v}_s = 0$.

The two-fluid model provides a basis for a discussion of persistent currents, particularly if we neglect magnetic effects. If there is a current-carrying state with $\vec{v}_s = 0$, scattering of individual particles can reduce $\vec{v}_n$ to zero, but such scattering will not change $\vec{v}_s$. Only a force which acts on all or a large fraction of the electrons can do so. A more complete discussion of persistent currents would require taking the Meissner effect into account and use of nonlocal relations.

Because of the very high attenuation of the normal component from interaction with the crystal lattice, phenomena such as second sound would be difficult to observe in superconductors.[2] It is possible that the peculiar results reported recently by Spiewak[6] on magnetic field dependence of high-frequency penetration may be associated with a counter flow of normal and superfluid components perpendicular to the surface in the penetration region. The wavelength of second sound in tin at the frequency she used (10^9 cps) is expected to be of the order of the penetration depth. However, use of local relations outlined above is questionable for this problem because, except very near T_c, the coherence distance is larger than the penetration depth.

[1]F. London, Superfluids (John Wiley and Sons, New York, 1954), Vols. 1 and 2.
[2]C. J. Gorter, Progress in Low-Temperature Physics (North Holland Publishing Company, Amsterdam, 1955), Chap. 1.
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[4]Bardeen, Cooper, and Schrieffer, Phys. Rev. 108, 1175 (1957).
[5]R. P. Feynman, Progress in Low-Temperature Physics (North Holland Publishing Company, Amsterdam, 1955), Chap. 2.
[6]M. Spiewak, Phys. Rev. Lett. 1, 136 (1958).

GENERATION OF PRISMATIC DISLOCATION LOOPS IN SILICON CRYSTALS

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The occurrence of helical dislocations in silicon was recently reported by the author.[1] They are formed when gold and copper are diffused simultaneously into the crystals at 1200°C, followed by quenching as in the method used for copper decoration.[2] Diffusion of gold alone forms helices which can be detected by etching. The detailed mechanism whereby these dislocations are formed has not yet been fully established. Similar helical dislocations have been observed in as-grown germanium crystals by Tweet.[3] In this case there is little doubt that climb induced by the condensation of vacancies is responsible for the phenomenon. For the purposes of this discussion it will be assumed that this process is also involved in the formation of helical dislocations in silicon crystals.

Helical dislocations have previously been observed in calcium fluoride crystals by Bontinck and Dekeyser.[4] It has been proposed that these dislocations result from climb processes in which vacancies are generated.[4-7]

Amelinckx et al.[6] have suggested that prismatic dislocation loops found in association with helices are formed by the cancellation of the screw component in the region of the loops by an interacting dislocation. Generation of similar loops by a mechanism involving only a single helical dislocation has been observed in silicon. Figure 1 shows a typical system which allows certain qualitative deductions about the generation of the dislocations to be made. The following mechanism is suggested:

At elevated temperature, there exists initially