When these conditions are required not merely to be valid on one hypersurface x^0 = constant but to remain valid as Ψ develops with time (we are using a Schrödinger picture), they give rise to two sets of conditions on Ψ, which turn out to be formally identical with the ϕ- and χ-equations, respectively, of Dirac's formalism. Moreover, the Hamiltonian derived from the modified Lagrangian is just the expression (2).

Thus, after applying to Ψ the unitary transformation which is the quantum-mechanical equivalent to Dirac's canonical transformation and making the substitution $\pi^{\mu\nu}(x^a)=i\delta/\delta g_{\mu\nu}(x^a)$, we get as the primary conditions [see Eq.(1)]

$$(\delta/\delta g_{\mu 0})\Psi = 0, \tag{7}$$

of which the solution is $\Psi = \Psi\{g_{rs}\}$.

We now consider the secondary conditions

$$\mathcal{H}_u\Psi = 0, \tag{8}$$

$$\mathcal{H}_L\Psi = 0. \tag{9}$$

The commutators of $\mathcal{H}_u$, $\mathcal{H}_L$ are readily evaluated from the expressions (3), (4) as $-i$ times the corresponding Poisson brackets: they are

$$[\mathcal{H}_u(x), \mathcal{H}_v(y)] = i[(\partial/\partial x^v)\{\mathcal{H}_u(x)\delta^{(3)}(x-y)\} - (\partial/\partial y^u)\{\mathcal{H}_v(y)\delta^{(3)}(x-y)\}], \tag{10}$$

$$[\mathcal{H}_u(x), \mathcal{H}_L(y)] = -i\mathcal{H}_{L,u}\delta^{(3)}(x-y). \tag{11}$$

Given arbitrary patch functions $f^u(x)$, we may rewrite (3) as

$$i\int d^3x\, f^u\mathcal{H}_u = \int d^3x\,(f^u g_{rs,u} + f^u{}_{,r}g_{us} + f^u{}_{,s}g_{ru})\delta/\delta g_{rs}\,.$$

We compare this expression with the variation induced in $g_{rs}(x)$, $g_{rs}'(x) - g_{rs}(x) = -(a^u g_{rs,u} + a^u{}_{,r}g_{us} + a^u{}_{,s}g_{ru})$, by an arbitrary infinitesimal variation of the coordinates, $x'^u = x^u + a^u(x)$. It is clear that $\mathcal{H}_u(x)$ is just the set of infinitesimal generators of the group of general coordinate transformations on the potentials g_{rs}. Equation (10) exhibits the structure constants of this group; Eq. (11) merely expresses the fact that $\mathcal{H}_L$ is a scalar density. The conditions (8) now tell us that Ψ is a functional of those combinations of the g_{rs} which are invariant under general coordinate transformations, that is, of the three eigenvalues of the Riemann tensor $R^{(3)}{}_{rs}$ at all points of the manifold.

The remaining problem, which will not be discussed here, is to use the last condition (9) to eliminate a further degree of freedom. The two remaining invariants of $R^{(3)}{}_{rs}$, on which Ψ will then depend, will correspond to the two independent states of polarization of a graviton.

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 326 (1958).
[2] P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958).

PROPOSAL FOR A SOURCE OF POLARIZED PROTONS

A. Abragam and J. M. Winter
Centre d'Etudes Nucléaires de Saclay,
BP2 à Gif-sur-Yvette,
Seine et Oise, France
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In a cyclotron, protons are produced by ionizing hydrogen atoms in the center of the magnet that is in a strong field where the electron spin $\vec{S}$ and the proton spin $\vec{I}$ are uncoupled. The component I_z of the proton spin along the field is then a good quantum number and it is generally assumed that it is left unchanged in the process of ionization of the hydrogen atoms. If these atoms arrive in the ionizing region in states where I_z has one value only, say $+\frac{1}{2}$, completely polarized protons could be available for acceleration.

The variation of the four energy levels of a hydrogen atom as a function of the applied field is represented in Fig. 1.

It is relatively easy to separate in an incoming beam of unpolarized hydrogen atoms by a Stern-Gerlach experiment those in states a and b from those in states c and d, because of the large difference in magnetic moment between the two groups of states. The further separation by the same method of the states a and b, necessary if atoms with $I_z = \frac{1}{2}$ only, are to be selected, although feasible, is far more difficult.

In the present proposal, once a beam of atoms distributed with equal populations between the states a and b has been prepared by a Stern-Gerlach device, all the atoms in the state b are transferred into the state d by means of a radio-frequency field $H_1\cos\omega t$ parallel to the field H_0, by the adiabatic passage method.[1] All proton spins are then in the state $I_z = \frac{1}{2}$.

The matrix element for the transition is

$$A\cos\omega t = 2\beta H_1(b|S_z|d)\cos\omega t = \sqrt{2}\,\beta H_1\,\Delta W[\Delta W^2 + (2\beta H_0)^2]^{-1/2}\cos\omega t, \tag{1}$$

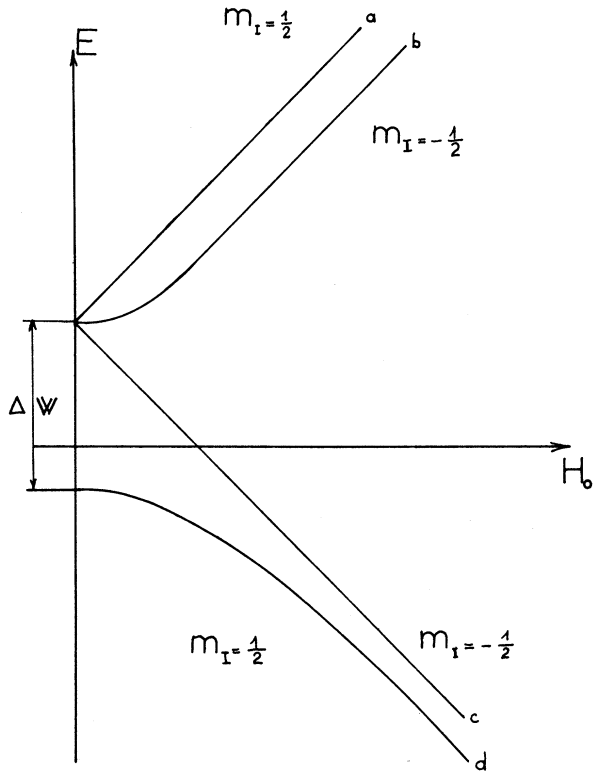

FIG. 1. Energy levels of a hydrogen atom in an applied magnetic field.

where β is a Bohr magneton and ΔW the zero-field splitting. A will have an appreciable value except if $2\beta H_0 >> \Delta W$.

The behavior of a system with two energy levels E_b and E_d perturbed by a Hamiltonian $\mathcal{H}_1$ such that $(b|\mathcal{H}_1|d) = A\cos\omega t$ with ω in the vicinity of $(E_b - E_d)/\hbar$ can be described as that of a fictitious spin $K = \frac{1}{2}$ with a gyromagnetic ratio γ', placed in a dc field H_0' perpendicular to a rotating field of amplitude H_1', those quantities being defined through

$$\gamma' H_0' = \omega_0, \quad \gamma' H_1' = A/2. \tag{2}$$

The interchange of the populations between the states b and d corresponds to the reversal of the polarization K_z of the fictitious spin K. The conditions for that reversal are[1]

(1) $dH_0'/dt << \gamma' H_1'^2$,

(2) $\Delta H_0' = H_0'(\text{in}) - H_0'(\text{out}) >> H_1$,

where $H_0'(\text{in})$ and $H_0'(\text{out})$ are the values of the fictitious dc field H_0' crossed by the spin K upon entering and leaving the radio-frequency region of length Δx, with the condition $\gamma' H_0'(\text{in}) < \omega < \gamma' H_0'(\text{out})$. Transposed to the real system, these conditions become

$$(1) \quad \frac{d}{dt}\frac{(E_b - E_a)}{\hbar} \cong \frac{\Delta(E_b - E_a)}{\Delta x \cdot \hbar}\frac{dx}{dt} << \beta^2 H_1^2/\hbar^2,$$

where dx/dt is the mean velocity of hydrogen atoms of the order of 2×10^5 cm/sec;

$$(2) \quad \Delta\omega = \Delta(E_b - E_d)/\hbar >> \beta H_1/\hbar .$$

The choice of the frequency ω and thus of the field H_0, such that $E_b(H_0) - E_d(H_0) = \hbar\omega$ is a compromise between the necessity to minimize the influence of stray fields from the cyclotron and to keep an appreciable value for the matrix element $(b|S_z|d)$; ω = 2400 Mc/sec seems to be reasonable. The length Δx being of the same order as the radio-frequency wave length λ = 12 cm, if one chooses $\Delta\omega \sim \omega/100$ the two inequality conditions (1) and (2) will be satisfied with H_1 = 1 gauss.

If the populations of the states b and d were equalized rather than interchanged by the rf field, the resulting proton polarization $\langle I_z\rangle/I$ would only be $\frac{1}{2}$.

[1]F. Bloch, Phys. Rev. 70, 460 (1946).

ASYMMETRY IN SCATTERING OF 150-Mev POLARIZED PROTONS IN NUCLEAR EMULSIONS*

B. T. Feld and B. C. Maglić
Physics Department and Laboratory for Nuclear Science,
Massachusetts Institute of Technology,
Cambridge, Massachusetts
(Received October 30, 1958)

The use of a volume of emulsion in the dual role of scatterer and detector (with the attendant large available solid angle and high angular resolution) appears to contain some unique features as a method for measuring the polarization of high-energy protons. We have in mind especially situations in which the proton beams are of low intensity, and/or in which the beam shape is unfavorable or poorly known; for example, the measurement of proton polarization in π-p scattering, photoproduction and photodisintegration reactions, etc.

Grigor'ev[1] has observed polarization in the