Table I. Comparison of the theoretical value of $x = V_I - V_C$ with its value $(1/3)|\mathrm{Re}V_C|$ obtained from the analysis of proton-carbon scattering by Levinson and Banerjee.[a] V_C is the ordinary, elastic optical potential; V_I, the optical potential appropriate to propagation preceding a "direct interaction." V_C data are from Riesenfeld and Watson.[b] E is the proton energy. All quantities in Mev.

E	$\lvert\mathrm{Im}V_C\rvert$	$(1/3)\lvert\mathrm{Re}V_C\rvert$	x
17	8.5	15.1	11.9
20	9	12.7	11.6
31.5	15	11.7	15.4
50	12.6	9.3	10.1
100	7.7	6.5	4.5
150	7.4	5.0	3.5
200	8.0	4.7	3.2

[a]See reference 2.

[b]W. B. Riesenfeld and K. M. Watson, Phys. Rev. 102, 1157 (1956).

velopment with the Levinson-Banerjee analysis of a single inelastic transition would have required calculating the transition from packet state γ, not known in detail, to a particular final state. However, since the impulse approximation applies at most energies of interest, detailed features of the nuclear states influence the scattering much less than does V_I, which alters the kinematics of the final collision, and hence the angular distribution.[7]

*Operated by Union Carbide Corporation for the U. S. Atomic Energy Commission.

[1]Nomenclature: inelastic collision means one in which the nucleus is left in a different energy eigenstate. Units: $\hbar = 1$ throughout.

[2]C. A. Levinson and M. K. Banerjee, Ann. Phys. 3, 67 (1958).

[3]K. M. Watson, Phys. Rev. 105, 1388 (1956).

[4]The inelastic part of ψ_{coh}, which is not coherent with the incident beam, concerns instead relative coherence defined by M. Lax, Revs. Modern Phys. 23, 287 (1951); see also reference 3.

[5]H. A. Bethe, Phys. Rev. 103, 1353 (1956).

[6]The convenience of defining the nuclear medium with respect to a packet state following certain inelastic collisions was pointed out by K. M. Watson, reference 3.

[7]$\overline{L}_\beta$ is the nuclear excitation accompanying correlation scattering in impulse approximation. An additional excitation $(2M)^{-1}\kappa^2$ would occur if either direct (noninterference) scattering or repeated inelastic collisions with β were considered. See G. C. Wick, Phys. Rev. 94, 1228 (1954), and T. K. Fowler (to be published).

INTEGRATION OF SECONDARY CONSTRAINTS IN QUANTIZED GENERAL RELATIVITY

Peter W. Higgs
Department of Mathematics, Imperial College, London, England
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Using his generalized Hamiltonian dynamics[1] and treating x^0 as the time parameter, Dirac[2] has recently shown that the primary constraints (ϕ-equations) in general relativity can be brought by a canonical transformation into the form

$$\pi^{\mu 0} \approx 0, \tag{1}$$

where $\pi^{\mu\nu}$ is the momentum conjugate to $g_{\mu\nu}$. He has shown moreover that the transformed Hamiltonian in the absence of matter, after Eq. (1) has been taken into account, is

$$H = \int d^3x \{(g^{00})^{-1/2}\mathcal{H}_L + g_{r0}e^{rs}\mathcal{H}_s\}, \tag{2}$$

in which

$$e^{rs}g_{st} = \delta_t{}^r,$$

$$\mathcal{H}_u = \pi^{rs}g_{rs,u} - 2(\pi^{rs}g_{ru})_{,s} \tag{3}$$

$$\mathcal{H}_L = K^{-1}(g_{ra}g_{sb} - \tfrac{1}{2}g_{rs}g_{ab})\pi^{rs}\pi^{ab} - KR^{(3)}, \tag{4}$$

where $-K^2 = \det g_{rs}$ and $R^{(3)}$ is the fully contracted Riemann tensor of the spatial manifold. The secondary constraints (χ-equations) are then

$$\mathcal{H}_u \approx 0, \quad \mathcal{H}_L \approx 0. \tag{5}$$

(The symbol $\approx$ denotes a "weak" equation. Greek indices run from 0 to 3; Latin indices from 1 to 3.)

If one wishes to quantize general relativity, an alternative procedure to that of Dirac is to render the Lagrangian nonpathological by adding to the Lagrangian density the term

$$\tfrac{1}{2}(-g)^{1/2}g_{\mu\nu}S^\mu S^\nu,$$

where $S^\mu = (g^{\mu\alpha}g^{\beta\rho} - \tfrac{1}{2}g^{\mu\rho}g^{\alpha\beta})g_{\alpha\beta,\rho}$, and then to restore the Einstein equations of motion for matrix elements of the potentials by imposing on the state functionals $\Psi\{g_{\mu\nu}(x^\alpha)\}$ the subsidiary conditions (harmonic coordinate conditions)

$$S^\mu\Psi = 0. \tag{6}$$

When these conditions are required not merely to be valid on one hypersurface x^0 = constant but to remain valid as Ψ develops with time (we are using a Schrödinger picture), they give rise to two sets of conditions on Ψ, which turn out to be formally identical with the ϕ- and χ-equations, respectively, of Dirac's formalism. Moreover, the Hamiltonian derived from the modified Lagrangian is just the expression (2).

Thus, after applying to Ψ the unitary transformation which is the quantum-mechanical equivalent to Dirac's canonical transformation and making the substitution $\pi^{\mu\nu}(x^a)=i\delta/\delta g_{\mu\nu}(x^a)$, we get as the primary conditions [see Eq.(1)]

$$(\delta/\delta g_{\mu 0})\Psi = 0, \tag{7}$$

of which the solution is $\Psi = \Psi\{g_{rs}\}$.

We now consider the secondary conditions

$$\mathcal{H}_u\Psi = 0, \tag{8}$$

$$\mathcal{H}_L\Psi = 0. \tag{9}$$

The commutators of $\mathcal{H}_u$, $\mathcal{H}_L$ are readily evaluated from the expressions (3), (4) as $-i$ times the corresponding Poisson brackets: they are

$$[\mathcal{H}_u(x), \mathcal{H}_v(y)] = i[(\partial/\partial x^v)\{\mathcal{H}_u(x)\delta^{(3)}(x-y)\} - (\partial/\partial y^u)\{\mathcal{H}_v(y)\delta^{(3)}(x-y)\}], \tag{10}$$

$$[\mathcal{H}_u(x), \mathcal{H}_L(y)] = -i\mathcal{H}_{L,u}\delta^{(3)}(x-y). \tag{11}$$

Given arbitrary patch functions $f^u(x)$, we may rewrite (3) as

$$i\int d^3x\, f^u\mathcal{H}_u = \int d^3x\,(f^u g_{rs,u} + f^u{}_{,r}g_{us} + f^u{}_{,s}g_{ru})\delta/\delta g_{rs}\,.$$

We compare this expression with the variation induced in $g_{rs}(x)$, $g_{rs}'(x) - g_{rs}(x) = -(a^u g_{rs,u} + a^u{}_{,r}g_{us} + a^u{}_{,s}g_{ru})$, by an arbitrary infinitesimal variation of the coordinates, $x'^u = x^u + a^u(x)$. It is clear that $\mathcal{H}_u(x)$ is just the set of infinitesimal generators of the group of general coordinate transformations on the potentials g_{rs}. Equation (10) exhibits the structure constants of this group; Eq. (11) merely expresses the fact that $\mathcal{H}_L$ is a scalar density. The conditions (8) now tell us that Ψ is a functional of those combinations of the g_{rs} which are invariant under general coordinate transformations, that is, of the three eigenvalues of the Riemann tensor $R^{(3)}{}_{rs}$ at all points of the manifold.

The remaining problem, which will not be discussed here, is to use the last condition (9) to eliminate a further degree of freedom. The two remaining invariants of $R^{(3)}{}_{rs}$, on which Ψ will then depend, will correspond to the two independent states of polarization of a graviton.

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 326 (1958).
[2] P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958).

PROPOSAL FOR A SOURCE OF POLARIZED PROTONS

A. Abragam and J. M. Winter
Centre d'Etudes Nucléaires de Saclay,
BP2 à Gif-sur-Yvette,
Seine et Oise, France
(Received October 29, 1958)

In a cyclotron, protons are produced by ionizing hydrogen atoms in the center of the magnet that is in a strong field where the electron spin $\vec{S}$ and the proton spin $\vec{I}$ are uncoupled. The component I_z of the proton spin along the field is then a good quantum number and it is generally assumed that it is left unchanged in the process of ionization of the hydrogen atoms. If these atoms arrive in the ionizing region in states where I_z has one value only, say $+\frac{1}{2}$, completely polarized protons could be available for acceleration.

The variation of the four energy levels of a hydrogen atom as a function of the applied field is represented in Fig. 1.

It is relatively easy to separate in an incoming beam of unpolarized hydrogen atoms by a Stern-Gerlach experiment those in states a and b from those in states c and d, because of the large difference in magnetic moment between the two groups of states. The further separation by the same method of the states a and b, necessary if atoms with $I_z = \frac{1}{2}$ only, are to be selected, although feasible, is far more difficult.

In the present proposal, once a beam of atoms distributed with equal populations between the states a and b has been prepared by a Stern-Gerlach device, all the atoms in the state b are transferred into the state d by means of a radio-frequency field $H_1\cos\omega t$ parallel to the field H_0, by the adiabatic passage method.[1] All proton spins are then in the state $I_z = \frac{1}{2}$.

The matrix element for the transition is

$$A\cos\omega t = 2\beta H_1(b|S_z|d)\cos\omega t = \sqrt{2}\,\beta H_1\,\Delta W[\Delta W^2 + (2\beta H_0)^2]^{-1/2}\cos\omega t, \tag{1}$$