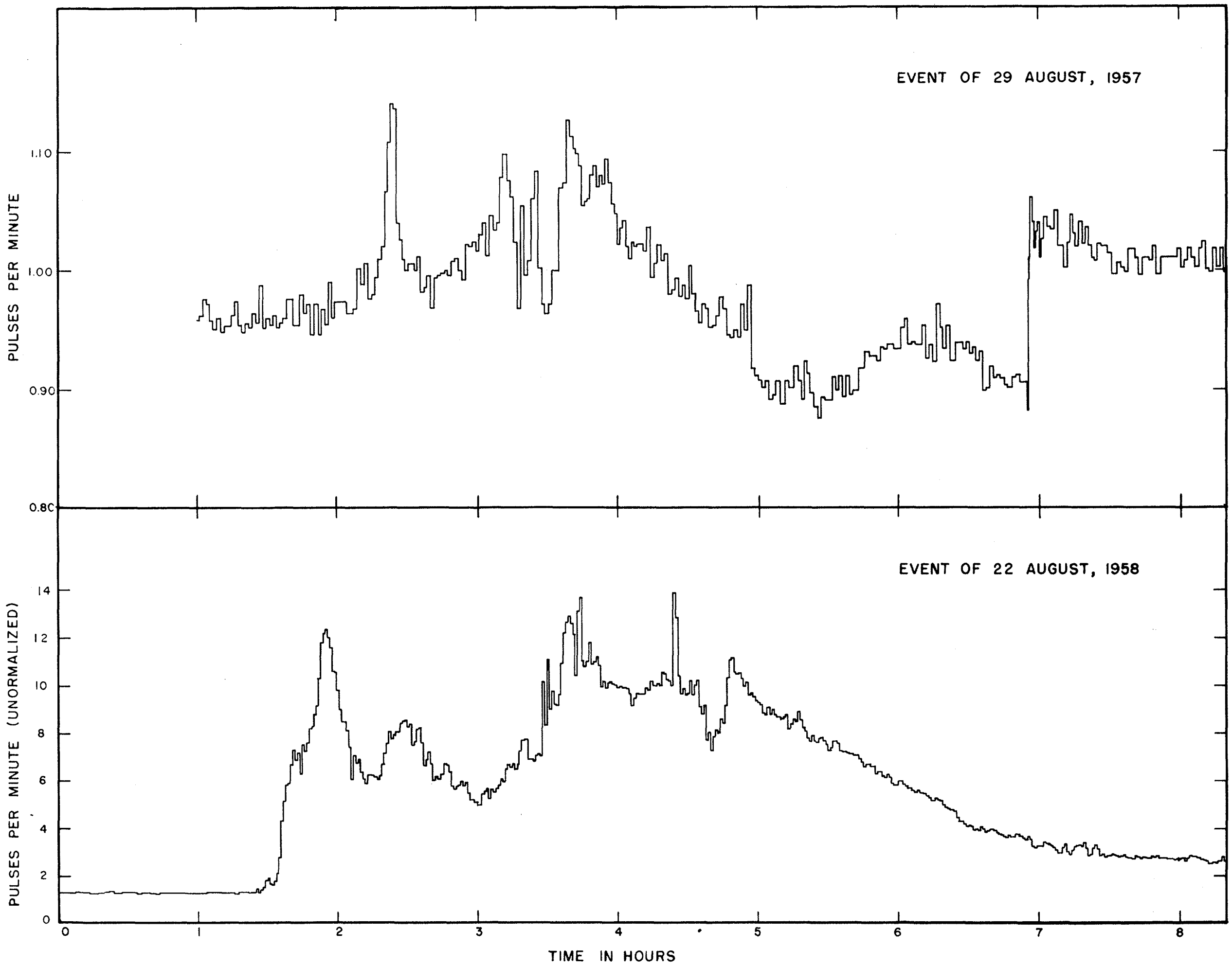

FIG. 2. Ion chamber response to x-ray event of August 29-30, 1957 (top figure), and proton event of August 22-23, 1958 (bottom figure).

was very little if any magnetic disturbance present during this event.

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[1]The author is grateful to Dorothy Trotter of the High Altitude Observatory and to the Canadian Defense Research and Telecommunications Establishment for information on this event.

[2]H. V. Neher, Progress in Cosmic-Ray Physics (North-Holland Publishing Company, Amsterdam, 1952), Vol. 1, p. 261.

[3]U. S. National Committee for the IGY, Summary of Advances in the U. S. IGY Program for Cosmic-Ray Research, June 13, 1958. Presented by S. E. Forbush at the Fifth General Assembly of CSAGI, Moscow, July 30-August 9, 1958.

[4]K. A. Anderson, Phys. Rev. 111, 1397 (1958).

DETERMINATION OF THE π-π INTERACTION STRENGTH FROM π-N SCATTERING*

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Many people have considered the role of the π-π interaction in π-N scattering, particularly in the pion production process.[1] Among the effects which can be qualitatively argued to be a result of the π-π interaction are the following:

(a) The nonresonant behavior of the $i=\frac{1}{2}$ S-wave phase shift at low energies, which indicates a fairly long range for the S-wave interaction.[2]

(b) The positive value of δ_{13} above ~200 Mev,[3] and the positive rise of the $i=\frac{1}{2}$ scattering amplitude in the region 200 to 500 Mev.[4] Taking into account interactions of only the Born approximation (Tamm-Dancoff), one finds that the P-wave amplitudes are predicted to be negative, and the positive D-wave amplitude is very small up to at least a Bev.[5]

(b′) The large value of the meson production cross section near threshold in the $i=\frac{1}{2}$ state,[6] but <u>not</u> in the $i=\frac{3}{2}$ state. In the latter case the observed production cross section is roughly in agreement with static model calculations, but these are a factor of about 5 too small in the $i=\frac{1}{2}$ state.[7]

(c) The maximum of the total cross section in the $i=\frac{1}{2}$ state at ~ 0.9 Bev (the "second maximum").[8] It was suggested that this reflected a resonance in the π-π scattering,[9] but it was subsequently pointed out that the spread of momentum of the self-field pions would make the resulting maximum in the π-N cross section very broad.[8] Further, there is no evidence that the final pair of pions tend to have the presumed resonant relative energy. But at this energy the final state resonant π-N interaction is probably important, as well as perhaps "multiple scattering" in the self field, so that the situation is too involved to allow ruling out of this mechanism. For instance, if the π-N resonant final state is indeed important, this may tend to select the momentum of the virtual pion, so that the effective momentum spread is really less.

(d) The large size of the high-energy total cross section, and the consequent backward peaking of the nucleon's angular distribution, in both elastic and inelastic events of low multiplicity.[1] This clearly indicates that the incident π interacts with the virtual pion cloud of the nucleon, rather than with a core.

A way of getting a quantitative estimate of the π-π interaction strength is by isolating the contribution of the one-pion exchange diagram (Fig. 1) from all others. One can attempt to do this by using the fact that this matrix element becomes infinite when the exchanged pion is real, i.e., for a momentum transfer Δ to the nucleon of about $i\mu$. One cannot of course reach this physically, but one can try to extrapolate the cross section as a function of Δ^2 to $\Delta^2 \approx -\mu^2$. This is just the principle of the threshold theorem, used for instance in the determination of the π-N coupling constant from π-N scattering. Chew[10] has recently suggested the application of the extrapolation procedure to a determination of the π-N coupling constant from N-N scattering. The present case is similar, but in order to deduce the dependence of the matrix element on momentum transfer, one must extract from the cross

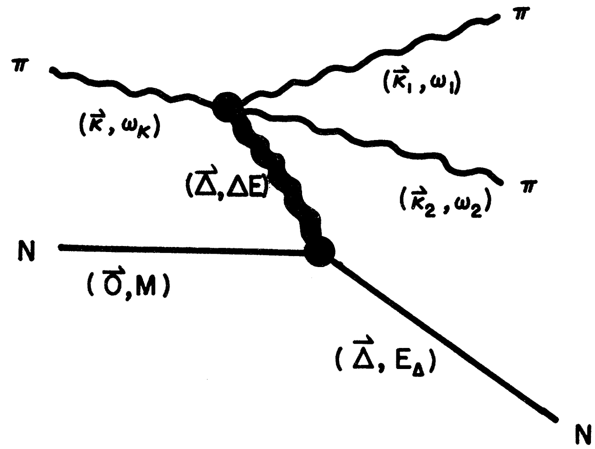

FIG. 1. Feynman diagram of the pole-containing matrix element. Beneath each line is written its momentum and energy.

section the phase space of the three-body final state.[11]

It is convenient to work in the laboratory system. We have, for the differential cross section for single pion production,

$$d\sigma=(2\pi)^4 |\mathfrak{M}|^2 d^3k_1 d^3k_2 \delta(\omega_1+\omega_2+E_\Delta-W), \qquad (1)$$

where $W=\omega_k+M$ the total energy, and we have put the velocity of the incident pion equal to unity. The pole of $\mathfrak{M}$ occurs at $T_\Delta=-\mu^2/2M$ where $T_\Delta=E_\Delta-M$, the kinetic energy of the final nucleon. It is convenient to describe the final state by the relative momentum of the final mesons (in their center of mass) $\vec{\xi}$ and the total momentum of the mesons $\vec{P}$ or, equivalently, the momentum $\vec{\Delta}$ of the nucleon. The pertinent Jacobian is

$$d^3k_1 d^3k_2 = d^3\xi d^3P \frac{1}{\omega_\xi \Omega_\xi}\left[\Omega_\xi^2 - \frac{(\vec{\xi}\cdot\vec{P})^2}{4\omega_\xi^2}\right],$$

where

$$\omega_\xi^2 = \xi^2+\mu^2, \Omega_\xi^2 = \omega_\xi^2 + P^2/4 . \qquad (2)$$

Integrating over the direction of $\vec{\xi}$ (the length is fixed by energy conservation), we find that Eq. (1) becomes

$$d\sigma = 8\pi^5 \langle |\mathfrak{M}|^2 \rangle v_\xi [(W-E_\Delta)^2 - v_\xi^2 P^2/3] d^3\Delta, \qquad (3)$$

where

$$v_\xi = \frac{\xi}{\omega_\xi} = \left(\frac{\vec{k}\cdot\vec{\Delta} - WT_\Delta - \frac{3}{2}\mu^2}{\vec{k}\cdot\vec{\Delta} - WT_\Delta + \frac{1}{2}\mu^2}\right)^{1/2}. \qquad (4)$$

The $\langle\ \rangle$ on $|\mathfrak{M}|^2$ indicate an average. Finally,

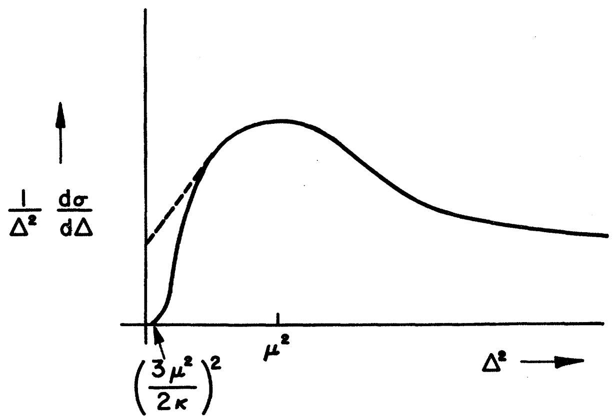

FIG. 2. Schematic sketch of the Δ dependence of the single pion production cross section. The deviation from the dashed line near threshold is the effect of the factor $C(\Delta)$, Eq. (5).

integrating over the direction of $\vec{\Delta}$ we find, for $k \gg \mu$ and $\Delta \ll M$ or k,

$$\frac{d\sigma}{d\Delta} \approx \frac{32}{3}\pi^6 \langle |\mathfrak{M}|^2 \rangle k^2 \Delta^2 C(\Delta) \tag{5}$$

where

$$C(\Delta) = \frac{(k\Delta - WT_\Delta - \frac{3}{2}\mu^2)^{3/2}}{k\Delta(k\Delta - WT_\Delta + \frac{1}{2}\mu^2)^{1/2}} . \tag{6}$$

Except very near the threshold, which is at $\Delta \approx \frac{3}{2}\mu^2/k$, the two pions are relativistic in their center-of-mass system, and $C(\Delta) \approx 1$.

The matrix element of the diagram of Fig. 1 (containing the pole) would be of the form

$$|\mathfrak{M}|^2 \sim \Delta^2/(\mu^2+\Delta^2)^2 \tag{7}$$

for small momenta, whereas other diagrams would contribute matrix elements which are roughly constant; thus the total squared matrix element would look something like Fig. 2. The size of the hump at $\Delta^2 = +\mu^2$ would be a measure of the strength of the π-π interaction.

Of course, the picture is distorted if other contributions to the single-pion production matrix elements interfere strongly with the pole term or have themselves a significant dependence on Δ in the region $\Delta \sim \mu$. The latter possibility is not a priori likely; all indications are that the nucleon is effectively smaller than μ^{-1}. Some evidence on this point is provided by Walker et al.,[12] Fig. 16. This exhibits $d\sigma/d\Delta$, which seems to be proportional to Δ^2 up to $\Delta \approx 500$ Mev/c. They have very few events at small Δ; in fact to investigate our point economically one must ignore events at $\Delta \lesssim 400$ Mev/c, say.

Because of the low energy of the recoil nucleon $\Delta = \mu (\approx 10$ Mev), it seems impractical to do the experiment of measuring $d\sigma/d\Delta$ with counters. The present techniques using diffusion cloud chambers, or perhaps bubble chambers, should be sufficient. One should discriminate against higher multiplicity events, although their inclusion might merely increase the "background" in Fig. 2.

As one goes to higher and higher energies, keeping Δ fixed, one expects that the matrix element of Fig. 1 remains constant (if the π-π cross section does), but all other diagrams become small[13] so that in principle it could be found exactly.

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*This work was supported by the U. S. Atomic Energy Commission.

[1] The most recent experimental papers, which contain references to earlier work, are W. D. Walker, Phys. Rev. 108, 872 (1957); G. Maenchen et al., Phys. Rev. 108, 850 (1957).

[2] See Drell, Friedman, and Zachariasen, Phys. Rev. 104, 236 (1956). With their cutoff [$\omega_{max} = 4.5\mu$], δ_1 goes through 90° at $k \approx 2.2\mu$. Thus to avoid this one needs to use a "Born approximation" which has a cutoff factor which decreases at energies small compared to M.

[3] See G. Puppi, The Eighth Annual International Conference on High-Energy Nuclear Physics, Geneva (unpublished.

[4] R. Sternheimer, Phys. Rev. 101, 384 (1956). See also reference 7.

[5] Chew, Goldberger, Low, and Nambu, Phys. Rev. 106, 1345 (1957).

[6] W. D. Walker et al., Bull. Am. Phys. Soc. Ser. II, 3, 104 (1958), and private communication from Dr. Walker.

[7] I must thank E. Kazes for drawing this to my attention. His is the latest calculation of pion production in the static model, and contains references to earlier work, Phys. Rev. 107, 1131 (1957).

[8] See, for instance, Cool, Piccioni, and Clark, Phys. Rev. 103, 1082 (1956).

[9] F. J. Dyson, Phys. Rev. 99, 1037 (1955); G. Takeda, Phys. Rev. 100, 440 (1955).

[10] G. F. Chew, University of California Radiation Laboratory Report UCRL-8283 (unpublished).

[11] We average over all final state parameters except Δ in order to "gain statistics" using a given number of events. One would of course learn more details about the π-π interaction if one could get away with more subdivision of the data.

[12] Walker, Hushfar, and Shephard, Phys. Rev. 104, 526 (1956).

[13] That is, except for those corresponding to "diffraction" meson production, in which the diffraction-scattered proton shakes off a meson. E. L. Feinberg and I.Pomeranchuk, Suppl. Nuovo cimento 4, 652 (1956). This cross section is very small.