percent or less to the value of χ at temperatures in the region of the critical point and above. (For lattices such as the plane triangular lattice which have no critical point, the relevant range is determined by the corresponding ferromagnetic Curie temperature.) Because of this the behavior of the susceptibility in the critical region is effectively determined by the energy alone. For plane lattices this is known exactly from the work of Onsager[5] and others,[6] and the expressions are valid at all temperatures. Consequently we may conclude that for loose-packed lattices, χ behaves in the critical region as $a + b(T-T_c)\log|T-T_c|$ where a and b are constants and T_c is the critical temperature. The energy is not known in closed form for three-dimensional lattices but the calculations of Wakefield[7] and of Domb and Sykes[8] indicate that the specific heat of a loose-packed lattice becomes infinite at T_c which again shows that the susceptibility should have a vertical tangent at the same temperature.

For close-packed lattices such as the plane triangular and the face-centered cubic, there is no specific heat singularity and the general result indicates that the susceptibility should not exhibit any singularity in the neighborhood of the corresponding ferromagnetic critical temperature.

The conclusion for plane lattices is borne out by an exact solution in the presence of a magnetic field of a particular class of two-dimensional Ising model in which normal Ising spins (with magnetic moments) are coupled together via "nonmagnetic spins" so yielding a type of "superexchange" interaction.

The generalization of the result (A) for spin $s > \frac{1}{2}$ has not been obtained. General arguments, however, indicate that similar physical conclusions should still be valid.

More detailed accounts of these investigations will be published in due course.

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CONDUCTION ELECTRON-MAGNETIC ION INTERACTION IN RARE EARTHS*

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The 4*f* electrons in the rare earths, which are responsible for the magnetic moments of the rare earth elements, are highly localized and presumably do not form a band; hence the direct interaction between 4*f* electrons on different atoms is expected to be fairly weak. The ion-ion interaction is presumably an indirect interaction via the conduction electrons in the manner suggested by Zener.[1] This conduction electron-magnetic ion interaction should also be manifested by the electrical resistivity.[2-4] It now appears that this interaction depends on the spin of the 4*f* shell and is involved in the electric and magnetic properties of the rare earth metals and in the superconductivity of alloys of the rare earths.

Electrical resistivity measurements on many of the rare earth metals have now been reported.[5-7] Figure 1 shows the electrical resistivity of Gd as a function of temperature. This is rather typical of the hexagonal close-packed rare earths Gd, Tb, Dy, Ho, Er, Tm, and Lu. The other rare earths do not show regions where the electrical resistivity varies linearly with temperature, so the present discussion is not pertinent for these elements. Following the proposal of Kasuya,[4] the electrical resistivity of a rare earth in the paramagnetic region might be ex-

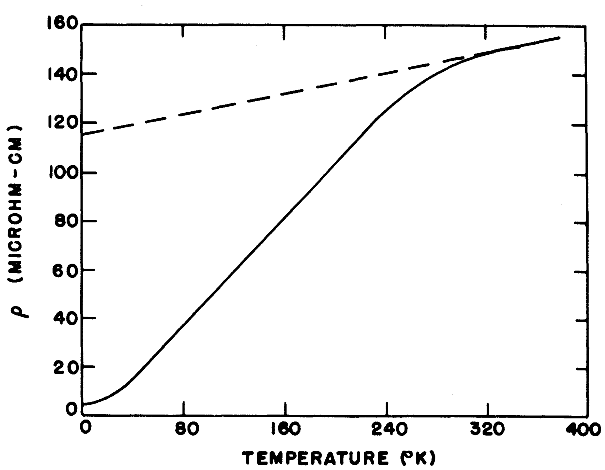

FIG. 1. The electrical resistivity of gadolinium as a function of temperature.

Table I. Resistivity values in microhm-cm.

Element	ρ_{res}	ρ_{corr}	ρ_{magn}
Gd	3.5	1.5	115.5
Tb	3.5	2.5	88.5
Dy	18.0	5.0	74.5
Ho	6.0	5.0	39.0
Er	8.5	4.5	25.4
Tm	28.7	4.4	10.2
Lu	12.0	4.9	1.6

pressed in the form $\rho=\rho_{th}+\rho_{res}+\rho_{magn}$ where ρ_{th} is the resistivity due to phonon scattering of electrons, ρ_{res} is the residual resistivity, and ρ_{magn} is the term arising from the magnetic disorder scattering of electrons.[8] ρ_{magn} is determined by extrapolating the resistivity in the paramagnetic region to 0°K, and subtracting the residual resistivity and a correction ρ_{corr} using the Grüneisen temperature dependence for the usual ideal resistivity. These quantities are shown in Table I. In Fig. 2 we have plotted ρ_{magn} as a function of $S(S+1)$. This indicates that an exchange interaction between the conduction electrons and the spin of the magnetic ions is one of the factors responsible for the scattering of conduction electrons. The ρ_{magn} term does not depend on J or gJ as one might expect.

Néel has previously noted that paramagnetic Curie temperatures of the rare earths follow a relation which can be deduced by assuming an interaction between spins only.[9] It is reasonable to assume that the exchange interaction between conduction electrons and the spin of magnetic

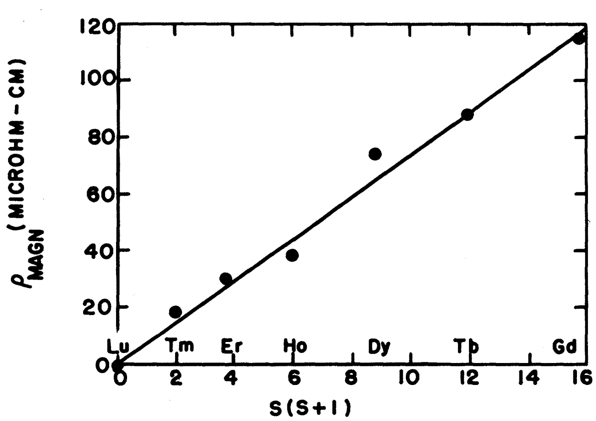

FIG. 2. The magnetic disorder contribution to the electrical resistivity as a function of $S(S+1)$ for the hexagonal close-packed rare earths Gd to Lu.

ions could result in an effective interaction between the spins of the magnetic ions.[1]

Recently it has been shown with alloys of rare earths and lanthanum that the depression of superconductivity is correlated with the spin of the solute atoms.[10] We believe that the conduction electron-magnetic ion interaction, which we have just discussed, is responsible for this shift. According to the BCS theory of superconductivity,[11] the superconducting transition temperature may be expressed as

$$T_c=(1.14/k)\langle\hbar\omega\rangle\exp[-1/N(0)V],$$

where $\langle\hbar\omega\rangle$ is the average energy of phonons interacting with electrons, $N(0)$ is the density in energy of electrons of one spin, and V is the average strength of the net electron-electron interaction (all quantities being defined at the Fermi surface). V may be expressed as

$$V=\left\langle\frac{2|M_K|^2}{\hbar\omega_K}-\frac{4\pi e^2}{\kappa^2}\right\rangle_{\text{Av}},$$

where $4\pi e^2/\kappa^2$ is the screened Coulomb interaction and $2|M_K|^2/\hbar\omega_K$ is the phonon induced effective electron-electron interaction. The criterion for superconductivity is that $V>0$. Since we will be concerned with solid solutions with the order of 1% solute atoms, we assume that $\langle\hbar\omega\rangle$ does not vary much. We also assume that $N(0)$ and $4\pi e^2/\kappa^2$ also are not changed much since the total number of conduction electrons per atom is unchanged and the volume change is very slight. These assumptions seem justified by experiments performed on similar alloys with solute atoms of zero spin.[12] Our proposal is that there is an additional effective electron-electron interaction which results from the conduction electron-magnetic ion exchange. V should then be expected to be a function of spin (for these alloys) as well as solute concentrations. In Figs. 3 and 4 we have shown the data of Matthias, Suhl, and Corenzwit[10] fitted to an empirical equation $N(0)V=[N(0)V]_{\text{La}}-a\delta[S(S+1)]^2$, where $[N(0)V]_{\text{La}}$ is the value for pure face-centered cubic lanthanum, a is the percent magnetic rare earth, S is the spin quantum number for the $4f$ electrons of the solute atoms, and δ is a proportionality constant independent of the solute atoms. The shift in transition temperature due to a volume change would be negligible for these alloys. The largest shift we would expect for 1% magnetic ion would be 0.1 degree.[12]

It is also possible that the shift may be explain-

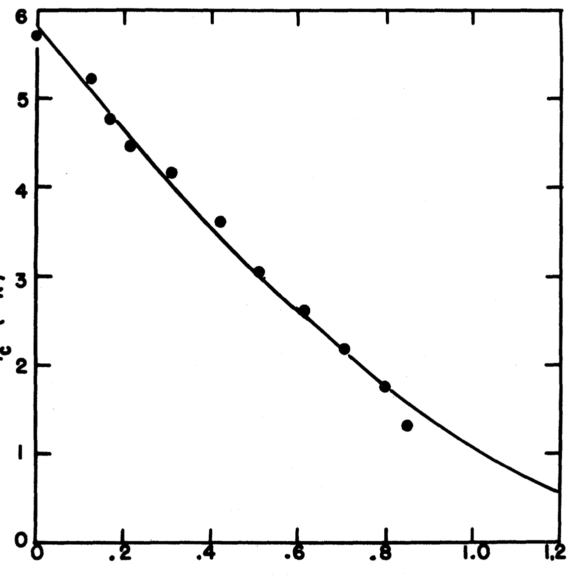

FIG. 3. Superconducting transition temperatures of La-Gd alloys. The points are the data of Matthias, Suhl, and Corenzwit and the solid curve is theoretical.

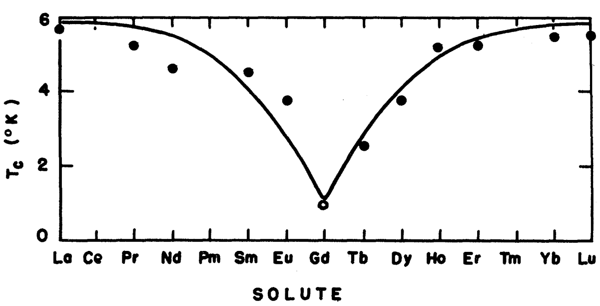

FIG. 4. Superconducting transition temperatures of alloys of La and 1% solute indicated. The points are the data of Matthias, Suhl, and Corenzwit and the solid curve is theoretical. The point for Gd is taken from the curve of Fig. 3.

ed in a manner suggested by Pippard[13] for impurity scattering of electrons.

In his model $|M_K|^2$ depends on κl in the same manner as the phonon mean free path (κ is the longitudinal wave number and l is the electronic mean free path). A decrease in l results in a decrease in $|M_K|^2$ and hence in T_c.

The ferromagnetic impurity effect has also been treated recently by Herring[14] and by Matthias and Suhl.[15]

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TEST OF TIME-REVERSAL INVARIANCE OF THE BETA INTERACTION IN THE DECAY OF FREE POLARIZED NEUTRONS*

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Jackson, Treiman, and Wyld[1] have pointed out that the electron momentum, $\vec{p}_e$, the antineutrino momentum, $\vec{p}_{\bar{\nu}}$, and the polarization of the nucleus, $\langle\vec{J}\rangle/J$, may exhibit, in the decay of oriented nuclei, a correlation of the form

$$1+\mathfrak{D}(\langle\vec{J}\rangle/J)\cdot(\vec{p}_e/E_e)\times(\vec{p}_{\bar{\nu}}/E_{\bar{\nu}})$$

if, and only if, the interaction responsible for beta decay is not invariant under time reversal.