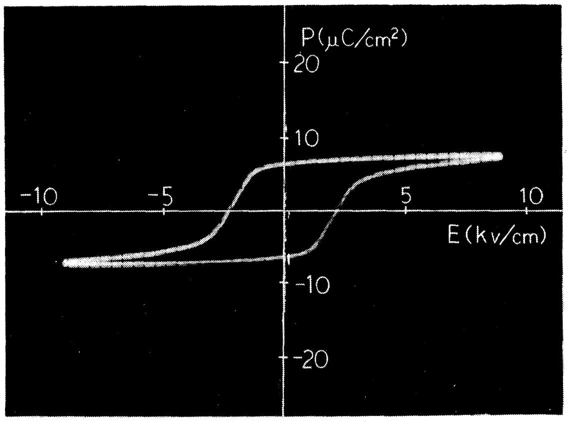
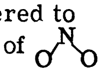


FIG. 2. Hysteresis loop (50 cps) in the direction of b axis at 143°C.

perature.

Judging from these experimental results, it is clear that $NaNO_2$ is ferroelectric along the b axis below the 160°C transition point, and the transition seems to be of the first kind, but even then the jump at the transition would be very small. The ferroelectricity is considered to come from the favorable arrangement of  groups in the crystal. The ordered arrangement of these groups causing the crystal to be polar in the low-temperature form seems to become disordered at the transition temperature.

Details of our studies will be reported shortly.

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SUSCEPTIBILITY OF THE ISING MODEL OF AN ANTIFERROMAGNET

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The susceptibility versus temperature curve for an antiferromagnet is usually characterized by the appearance of a maximum and the precise relationship of this maximum to the associated anomaly in the specific heat is of great interest. The elucidation of this problem using even a comparatively simple model of an antiferromagnet, such as the Ising model, is difficult and the behavior in the critical region has long been a subject of speculation. The work summarized below, however, leads to the conclusion that the susceptibility, χ, versus temperature, T, plot for an antiferromagnet should have a vertical tangent (χ increasing with T) at the same temperature at which the specific heat anomaly occurs. Since χ must vanish as T becomes infinite, this form of the singularity implies that the observed maximum in χ occurs above the true critical temperature.

These conclusions are based on a study of the susceptibility of Ising models. No exact closed expressions for the susceptibility of two- or three-dimensional Ising models have yet been given but series have been published by various authors.[1-3] The best method of obtaining these seems to be that employed by Oguchi[2] and developed by Domb and Sykes.[3] Extrapolation of these series has been attempted, most recently by Park,[4] but it has proved difficult to draw firm conclusions in the critical region. We have been able to rewrite the general expansion in such a way that the major part of the susceptibility is known if the configurational energy of the lattice is known. The remaining part has a much simpler configurational interpretation than the original expansion. In terms of the antiferromagnetic "high-temperature" variable $v = \tanh |J|/kt$, where J is the interaction energy between parallel spins, we find that the susceptibility per spin may be written

$$\chi(v) = \frac{m^2}{kT}[1+(q-1)v]^{-2}\{1+(q-2)v+v^2 - 2v\left|\frac{U(v)}{J}\right| + 8\sum_r(-1)^r d_r v^r\}, \qquad \text{(A)}$$

where m is the magnetic moment per spin, q is the coordination number, and $U(v)$ is the configurational energy per spin. The positive coefficients d_r enumerate a well defined class of closed graphs on the lattice with at most two odd vertices; d_0 to d_4 vanish identically on all lattices and d_5 and d_6 vanish on most loose-packed lattices.

By using this result, the series expansion for the susceptibility of the plane triangular lattice has been extended up to the term in v^{12}, that of the plane quadratic lattice up to the term in v^{14}, and that of the honeycomb lattice up to the term in v^{24}. The general result holds equally for three-dimensional lattices and the corresponding series can also be considerably extended.

Numerical evaluation of the function defined by $\Sigma(-1)^r d_r v^r$ shows that it contributes only a few percent or less to the value of χ at temperatures in the region of the critical point and above. (For lattices such as the plane triangular lattice which have no critical point, the relevant range is determined by the corresponding ferromagnetic Curie temperature.) Because of this the behavior of the susceptibility in the critical region is effectively determined by the energy alone. For plane lattices this is known exactly from the work of Onsager[5] and others,[6] and the expressions are valid at all temperatures. Consequently we may conclude that for loose-packed lattices, χ behaves in the critical region as $a + b(T-T_c)\log|T-T_c|$ where a and b are constants and T_c is the critical temperature. The energy is not known in closed form for three-dimensional lattices but the calculations of Wakefield[7] and of Domb and Sykes[8] indicate that the specific heat of a loose-packed lattice becomes infinite at T_c which again shows that the susceptibility should have a vertical tangent at the same temperature.

For close-packed lattices such as the plane triangular and the face-centered cubic, there is no specific heat singularity and the general result indicates that the susceptibility should not exhibit any singularity in the neighborhood of the corresponding ferromagnetic critical temperature.

The conclusion for plane lattices is borne out by an exact solution in the presence of a magnetic field of a particular class of two-dimensional Ising model in which normal Ising spins (with magnetic moments) are coupled together via "nonmagnetic spins" so yielding a type of "superexchange" interaction.

The generalization of the result (A) for spin $s > \frac{1}{2}$ has not been obtained. General arguments, however, indicate that similar physical conclusions should still be valid.

More detailed accounts of these investigations will be published in due course.

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CONDUCTION ELECTRON-MAGNETIC ION INTERACTION IN RARE EARTHS*

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The $4f$ electrons in the rare earths, which are responsible for the magnetic moments of the rare earth elements, are highly localized and presumably do not form a band; hence the direct interaction between $4f$ electrons on different atoms is expected to be fairly weak. The ion-ion interaction is presumably an indirect interaction via the conduction electrons in the manner suggested by Zener.[1] This conduction electron-magnetic ion interaction should also be manifested by the electrical resistivity.[2-4] It now appears that this interaction depends on the spin of the $4f$ shell and is involved in the electric and magnetic properties of the rare earth metals and in the superconductivity of alloys of the rare earths.

Electrical resistivity measurements on many of the rare earth metals have now been reported.[5-7] Figure 1 shows the electrical resistivity of Gd as a function of temperature. This is rather typical of the hexagonal close-packed rare earths Gd, Tb, Dy, Ho, Er, Tm, and Lu. The other rare earths do not show regions where the electrical resistivity varies linearly with temperature, so the present discussion is not pertinent for these elements. Following the proposal of Kasuya,[4] the electrical resistivity of a rare earth in the paramagnetic region might be ex-

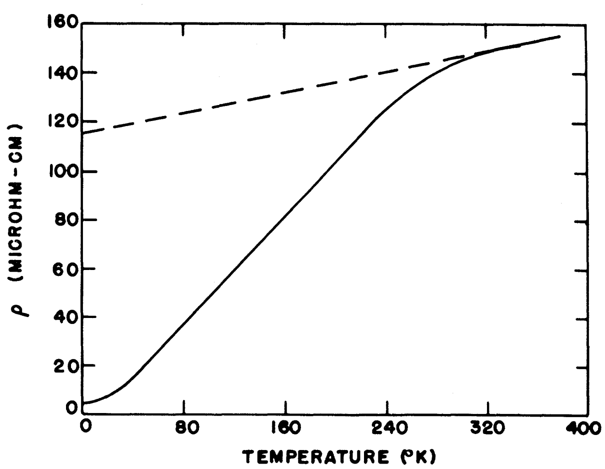

FIG. 1. The electrical resistivity of gadolinium as a function of temperature.