conducive to high superconducting[2] transition temperatures. Correlation of these shifts with the superconducting potentialities is lost when one considers that the critical temperatures[3] are Nb_3Sn, 18.05°K; V_3Sn, 6°K; and V_3Ga, 17°K; while for Nb_3Ga, which showed no NMR shift, it is[4] 16.8°K. The mechanism responsible for these shifts is not presently understood.

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THEORY OF GALVANOMAGNETIC EFFECT AT HIGH MAGNETIC FIELD

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According to the general theory of irreversible processes[1] the conductivity tensor is expressed as

$$\sigma_{\mu\nu} = \int_0^\infty dt \int_0^\beta d\lambda \, \langle j_\nu(-i\hbar\lambda) j_\mu(t) \rangle , \qquad (1)$$

where β is $1/kT$ and

$$j_\nu = \int \psi^\dagger v_\nu \psi \, d\vec{r} \qquad (2)$$

is the current density operator, $\psi^\dagger$ and ψ being quantized electron wave functions normalized per unit volume. v_ν is the one-electron operator for the velocity. The average is taken over the equilibrium in the absence of the electric field. The best way of treating the electronic conduction in magnetic field is to start from this expression, because the transport equation of usual Boltzmann-Bloch type may not be used particularly in strong magnetic field, where the condition

$$\omega_0 \tau_\gamma \gg 1 \qquad (3)$$

is satisfied. Here ω_0 is a measure of cyclotron frequencies and τ_γ that of the relaxation time. Under this condition, where electrons complete cyclotron cycles before they get scattered, and at low temperatures, the quantum-mechanical effect appears in transport properties as de Haas-van Alphen type oscillations and further as some asymptotic behaviors at extremely strong magnetic fields where only the lowest Landau levels are occupied (quantum limit).

Let us consider crystal electrons which are described by the energy function $E_0(\vec{p})$ in the absence of magnetic field, $\vec{p}$ being the crystal momentum. From general considerations, it is seen that the cyclotron motion may be represented by the set of canonical variables (ξ, η), (X, Y), and (p_z, z) which satisfy the commutation rules

$$[\xi, \eta] = [Y, X] = l^2/i, \; l^2 \equiv \hbar c/eH, \; [p_z, z] = \hbar/i. \qquad (4)$$

Here z is the Cartesian coordinate of an electron in the direction of the magnetic field. (ξ, η) may be called the relative coordinates and (X, Y) the center coordinates of cyclotron motion, because the Cartesian coordinates in the projection normal to the magnetic field are

$$x = X + \xi, \; y = Y + \eta , \qquad (5)$$

and (ξ, η) are defined by

$$\xi = (c/eH)\pi_y , \eta = -(c/eH)\pi_x , \qquad (6)$$

where

$$\pi_x = p_x + (e/c)A_x , \; \pi_y = p_y + (e/c)A_y$$

are the noncommutative quasimomentum components in magnetic field.

The effective Hamiltoniam of an electron is, if the interband effect is neglected, given by[2]

$$\mathcal{H} = E_0(\pi_x, \pi_y, p_z) + U(x, y, z),$$

where U is the scattering potential. The above introduced definitions and the Hamiltonian lead to the equation of motion,

$$\dot{X} = \frac{c}{eH}\frac{\partial U}{\partial y}, \; \dot{Y} = -\frac{c}{eH}\frac{\partial U}{\partial x}, \qquad (7)$$

which is valid both classically and quantum-mechanically. So X and Y are constant if U is zero, and therefore ξ and η define the cyclotron motion relative to the center (X, Y). Consequently, the velocity in (2) is divided into two parts,

$$v_x = \dot{X} + \dot{\xi}, \; v_y = \dot{Y} + \dot{\eta} ,$$

and the conductivity tensor, Eq. (1), will then consist of four components.

However, a great simplification results if the Fermi surface is not touching or not too close to the zone boundary, so that every electron (or hole) may be regarded as making a certain cyclotron motion and further the broadening effect discussed by Harper and Zilberman[3] may well be neglected. This assumption eliminates the possibility of infinitely extended orbits, but it may be taken into account, if necessary, by some modification of the present treatment. By their assumed nature, the variables ξ and η are bounded. From this we can show that

$$\sigma_{xx} = \frac{e^2}{2kT}\int_{-\infty}^{\infty} \langle \{ \dot{X}(0)\, \dot{X}(t) \} \rangle \, dt, \qquad (8)$$

etc., for the symmetric part of the conductivity tensor, and

$$\sigma_{xy} = -\frac{nec}{H} + e^2 \int_0^{\infty} dt \int_0^{\beta} \langle \dot{Y}(-i\hbar\lambda)\, \dot{X}(t) \rangle \, d\lambda \, , \qquad (9)$$

for the antisymmetric part. Here $\dot{X}$ and $\dot{Y}$ are the current density components due to the center motion expressed similarly to (2). The relative current $(\dot{\xi}, \dot{\eta})$ has disappeared from these expressions, so that conduction may be thought in terms of the migration of the center of cyclotron motion.

Equation (8) is a direct expression of the Einstein relation. Figure 1 shows the Fourier spectra of various correlation functions of velocity components. By the general theory[1] each of these Fourier components represents its contribution to the dynamical conductivity at frequency ω. Thus we see that only the static conductivity can be represented by the center motion.

The successive displacements of the center caused by successive scattering processes are generally correlated. If this correlation is fully taken into account, Eqs. (8) and (9) are always exact, even for weak fields. The condition of strong fields, (2), will however introduce an essential simplification, because the above mentioned correlation will then become negligible. For instance, Eq. (8), will then be simplified to

$$\sigma_{xx} = \frac{n e^2}{kT} \nu \, \langle (\Delta X)^2 \rangle \, ,$$

where n is the electron density, ν the frequency of collision per unit time, and ΔX the displacement of the center in a single scattering process. So the problem is reduced essentially to calculation of the scattering probability of an electron by a phonon or an impurity in the presence of a magnetic field.

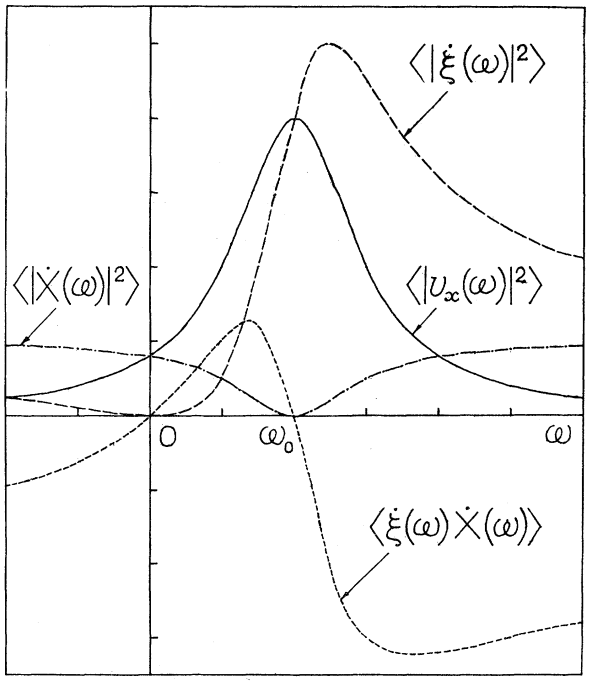

FIG. 1. Fourier spectra of various electron velocity components. The quantities ξ, X, and v are defined in the text.

The calculation of magnetoresistance in the above formulation is rather easy for the case of phonon scattering, but is more complicated for the impurity scattering case. The simplest Born approximation will cause a bad divergence and so we must go a step further to use at least some sort of damping-theoretical treatment.

In this way we could formulate a rather clear-cut theory of the galvanomagnetic effect at high magnetic fields and give foundations and deeper insights to the former theories of Titeica,[4] Pomeranchuk and Davydov.[5] The recent theory of Argyres[6] is also examined from this point of view. It seems that his method is not adequate in the quantum limit.

The details of this work will shortly appear in the Journal of Physical Society of Japan.

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DENSITY OF NEUTRON-IRRADIATED LITHIUM FLUORIDE CRYSTALS

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Density measurements of neutron-irradiated lithium fluoride crystals have been made by Binder and Sturm,[1,2] Cohen,[3] and Senio and Tucker.[4]

Binder and Sturm limited their study to thermal neutron doses up to 6×10^{16} *nvt*. Cohen and Senio and Tucker used thick crystals, which implies corrections for compensation of the considerable self-screening. Cohen found that the density became approximately constant at 6×10^{17} *nvt*. The results of Senio and Tucker are only of an exploratory nature.

No accurate density measurements on small samples are available for does higher than 1×10^{17} *nvt*.

X-ray studies by Perio, Tournarie, and Gance,[5] Lambert and Guinier,[6-8] and Smallman and Willis[9] have shown that (1) a maximum exists in the lattice parameter versus dose curve at about 2×10^{17} *nvt* thermal neutrons; (2) below the maximum only isolated Frenkel defects exist; (3) beyond the maximum vacancies form small clusters and lithium atoms coagulate into platelets between the (100) planes.

A comparison of density and parameter data at and beyond the maximum presented some interest. We directed therefore our attention particularly to that region.

Density was measured by flotation of the samples in suitable mixtures of bromoform, hexanol and pentanol. By comparison with standard floats of fused silica, and applying a technique described earlier,[10] an accuracy of 0.001% was readily attained. A U-shaped flotation chamber was used, one tube containing the crystals and the other the standard float.

This method permits the use of small crystals (platelets of about 5×5 mm, of thickness varying between 0.3 and 0.7 mm). In this way self-screening was kept small.

The platelets were cleaved from Harshaw single crystals and irradiated in vacuum in the graphite reactor *BR*1 at reactor temperature (about 80°C). Thermal neutron doses were measured with indium monitors placed with the samples.[11]

Results are given in Fig. 1. A logarithmic scale is used because of the high density decrease (up to 23% of the original value) which saturates at 1.2×10^{18} *nvt*. High stresses are generated during irradiation, even in crystals this thin. For doses higher than 1×10^{18} *nvt* the samples in some cases break into four or five fragments. Fragments of the same crystal have about the same density (differences are smaller than 0.5%). Black circles in the curve represent the mean value of the fragments of samples which broke during irradiation.

The spread of the measured points is not due to inaccurate density measurements, but is mainly caused by quite large uncertainties in the dose measurements. Small differences in crystal dimension and irradiation temperature also influence the density change for a given dose.

The results indicate that clusters of point defects affect lattice parameter and density in a very different way (see Seeger[12]). There are some indications that the very large density changes are partly due to the escape of fluorine out of the crystals.

Combined measurements on the annealing behavior of density and lattice parameter have been started. Preliminary density results on crystals

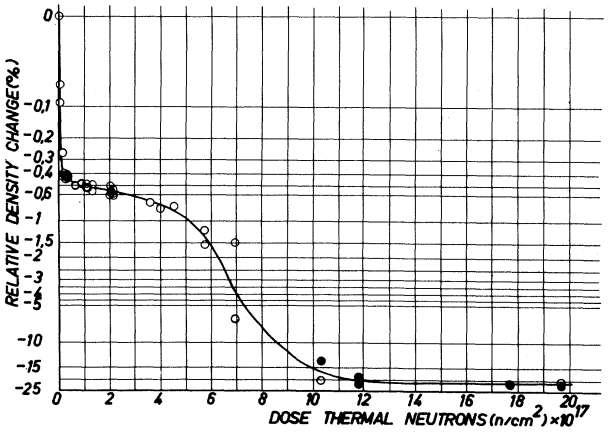

FIG. 1. Relative density change of neutron-irradiated lithium fluoride crystals as a function of dose (black circles refer to mean values of fragments of a crystal which broke during irradiation).