sinki, Finland.

[1] Containing 0.09% C, 0.26% Fe, balance B, obtained from A. D. Mackay, Inc., New York.

[2] Bronner, Ehlers, Eukel, Gordon, Marker, Voelker and Fink (to be published); for a detailed report on this accelerator see Annual Progress Report, U. S. Atomic Energy Commission Contract, University of Arkansas, March 1, 1958 unpublished), pp. 1-32.

[3] To be described in detail elsewhere.

[4] W. F. Hornyak and C. C. Lauritsen, Phys. Rev. 77, 160 (1950).

[5] A. M. Lawrance, Proc. Cambridge Phil. Soc. 35, 304 (1939).

[6] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. 83, 517 (1951).

SPACINGS OF NUCLEAR ENERGY LEVELS*

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Recently, attempts have been made to discover the law which governs the distribution of spacings of adjacent energy levels, of the same spin and parity, of highly excited nuclei.[1-3] It was pointed out by Wigner[4,5] and also by Landau and Smorodinsky[6] that one expects, on the basis of the "crossing theorem" for the eigenvalues of Hermitian matrices,[7] that the probability density for very small spacings is proportional to the spacing itself. If one retains the same description for all spacings (i.e., a Poisson process in the square of the spacing), one obtains the distribution law

$$p(x) = \frac{\pi}{2} x \exp\left(-\tfrac{1}{4}\pi x^2\right), \qquad (1)$$

in which $\underline{x}$ is the value of a particular spacing divided by the mean value. Indeed, a careful analysis of the experimental data indicates that small spacings (say $x < 0.3$) occur relatively infrequently.[2,3]

Wigner has suggested the following statistical model for obtaining the correct distribution law for all values of $\underline{x}$. There exists a representation in which spin and parity are diagonal, and in which the matrix elements of the energy are randomly distributed according to a frequency function which is symmetric about the value zero. (One of the best known examples of such a distribution is a Gaussian with zero mean.) Furthermore, Wigner conjectures that the distribution law for $\underline{x}$ does not depend sensitively on the precise distribution law for the maxtrix elements[8] and that the correct p(x) is given approximately by Eq. (1).

In order to study these ideas we have resorted to a numerical analysis, which is easily carried through with modern computing machinery. We have considered a set of 200 real symmetric matrices of order 20. The matrix elements were chosen at random from a symmetric uniform distribution. (It can be shown rigorously that the distribution law for the relative spacing $\underline{x}$ is independent of the dispersion of the uniform distribution.) After diagonalization, the 19 spacings resulting from one particular matrix are divided by the mean spacing for that matrix to yield 19 values of $\underline{x}$. In this way all 3800 values of $\underline{x}$ may be considered together (in exact analogy with the procedure adopted in the treatment of the experimental data), and the result is shown in the form of a histogram in Fig. 1, which also contains a plot of p(x).

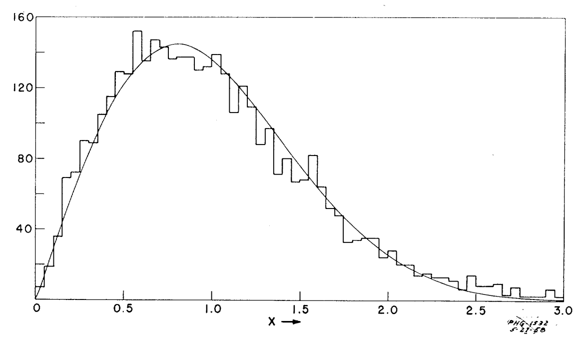

FIG. 1. The results for the uniform distribution of matrix elements as shown in the form of a histogram. The ordinate represents the number of spacings per 0.05 unit of $\underline{x}$. Fourteen spacings had values exceeding 3. The distribution given by Eq. (1), suitably normalized by the factor 3800/20, is plotted for comparison.

The above calculations were repeated, with the matrix elements selected at random from a Gaussian distribution $(0, \sigma)$. Again, the result is rigorously independent of σ and turns out to be quite similar to the histogram of Fig. 1.

Thus, Wigner's conjectures are very well confirmed within the limited scope of our calculations. Our results, based on two different distributions of the matrix elements, are well described by the function (1), except for very large spacings ($x > 2.5$). We find about three times as many spacings ($x > 2.5$) as are predicted by Eq. (1), and there is some indication of this also in the experimental data.

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[1]Fujimoto, Fukuzawa, and Okai, Progr. Theoret. Phys. Japan 16, 246 (1956).

[2]I. I. Gurevich and M. I. Pevsner, J. Exptl. Theoret. Phys. U.S.S.R. 31, 162 (1956) [translation: Soviet Phys. JETP 4, 278 (1957)]; also Nuclear Phys. 2, 575 (1956/57).

[3]J. A. Harvey and D. J. Hughes, Phys. Rev. 109, 471 (1958).

[4]E. P. Wigner, Proceedings of Conference on Neutron Physics by Time-of-Flight, Gatlinburg, Tennessee, 1956, reported in Oak Ridge National Laboratory Report ORNL-2309 (unpublished).

[5]E. P. Wigner, Proceedings of the International Conference on Neutron Interactions with Nuclei, Columbia University, 1957 (unpublished).

[6]L. Landau and Y. Smorodinsky, Lectures on the Theory of the Atomic Nucleus, Moscow, 1956 (unpublished).

[7]J. v. Neumann and E. P. Wigner, Physik. Z. 30, 467 (1929).

[8]In this connection see the related problem discussed by E. P. Wigner, Ann. Math. 67 (1958).

DIVERGENCELESS CURRENTS AND K-MESON DECAY*

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The striking successes of the V-A theory for weak interactions raise a difficult point of principle. The chirality invariance[1] which led to the universal V-A four-fermion interaction was postulated for the bare fermions and it is not clear why the strong interactions modify the results so little for the physical fermions. The evidence from β decay indicates that the pion renormalization effects are approximately the same (within 15%) for the V and A parts of the interaction. The fact that the coupling constants for μ meson and O^{14} decay are so close (within a few percent) implies that the renormalization effects are essentially nonexistent for the V interaction (and à fortiori for the A interaction). It appears therefore as if the four-fermion interaction actually couples the positive chiral states of the physical fermions.

In order to explain the remarkable agreement between the coupling constants for μ-meson and O^{14} decay, Feynman and Gell-Mann[2] have proposed that the ordinary (i.e., strangeness-conserving) V part of the interaction be described in terms of a divergenceless current. This would eliminate pion renormalization effects and has experimental consequences which can be tested.[3] It seems difficult to extend this attractive idea to other (i.e., strangeness-nonconserving) lepton interactions, since we do not know how to write down any other divergenceless currents than the ordinary vector one. This is hardly a conclusive argument, and one might suspect that if we understood the strong interactions and the origin of particle masses better, that we could construct such currents. It seems worthwhile, therefore, to look for experiments that can test directly whether the "current" responsible for a particular decay process is divergenceless or not. In this connection, it is to be noted that the currents for ordinary (e,ν) and (μ,ν) A interactions are not divergenceless, since $\pi \to \mu + \nu$ does occur, and since there is no enormous pseudoscalar component in beta-decay.[4] We wish to point out that one may test the role of divergenceless currents in strangeness nonconserving (μ,ν) interactions by looking for certain striking effects in the angular correlations in $K_{\mu 3}$ decay.

If we assume that only positive chiral states are coupled, then all (μ,ν) interactions with $\Delta S = \pm 1$ arise from a weak-interaction Hamiltonian

$$H_W = \mathcal{G}_\lambda [\bar{\psi}_\mu \alpha^\lambda (1+\alpha_5)\psi_\nu] - \mathcal{G}_\lambda^\dagger [\bar{\psi}_\nu \alpha^\lambda (1+\alpha_5)\psi_\mu]. \quad (1)$$

Here $\mathcal{G}_\lambda$ is an unknown function of baryon and meson fields with the transformation properties under proper Lorentz transformations of a vec-