separations, and to Mr. E. Goeler, who prepared the Illiac least squares program used for analyzing the data.

*Supported in part by the U. S. Atomic Energy Commission.

†Now at Convair Scientific Research Laboratory, San Diego, California.

[1] J. Bardeen and C. Herring, Atom Movements (American Society for Metals, Cleveland, 1951).

[2] Hoffman, Turnbull, and Hart, Acta Metallurgica 3, 417 (1955).

[3] A. B. Lidiard, Phil. Mag. 46, 1218 (1955).

[4] K. Compaan and Y. Haven, Trans. Faraday Soc. 52, 786 (1956); Proceedings of the Third International Conference on Reactivity of Solids, Madrid, April, 1956 (unpublished).

[5] A. D. LeClaire and A. B. Lidiard, Phil. Mag. 1, 518 (1956).

[6] C. W. McCombie and A. B. Lidiard, Phys. Rev. 101, 1210 (1956).

[7] A. B. Lidiard, Encyclopedia of Physics (Springer-Verlag, Berlin, 1957), Vol. XX, chapter on "Ionic Conductivity."

[8] E. Koch and C. Wagner, Z. physik. Chem. B38, 295 (1937); F. Seitz, Acta Cryst. 3, 355 (1950).

[9] C. Wert, Phys. Rev. 79, 601 (1950).

[10] W. Jost and A. Widmann, Z. physik. Chem. B29, 247 (1935).

[11] We have been informed by Dr. O. N. Salmon, of General Electric Knolls Atomic Power Laboratory, that in measurements of the temperature dependence of H^1 and H^3 diffusion in Pd and in stainless steel, slight deviations from Eq. (11) were observed. These experiments are described in two reports of Knolls Atomic Power Laboratory: D. Randall and O. N. Salmon, Knolls Atomic Power Laboratory Report KAPL 904, 1953; (unpublished) O. N. Salmon and D. Randall, Knolls Atomic Power Laboratory Report KAPL 984, 1954, (unpublished). We thank Dr. Winfield W. Tyler for informing us of this work.

[12] W. A. Johnson, Trans. Am. Inst. Mining Met. Engrs. 166, 114 (1946).

[13] M. Chemla, Ann. phys. 1, 959 (1956).

[14] C. T. Tomizuka and L. M. Slifkin, Phys. Rev. 96, 610 (1954).

[15] Kuper, Letaw, Slifkin, Sonder, and Tomizuka, Phys. Rev. 96, 1224 (1954).

SIMPLE APPROXIMATION FOR THE NON-UNIQUE FIRST FORBIDDEN TRANSITION*

Tsuneyuki Kotani† and Marc Ross

Department of Physics, Indiana University, Bloomington, Indiana

(Received July 28, 1958)

It is well known that the energy spectra of many nonunique first forbidden β transitions have roughly statistical shapes. That is, the energy shape correction factor, C, is constant. This can be understood by expanding the transition probability in powers of the nuclear radius ρ.[1] The leading terms are associated with the Coulomb factor $\xi \equiv \alpha Z/2\rho \approx Z/A^{\frac{1}{3}}$ and they give constant C.[2] The approximation of keeping only the leading terms in powers of ρ (i.e., in descending powers of ξ) we call the "ξ-approximation." (This approximation has been expressed by $\xi \gg W_0$, where W_0 is the maximum β-ray energy.[2]) In this letter we shall examine the accuracy of the ξ-approximation and show its consequences for other observables such as the longitudinal polarization and β-γ correlations.

Let us assume that the (local) V and A interactions[3,4] and the two-component theory of the neutrino, $C_V = C_V'$ and $C_A = C_A'$,[5] characterize the β decay. In the ξ-approximation the shape correction factor is energy independent:

$$C = |V|^2 + |Y|^2, \tag{1}$$

where

$$V = \xi' v_0 + \xi w_0 , \tag{2}$$

and

$$Y = \xi' y_0 - 2\xi(u_0 + x_0)(1+\gamma_1)^{-1}. \tag{3}$$

Here the subsidiary parameters have the following significance[2]:

$$\eta = -C_V \int \vec{r} R(-2), \; \eta w_0 = C_A \int \vec{\sigma}\cdot\vec{r} R(1),$$

$$\xi'\eta v_0 = C_A \int i\gamma_5 R(-1), \; \eta u_0 = C_A \int i\vec{\sigma}\times\vec{r} R(1),$$

$$\xi'\eta y_0 = -C_V \int i\vec{\alpha} R(-1), \; \eta x_0 = -C_V \int \vec{r} R(1). \tag{4}$$

The energy-independent quantity $R(\kappa)$ entering in the matrix element involves the ratio of the actual electron wave function to the point charge function evaluated at the nuclear surface (and thus R depends on the electron angular momentum quantum number κ shown). We do not need to know the details of R, if we do not evaluate the nuclear matrix elements themselves. In this way, finite nuclear size effects are taken into account, in the ξ-approximation.

The parameters $u_0, \ldots, y_0$ are defined so they may be of order unity (except, of course, that some may vanish due to selection rules). For this purpose, a parameter, ξ', is introduced to relate the relativistic to the nonrelativistic matrix elements. It has been theoretically predicted that

$$\xi' = \xi\lambda, \text{ with } \lambda = 1,^{6} \text{ or } 2.^{7} \tag{5}$$

If the next term in the ξ-expansion is included, the shape correction factor has the form

$$C = k(1 + aW + b/W), \tag{6}$$

where k, a and b are independent of the β-ray energy, W. (Full expressions for the constants k, a, and b are given, for example, by Kotani and Ross.[8]) If (5) is assumed, a and b are of order $1/\xi$. Thus, it may be expected that the ξ-approximation is poor and the shape correction factor is energy dependent for low-Z nuclei. For example, in the case of K^{42}, $\xi = 7.1$ and $W_0 = 4.9$. But experimental results do not show any energy dependence of C.[9-11] This may be explained by some accidental cancellation among the nuclear parameters which are included in a. If so, other observable quantities, which have different combinations of nuclear matrix elements from a, should be examined.

One of these observables is the β-γ directional correlation:

$$N = 1 + \epsilon(3\cos^2\theta - 1)/2, \tag{7}$$

where θ is the angle between β and γ. The coefficient ϵ has the form

$$\epsilon = (p^2/W)(R_3/C) + \alpha Z p(I_3/C), \tag{8}$$

neglecting $\alpha ZW/p$ terms.[12] Here R_3 and I_3 are energy-independent combinations of the nuclear parameters[8] (if the time-reversal assumption is correct for the weak interaction, I_3 vanishes). If $\xi' = \xi\lambda$, R_3/C is of order $(1/\xi)$. Even though R_3 includes some numerical constants (less than unity) which depend on nuclear spins, the experimental results for Cl^{38},[13] K^{42},[11] and As^{76},[14] seem very small ($|R_3/C| \approx 0.01$ to 0.03). It may be natural, then, to conclude that the next term in the ξ-expansion is only a few percent even for small Z. So it seems favorable to use $\xi' = \xi\lambda$ with $\lambda \gtrsim 2$, or to assume that ξ' does not become small for low Z as indicated by (5).

These data are not sufficient, however, to yield a definite conclusion. We hope that attempts will be made to observe other deviations or lack of them from the ξ-approximation in the same decays. We present, now, some features of the ξ-approximation for several observables. We can state that, for every observable normally occurring in the allowed transition, the expression for the same quantity in the nonunique first forbidden transition can be obtained in the ξ-approximation by using the substitution

$$C_F M_F \to \eta V,$$
$$C_{GT} M_{GT} \to \eta Y, \tag{9}$$

where C_F, M_F, C_{GT}, and M_{GT} are coupling constants and matrix elements for the Fermi and Gamow-Teller transitions, respectively (thus, allowed transition experiments suggested for testing general properties of the β-decay can be carried through for first forbidden transitions[15]).

If we include the next term in the ξ-expansion, the allowed-transition expressions no longer apply.[16] In this case, the longitudinal polarization, for example, can be written[8] (for β^--decay)

$$P_L = -(p/W)\left[1 + (R_1/NC)(1/W) + \alpha Z(I_1/NC)(1/p)\right], \tag{10}$$

where R_1 and I_1 are energy independent, I_1 vanishes if there is time-reversal invariance, and N is defined by Eq. (7). Neglecting $(\alpha Z)^2$ terms,

$$R_1/C = -b/(1 + aW + b/W)$$

in the notation of (6).

Empirical determination of the accuracy of the ξ-approximation can be made by observing the deviation of $|P_L|$ from p/W, by observing the deviation from p/W of the energy dependence of circular γ-ray polarization, and by observing the energy dependence of ϵ compared to Eq. (8).[8,12]

It should be noted that the ξ-approximation is not good in special cases where there is a large cancellation among nuclear parameters, (3), as in RaE, or in very high-energy decays where $W_0 \approx \xi$, as in the C^{15} case. Also there are some examples, aside from these extreme cases, which show small deviations from the ξ-approximation in the energy spectrum, e.g., A^{111},[17] Sb^{124},[18] and Re^{186}.[19] In these decays, the measurement of the deviation from the ξ-approximation of various observables mentioned above gives some useful information about the nuclear matrix elements.

---

*Supported in part by the National Science Foundation.

†On leave from Tokyo Metropolitan University, Setagaya, Tokyo.

[1] We use units $\hbar = c = m_e = 1$.

[2] See, for example, H. Mahmond and E. Konopinski, Phys. Rev. 88, 1266 (1952).

[3] Herrmannsfeldt, Maxson, Stähelin, and Allen,

Phys. Rev. 107, 641 (1957); Goldhaber, Grodzins, and Sunyar, Phys. Rev. 109, 1015 (1958).

[4] Burgy, Krohn, Novey, Ringo, and Telegdi, Phys. Rev. 110, 1214 (1958).

[5] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413 (1957); also see, for example, C. S. Wu, Proceedings of the Rehovoth Conference on Nuclear Structure (Interscience Publishers, New York, 1957), p. 346.

[6] T. Ahrens and E. Feenberg, Phys. Rev. 86, 64 (1952); T. Ahrens, Progr. Theoret. Phys. Japan 18, 331 (1957).

[7] D. L. Pursey, Phil. Mag. 42, 1193 (1951); M. E. Rose and R. Osborn, Phys. Rev. 93, 1315 (1954).

[8] T. Kotani and M. Ross, Progr. Theoret. Phys. (to be published).

[9] Pohm, Waddell, and Jensen, Phys. Rev. 101, 1315 (1956).

[10] Koerts, Schwarzschild, Gold, and Wu, Phys. Rev. 95, 612(A) (1954).

[11] R. Steffen, Proceedings of the Rehovoth Conference on Nuclear Structure (Interscience Publishers, New York, 1957), p. 416.

[12] Coulomb corrections to (8) are given in T. Kotani and M. Ross, Phys. Rev. (to be submitted).

[13] P. C. Marq and M. de Hemptinne, Nuclear Phys. 2, 160 (1956).

[14] H. Rose, Phil. Mag. 44, 739 (1953).

[15] Alder, Stech, and Winther, Phys. Rev. 107, 728 (1957); M. Morita and R. S. Morita, Phys. Rev. 109, 2048 (1958); F. Boehm and A. Wapstra, Phys. Rev. 109, 456 (1958).

[16] It should be noted that, although the allowed and forbidden transitions differ here in important respects, there are small corrections in the allowed transition similar to the $1/\xi$ terms in the forbidden transition.

[17] R. Robinson and L. Langer (private communication).

[18] Langer, Lazar, and Moffat, Phys. Rev. 91, 338 (1953).

[19] Porter, Freedman, Novey, and Wagner, Phys. Rev. 103, 921 and 942 (1956).

PHOTOPRODUCTION OF SINGLE POSITIVE PIONS FROM HYDROGEN IN THE 500-1000 Mev REGION*

F. P. Dixon and R. L. Walker
California Institute of Technology,
Pasadena, California
(Received July 21, 1958)

A wedge type magnetic spectrometer has been employed to analyze positive pions from the reaction $\gamma + p \rightarrow \pi^+ + n$ produced in a liquid hydrogen target by the bremsstrahlung beam of the Caltech Synchrotron. The spectrometer magnet focussed charged particles from the target into a system of three scintillation counters. The pulse heights from these counters were monitored so that particles with greater ionization than the pions were not detected. The peak energy of the bremsstrahlung was adjusted so that pions from reactions other than single-pion production were energetically impossible at the angle of observation and in the energy range accepted by the spectrometer.

A fourth scintillation counter was placed between the hydrogen target and the entrance aperture of the magnet. This front counter served to eliminate the cosmic-ray background and to reduce the background from radiation shields and the outer Mylar window of the hydrogen target, when a properly delayed coincidence was required with the rear counter system. Under these conditions the empty-target background was only 3% of the full-target counting rate, in general, and only 9% at the most forward laboratory angle, 13.1°. The experimental arrangement was similar to the schematic diagram shown in Fig. 1 of the K^+ report by Donoho and Walker,[1] except that in the present experiment no absorber was used between the front counter and the target or between the first two counters in the rear.

The data obtained from this measurement have been corrected for empty-target background, pion decay in flight, nuclear absorption of the pions, and muon contamination from π-μ decay. The pion decays before the counters resulted in the loss of 12 to 22% of the pions. However, the muon correction of 4 ± 2 to $7 \pm 3\%$ based on a preliminary estimate resulted in a net correction of 8 to 15% for decay and muon contamination. It is estimated that a more exact calculation of the muon contribution will not result in more than a three percent change in the cross sections presented in this report. The effect of the half-inch Pb absorber between the last two counters was measured, and the absorption in the counters and target was calculated using the geometrical cross sections. The absorption correction was typically 14%. Reversal of the magnetic field demonstrated that counts from electrons made a negligible contribution to the hydrogen counting rate, so no correction was needed for positron contamination in the present results.

The absolute cross sections in this report are assigned errors including the statistical standard deviations compounded with the errors in the corrections indicated above. The calibration of the copper-air ion chamber used as a beam mon-