





Impact of harmonic inflow variations on the size and dynamics of the separated flow over a bump

Himpu Marbona ^{1,2,*} Daniel Rodríguez ¹
Alejandro Martínez-Cava ^{1,3} and Eusebio Valero ^{1,2}

¹Universidad Politécnica de Madrid, Plaza Cardenal Cisneros 3, E-28040 Madrid, Spain

²Center for Computational Simulation, Universidad Politécnica de Madrid, Campus de Montegancedo, Boadilla del Monte, E-28660 Madrid, Spain

³Instituto Universitario “Ignacio Da Riva” (IDR/UPM), Universidad Politécnica de Madrid, Plaza Cardenal Cisneros 3, E-28040 Madrid, Spain



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The separated flow over a wall-mounted bump geometry under harmonic oscillations of the inflow stream is investigated with direct numerical simulations. The bump geometry gives rise to streamwise pressure gradients similar to those encountered on the suction side of low-pressure turbine (LPT) blades. Under steady inflow conditions, the separated-flow laminar-to-turbulent transition is initiated by self-sustained vortex shedding due to Kelvin-Helmholtz (KH) instability. In LPTs the dynamics are further complicated by the passage of the wakes shed by the previous stage of blades. The wake-passing effect is modeled here by introducing a harmonic variation of the inflow conditions. Three inflow oscillation frequencies and three amplitudes are considered. The frequencies are comparable to the wake-passing frequencies in practical LPTs. The amplitudes range from 1% to 10% of the inflow total pressure. The dynamics of the separated flow are studied by isolating the flow components that are respectively coherent with and uncorrelated to the inflow oscillation. Three scenarios are identified. The first one is analogous to the steady inflow case. In the second one, the KH vortex shedding is replaced during a part of the inflow period by the formation and release of a large vortex cluster. The third scenario consists solely of the periodic formation and release of the vortex cluster; it leads to a consistent reduction of the separated flow length over the entire period compared to the steady inflow case and would be the most desirable flow condition in a practical application.

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I. INTRODUCTION

Laminar boundary layer separation is a ubiquitous phenomenon present in several aeronautical applications, such as low-pressure turbines (LPTs) at high-altitude flight [1–3] and unmanned aerial vehicle (UAV) or microaerial vehicle (MAV) wings at steady and pitching conditions [4]. Separated flow is associated with detrimental effects on aerodynamics and performance; flow control strategies that lead to reductions in the size of the separated flow region are thus a way forward towards improving their efficiency [5,6].

*himpu.marbona@upm.es

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The laminar-to-turbulent transition process has a dominant role in the reattachment of the separated flow, thus determining the size and dynamics of separation bubbles [1,7–9] and also the structural loading of LPT blades (e.g., [10]). The details of the transition process in separated flows where the incoming flow and the aerodynamic surface are under steady conditions and low environmental disturbance levels are relatively well known. In scenarios representative of leading edge separation on airfoils and LPT blades with a moderate load, the transition is initiated by Kelvin-Helmholtz (KH) instability of the separated shear layer, that amplifies small-amplitude disturbances existing in the pre-separated boundary layer [8,11], quickly leading to the formation of spanwise vortices and triggering transition via nonlinear interactions of these vortices [12]. Alternative or complementary transition scenarios have been proposed for stronger adverse pressure gradients, that involve the onset of absolute inflectional instability [13–15], spanwise modulation of the recirculation region due to a self-excited global mode [16–19], or a combination of them [20–22]. The transition process can be altered significantly in the presence of moderate to elevated levels of free-stream disturbances. The incoming turbulence generates streaky structures in the attached boundary layer upstream of separation, referred to as Klebanoff modes [23], which prevent the formation of spanwise-homogeneous KH vortices. The interaction between the streaks and the inflectional instability leads to the formation of short-span KH structures which enhance momentum transfer in the wall-normal direction and result in a faster reattachment compared to cases with low free-stream disturbances [3,9,24,25].

The flow dynamics on the suction side of LPT blades, and especially the transition process, are complicated further by the inherent unsteadiness of the multistage machines, as the wakes shed by one blade stage convect through the downstream passages periodically disturbing the flow conditions of the next stage. These disturbances excite the formation of intense vortical structures that shed and pull fluid from the recirculation region downstream, thus temporally reducing the separated flow extent. Subsequently, the separation bubble regenerates, increasing in size until it reaches a stationary value or the influence of the next wake impacts the flow. The transition to turbulence is thus multimodal, promoted by periodic fluctuations of the incoming free-stream flow and the own instabilities of the separated flow [26]. Wake-induced transition is very sensitive to the combined effects of the adverse pressure gradient and intensity, temporal dependence, and frequency of the inflow free-stream fluctuations. Several investigations have addressed the impact of the wake-passing period on the length of the separated flow region, modeling the periodic passage of wakes either as an inlet velocity deficit localized in space and time [3,27–29] or as a harmonic change of the cross-sectional inlet conditions [30–32]. While the two approaches present some differences, their results agree qualitatively on the impact of the wake-passing frequency and intensity on the flow. A dimensionless frequency F , often referred to as reduced frequency, is defined based on the characteristic free-stream velocity U and a streamwise length L representative of the extent of the adverse pressure gradient region in the absence of wakes as $F = f_{\text{in}}L/U$, where f_{in} is the dimensional wake-passing frequency. The reduced frequency is thus the ratio of the convective time scale and the period of the imposed free-stream variation. Ambiguity exists in the practical characterization of F : First, the length of the steady-flow separated flow region is problem dependent and only known *a posteriori*. Alternatively, other definitions of L are preferred in the literature, like the length of the LPT blade or flat-plate used in experiments (e.g., [3,30,32]), as they can be rigorously quantified. Second, under unsteady inflow conditions, both the free-stream velocity and the streamwise pressure gradients change continuously and can differ significantly from the values obtained under the steady inflow conditions. Owing to this ambiguity, the order of magnitude, rather than the exact value of F , is relevant in the discussion of the physics.

Wake-passing reduced frequencies F close to but slightly lower than 1 are representative of aero-engine LPTs [3]. For low F , the wake passing period is long compared with the characteristic times of both the KH-related vortex shedding and the regeneration of the separation bubble; the impact on the time-average separated flow and aerodynamic performance is expected to be comparatively weak. For F above 1, successive wakes pass by the separated flow region before the separation bubble has time to fully regenerate, which leads to significantly shorter time-averaged bubbles.

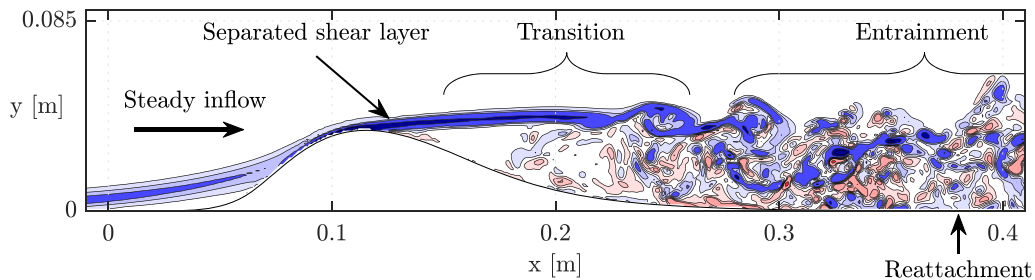


FIG. 1. Spanwise vorticity field around a wall-mounted bump under steady inflow conditions.

These observations agree with investigations on active flow control of separated flows by means of periodic excitation using wall suction and blowing or geometries with moving parts, reviewed by Greenblatt and Wygnanski [5], which conclude that the forcing frequency that minimizes the size of the separated flow is $F \sim 1$. Interestingly, this frequency is typically lower than that of the KH instability and scales with the global length of the separated flow rather than with the local properties of the separated boundary layer.

The present work studies the separated flow formed over a wall-mounted bump geometry that reproduces some characteristics of the suction side of a LPT blade under periodic fluctuations of the inflow stream. Similar geometries have been employed in the past both experimentally and numerically to study the fundamental physics of separated flow and possible means for flow control [20,33–38]. Direct numerical simulations are carried out for ten different inflow conditions. The first one is a steady inflow prescribed by a constant pressure difference between the inflow and outflow that results in the transition scenario shown in Fig. 1. Under steady inflow conditions, the laminar boundary layer separates just upstream of the bump summit. The separated shear layer sustains a self-excited shedding of spanwise-dominant KH vortices, followed by their breakdown in smaller three-dimensional structures and an abrupt transition to turbulence. The entrainment provided by turbulence leads to the mean flow reattachment.

The other cases study different scenarios of wake-induced transition. The large-scale action of passing wakes is modeled as a harmonic fluctuation of the inflow conditions (total pressure and bulk velocity) in a manner analogous to Refs. [30–32]. Lou and Hourmouziadis [30] present results for a single value of the reduced frequency lying in the limit $F \ll 1$. Wissink [31] considers four cases with $F \sim 1$ –6 and oscillations of the inflow velocity being up to 20% of the mean value. While the four cases simulated by [31,32] recover qualitatively the same flow dynamics, they are clearly different from those in the experiments of [30], emphasizing the existence of different dynamical scenarios depending on F and possibly on the amplitude of the harmonic inflow oscillations. The main objective of the present work is to unveil and fully characterize the possible physical scenarios and their impact on the size of the separated flow. To cover the parametric space, a matrix of nine cases is defined with three frequencies and three fluctuation amplitudes. It is anticipated here that three different scenarios will be identified.

The rest of the paper is organized as follows. Section II describes the geometry, computational domain, boundary conditions, and numerical methods used in the simulations. Section III presents and discusses the results. A qualitative description of the flow field evolution and mean flow properties are given in Secs. III A and III B. The triple decomposition proposed by Hussain and Reynolds [39] is applied to separate flow components that are coherent (in-phase) with the inflow oscillation from those occurring randomly. Section III C describes the phase-averaged fields for three representative cases. The phase-averaged data provides information on the impact of the inflow oscillation over the length of the separated flow and how it evolves over the inflow period, which is discussed in Sec. III D. Section III E presents the flow component that is incoherent (i.e., uncorrelated) with the inflow oscillation. Monitoring this component sheds light on the vortex dynamics that ultimately

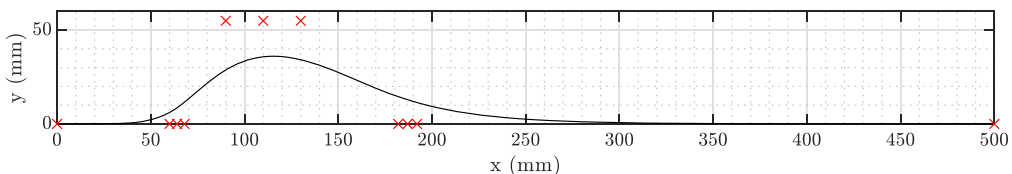


FIG. 2. Definition of the wall-mounted bump geometry. The symbols show the location of the control points defining the Bezier curves.

govern the behavior of the separated flow. Section III F presents frequency spectra at different probe locations. In combination, these results show that the separated flow subject to inflow oscillations can present three different scenarios regarding the flow dynamics and their impact on the length of the separated flow region. These scenarios are thus fully characterized in Sec. III and discussed in Sec. IV, along with their connection with active flow control strategies. To verify that present findings are general and not exclusive to the particular geometry of the wall-mounted bump used, results of an analogous study considering the related yet different NASA hump geometry [33,40] are presented in Appendix C.

II. NUMERICAL APPROACH

A. Geometry and domain

The geometry of interest is shown in Fig. 2. This geometry is based on the experimental setup used by Saavedra and Paniagua [37,38], consisting of a plane channel with a bump protruding from one of the walls. The bump geometry is defined by Bezier curves using 11 control points, to ensure continuity of the surface up to the second derivatives. The bump maximum height (summit) and the channel height are 0.036 and 0.17 m, respectively, resulting in a throat width of 0.134 m and a blockage ratio of 21.2%. This relatively large blockage ratio impacts the pressure gradient over the bump and especially near the bump summit. However, for the related geometry of the NASA hump, Greenblatt *et al.* [40] concluded that reducing the blockage ratio indeed modifies the wall pressure distribution but does not affect the separation and reattachment locations nor the root-mean-squared pressure distribution.

The computational domain and a representative mesh used are shown in Fig. 3. The channel is considered to be homogeneous in the spanwise direction. The spanwise size of the computational domain is set as $L_z = 0.08$ m (more than twice the bump height), and periodic boundary conditions are imposed on the lateral walls. For comparison, the larger spanwise domain used by Wissink

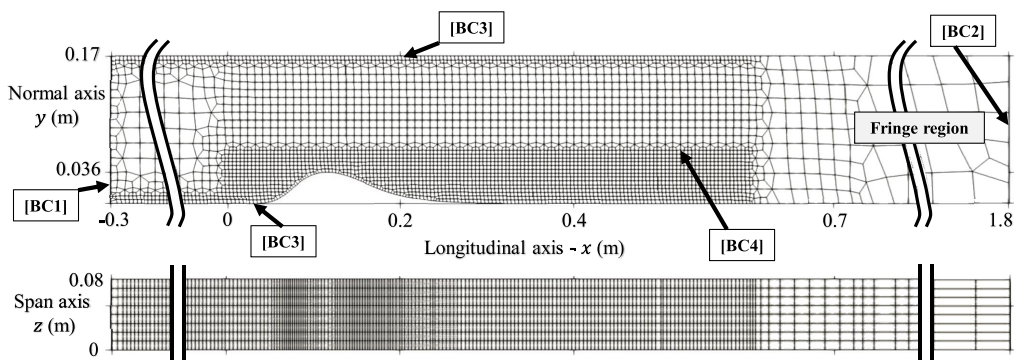


FIG. 3. Computational domain and representative mesh for spectral element computations.

TABLE I. Boundary conditions.

[BC1] Inlet	Total pressure p_t as in Eq. (1) and total temperature $T_t = 291.2$ K
[BC2] Outlet	Static pressure: $p = 101\,325$ Pa
[BC3] Wall	No-slip, adiabatic
[BC4] Sides	Periodicity

and Rodi [31] is $L_z = 0.12L$, where their L is comparable to the length of the bump. In the present geometry, the bump length is $L \sim 0.25$ m, yielding a ratio $L_z/L \sim 0.3$, which is approximately three times larger than that of Wissink and Rodi. Furthermore, this spanwise domain is considered to be sufficiently large since it accommodates several of the major spanwise structures, as shown below in Figs. 7 and 8.

The computational domain is extended in the streamwise direction with respect to the reference experiments and simulations to minimise the impact of the boundary conditions and to allow for the introduction of a Fringe region before the downstream boundary. The inlet section (denoted by BC1) is located 0.3 m upstream of the beginning of the bump, i.e., upstream of the first control point in the Bezier curves as shown in Fig. 2. This relatively long distance between the inlet section and the bump is intended to allow for the development of the incoming boundary layer.

A high-order mesh generation tool called the High Order Hex-Quad Mesher (HOHQ) [41] is used to generate a mesh suitable for spectral element computations. The curvature of the elements is represented using a fifth-order polynomial. Local mesh refinement is done in the regions adjacent to the wall and around and downstream of the bump, where the strongest velocity gradients are expected to appear. Section IID summarizes the mesh refinement studies. The mesh finally used consists of 31 640 rectangular elements with eight elements in the spanwise, z direction, and a third-order polynomial in all the elements and directions.

B. Simulation setup and boundary conditions

Table I lists the boundary conditions. The flow is driven by a pressure gradient imposed through the inflow and outflow boundary conditions. The static pressure $p_{\text{Ref}} = 101\,325$ Pa is imposed at the outlet and used as reference pressure. Uniform values of total pressure p_t and temperature T_t are prescribed at the inlet. A reference case is defined in which the inlet total pressure is constant and equal to $p_t = 105\,319$ Pa and the inlet total temperature is $T_t = 291.2$ K; this case is referred to as the *steady inflow* case. Under these conditions, the mean velocity at the inlet section is $U_{\text{Ref}} = 68.158$ m/s, the static temperature at inflow (which is taken as reference temperature) is $T_{\text{Ref}} = 288.9$ K, and the resulting flow has (unit length) Reynolds number $\text{Re} = \rho_{\text{Ref}} U_{\text{Ref}} L_{\text{Ref}} / \mu_{\text{Ref}} \approx 100\,000$ and Mach number $M = U_{\text{Ref}} / c_{\text{Ref}} = 0.2$, where ρ_{Ref} , μ_{Ref} , and c_{Ref} are, respectively, the density, dynamic viscosity, and speed of sound based on the reference temperature and pressure (T_{Ref} and p_{Ref}), and L_{Ref} is an arbitrary reference length which is chosen as $L_{\text{Ref}} = 1$ m.

In the other cases, referred to as *harmonic inflow* cases, a periodic variation of the total pressure at the inlet is imposed, while the total temperature remains constant as in the steady inflow case. The total pressure at the inlet is defined as

$$p_t(t^*) = p_{t,\text{steady}} \{1 + A_{\text{in}} \sin[2\pi f_{\text{in}}^*(t^* - t_0^*)]\}, \quad (1)$$

where $p_{t,\text{steady}}$ is the total pressure of the steady inflow case, A_{in} is the amplitude of the harmonic oscillation, f_{in}^* is a dimensionless frequency, t^* is a dimensionless time, and t_0^* is a reference instant. Dimensionless velocity, time, and frequency are defined using U_{Ref} and the unit length (L_{Ref}),

$$u^* = \frac{u}{U_{\text{Ref}}}, \quad t^* = t \frac{U_{\text{Ref}}}{L_{\text{Ref}}}, \quad f^* = f \frac{L_{\text{Ref}}}{U_{\text{Ref}}}. \quad (2)$$

Note that the definition of f^* is not the same as the reduced frequency F discussed in Sec. I. The definition of F is based on the representative length of the separated flow region, which is not known *a priori*. Also, the specification of f^* based on the unit length simplifies the data temporal sampling and subsequent analyses. Nine cases with harmonic inflow variation are considered, comprising three amplitudes ($A_{\text{in}} = 0.01, 0.05, \text{ and } 0.1$) and three different frequencies ($f_{\text{in}}^* = 0.5, 1, \text{ and } 2$).

Early simulations showed the presence of nonphysical disturbance reflections from the outlet boundary which impacted notably the dynamics of the separated flow. To minimize their upstream influence, a Fringe region is applied as in Spalart [42], Spalart and Watmuff [43], Nordström *et al.* [44]. This approach imposes a forcing term into a bounded spatial region extending upstream from the outlet, to drive the flow field towards prescribed values and effectively dampen the flow fluctuations before they reach the outlet boundary. The forcing term takes the form

$$\mathbf{s} = -\bar{\lambda} F \left(\frac{x - x_{\text{start}}}{\Delta_{\text{rise}}} \right) (\mathbf{q} - \mathbf{q}_{\text{target}}), \quad (3)$$

where $\bar{\lambda}$ is a constant controlling the strength of the forcing, $F(r)$ is a smooth function describing the spatial structure of the forcing term, x_{start} is the coordinate where the Fringe region starts, Δ_{rise} is the length of the Fringe region, \mathbf{q} is the vector of fluid variables, written in conservative variables (as described in Sec. II C), and $\mathbf{q}_{\text{target}}$ is the prescribed flow to be recovered at the outlet. Following Spalart and Watmuff [43], the function $F(r)$ is defined as

$$F(r) = \begin{cases} 0, & r \leq 0, \\ 1/[1 + \exp(-\frac{1}{r-1} + \frac{1}{r})], & 0 < r < 1, \\ 1, & 1 \leq r. \end{cases} \quad (4)$$

The reference values of the density ρ_{Ref} and temperature T_{Ref} , and the streamwise velocity of the steady inflow are imposed for the target values $\mathbf{q}_{\text{target}}$, together with the vanishing of the wall-normal and transversal velocity components. The Fringe parameters used in the simulations have been determined from different tests (not reproduced here for the sake of brevity) and ensure that the flow field does not fluctuate at the outlet and has negligible upstream effects. The Fringe forcing is very effective in dampening vortical disturbances and short Fringe regions are typically used for incompressible flow. Conversely, present computations consider compressible flow at relatively low Mach number, and consequently additional disturbances of acoustic nature are introduced. Previous tests simulating plane channels under the same inflow conditions as the wall-mounted bump showed that Fringe regions comprising at least one acoustic wavelength were required to effectively damp the acoustic reflections and prevent acoustic feedback with the current numerical setup. The Fringe region used starts a sufficient distance from the bump summit, i.e., at $x_{\text{start}} = 0.7$ m and extends to the outlet located at $x_{\text{outlet}} = 1.7$ m. The parameter Δ_{rise} is set equal to $x_{\text{outlet}} - x_{\text{start}}$ in order to provide the lowest gradient possible, and $\bar{\lambda} = 400$.

C. Computational methods

Direct numerical simulations are performed using the in-house discontinuous Galerkin spectral element code HORSES3D [45]. The flow variables are made dimensionless using $L_{\text{Ref}}, T_{\text{Ref}}, p_{\text{Ref}}$ and the reference velocity U_{ref} for the steady inflow case, resulting in a unit-length Reynolds number $\text{Re} = 100\,000$ and Mach number 0.2, as described in Sec. II B. The dimensionless compressible Navier-Stokes equations in conservative form take the form

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot (\mathbf{f}^a - \mathbf{f}^v) = \mathbf{s}, \quad (5)$$

where $\mathbf{q} = [\rho, \rho u, \rho v, \rho w, \rho E]^T$ are the conservative variables, E is the specific internal energy and \mathbf{s} is a source or volumetric forcing term. Advective (\mathbf{f}^a) and viscous (\mathbf{f}^v) fluxes are expressed in primitive variables as Eqs. (6) and (7), respectively:

$$\mathbf{f}_1^a = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ \rho uw \\ u(\rho E + p) \end{bmatrix}, \quad \mathbf{f}_2^a = \begin{bmatrix} \rho v \\ \rho uv \\ p + \rho v^2 \\ \rho vw \\ v(\rho E + p) \end{bmatrix}, \quad \mathbf{f}_3^a = \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ p + \rho w^2 \\ w(\rho E + p) \end{bmatrix}, \quad (6)$$

$$\mathbf{f}_1^v = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ v_i \tau_{1i} + \kappa \partial_x T \end{bmatrix}, \quad \mathbf{f}_2^v = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ \tau_{yz} \\ v_i \tau_{2i} + \kappa \partial_y T \end{bmatrix}, \quad \mathbf{f}_3^v = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \tau_{zz} \\ v_i \tau_{3i} + \kappa \partial_z T \end{bmatrix}. \quad (7)$$

The equation of state for ideal gas takes the form

$$p = (\gamma - 1)\rho \left[E - \frac{u^2 + v^2 + w^2}{2} \right], \quad (8)$$

and Sutherland's law is used for the dynamic viscosity

$$\mu = \frac{1 + T_{\text{suth}}/T_{\text{Ref}}}{T + T_{\text{suth}}/T_{\text{Ref}}} T^{\frac{3}{2}}, \quad (9)$$

where $T_{\text{suth}} = 110.4$ K. The dimensionless thermal conductivity is expressed as

$$\kappa = \frac{\mu}{(\gamma - 1)\text{Pr} M}, \quad (10)$$

where Pr is the Prandtl number, assumed to be constant and equal to 0.72. The stress tensor, using the Stokes hypothesis, is defined as

$$\boldsymbol{\tau} = \mu((\nabla \mathbf{v})^T + \nabla \mathbf{v}) - \frac{2}{3}\mu(\nabla \cdot \mathbf{v})\mathbf{I}. \quad (11)$$

The simulations are performed using third-order polynomials with Gauss nodes. Time integration is performed with an explicit third-order Runge-Kutta scheme. A standard discontinuous Galerkin discretization of the inviscid fluxes is done using Roe's method for the Riemann problem and the Bassi-Rebay 1 scheme is used for the discretization of the viscous fluxes. Further details on the numerical implementation can be found in Ferrer *et al.* [45] and references therein.

D. Convergence study

Preliminary simulations are performed to study the robustness of the results with respect to the spatial discretization and numerical parameters. The case with steady inflow is considered. Because the nominal Reynolds number is the same for all cases, the spatial resolution requirements are considered to be analogous. Similarly, the period of the inlet fluctuations is much longer than the characteristic times associated with the KH vortices (see Appendix A) and the series of increasingly smaller vortices typical of turbulent flow and thus the temporal resolution requirements are the same for all cases.

Spectral element methods allow the spatial discretization to be refined either by reducing the size of the elements (h -refinement) or by increasing the order of the polynomial used in the evaluation of derivatives within each element (p -refinement). A baseline mesh is defined based on a regular element size and different levels of zonal refinement, as shown in Fig. 3. To study the p -refinement, second-, third-, and fourth-order polynomials are used together with the baseline mesh. To study the h -refinement, two additional meshes were built with the element size respectively decreased (*fine mesh*) and increased (*coarse mesh*) by $\sim 20\%$ in all directions with respect to the baseline mesh.

TABLE II. Convergence study of spatial and temporal resolutions. h/h_{ref} : relative spatial mesh size; p : order of the polynomial discretizing each mesh element; DOF: degrees of freedom of the spatial discretization; Δt^* time step; L_s : mean length of the recirculation region; N_s : total number of time steps simulated after initial transient; N_{ss} : number of time steps per segment used in the computation of the PSD.

Discretization	h/h_{ref}	p	DOF	Δt^*	L_s [m]	Relative cost	N_s	N_{ss}
Baseline	1	3	2 024 960	3×10^{-5}	0.2782	1	2 410 000	33 332
Fine	0.8	3	5 012 480	5×10^{-6}	0.2818	13.82	2 400 000	200 000
Coarse	1.2	3	1 343 616	3×10^{-5}	0.2836	0.60	1 350 000	33 332
Baseline + low p	1	2	854 280	3×10^{-5}	0.2931	0.68	1 000 000	33 332
Baseline + high p	1	4	3 955 000	2×10^{-5}	0.2858	2.31	550 000	50 000

The number of spanwise elements is six, eight, and ten in the *coarse*, *baseline*, and *fine* meshes, respectively. Additionally, increasing the polynomial order p also increases the effective spanwise resolution, enabling the assessment of results convergence with respect to spanwise discretization. The parameters of the five discretizations tested are given in Table II, together with the number of spatial degrees of freedom (DOF) and the time step Δt^* used.

The mean length of the separated flow region L_s is compared for the different discretizations. The h -refinement study shows that L_s changes are below 2%, while the relative computational cost increases an order of magnitude for the finer mesh. The effect of the polynomial order p is found to be stronger. However, increasing the order from $p = 3$ to $p = 4$ results in a change of L_s below 3%.

The power spectra density (PSD) of the spanwise vorticity at different probe locations are computed and shown in Fig. 4. Power spectral densities are estimated using Welch's method [46]. For each spatial discretization, different time-step sizes Δt^* are used and varying total numbers of snapshots N_s are stored. The PSD is computed by averaging the periodograms of segments of N_{ss} snapshots each with 50% overlap and a Hamming window. The segment length N_{ss} is chosen separately for each simulation to ensure that the frequency bin $\Delta f^* = 1$.

The location of the probes is illustrated in Fig. 5 and given in Table III, together with other locations that are used throughout the paper. Probes 1, 2, and 3 are located along the mean shear layer. The reference point and Probe 4 will be referred to in Sec. III. Probe 1 is located relatively close to the separation point, upstream of the region where the first KH vortices are observed. Probe 2 is in the region in which KH vortices are shed. Probe 3 is located downstream, where

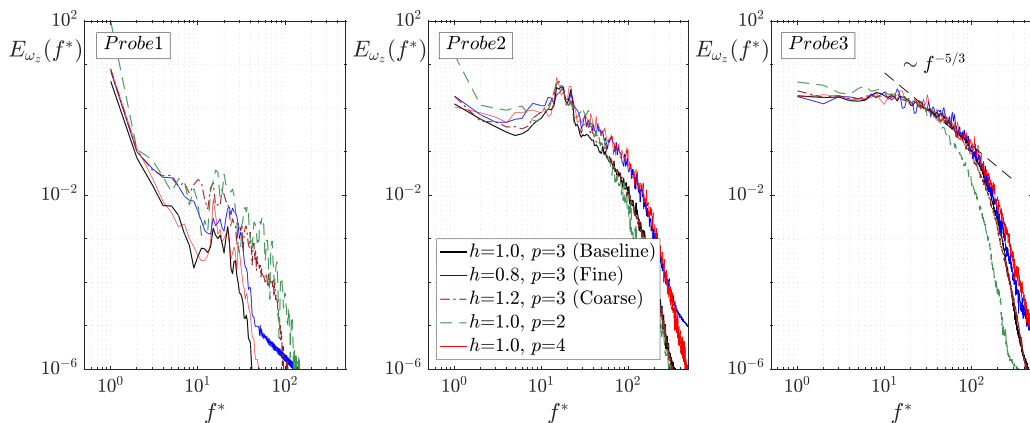


FIG. 4. PSD of spanwise vorticity at selected probe locations for different spatial and temporal discretizations.

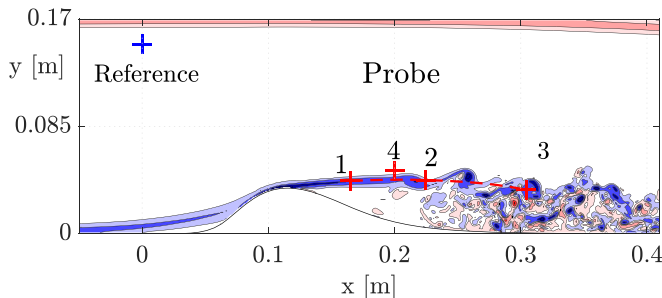


FIG. 5. Location of the reference point and sampling probes.

vortical structures of a broad range of scales are already present. For all five spatial resolutions, the energy spectra are roughly the same except for the case with second-order polynomial, coarser mesh, and at higher frequencies. These frequencies are associated with small scales and present very low energy levels, and the deviation of the spectra may indicate that the turbulent cascade is not fully resolved. Similarly, the y^+ coordinate of the first internal node of the wall-adjacent element takes values as large as 2 at certain locations, indicating that the baseline discretization may not fully resolve the attached boundary layer there. However, the focus of present simulations is to resolve the flow structures dominating the separated flow dynamics, which correspond to comparatively larger scales than the wall viscous sublayer, lower frequencies than those potentially under-resolved in the frequency spectra, and are nevertheless robustly captured by the baseline spatial discretization and temporal resolution (cf. Table II). This convergence study shows that further spatial refinement can improve the resolution of finer turbulent scales, at the cost of significantly more expensive simulations, but does not result in significant changes in the physics of interest.

III. RESULTS

Table IV summarizes the different simulations performed in this work, comprising the reference case that features a steady inflow and nine cases in which the inflow has a harmonic component of different frequency f_{in}^* and amplitude A_{in} . All the harmonic inflow cases use the same time step $\Delta t^* = 2 \times 10^{-5}$. The initial transient, different for each case but comprising several flow-through times, was discarded and the following simulation sampling time $t_{data}^* = 50$ was collected for the subsequent analysis.

The steady inflow case is considered as the baseline. The length of the separated flow region, defined as the streamwise distance from the time-averaged separation to reattachment points, is $L_{s,steady} = 0.2782$ m. The Reynolds number based on the mean inflow velocity and $L_{s,steady}$ is $Re_{L_s} = 27\,000$, which is comparable to the works by, e.g., Embacher and Fasel [47], Kurelek *et al.* [48], Dellacasagrande *et al.* [49]. The Reynolds number based on the maximum height of the recirculation region, approximated here as the height of the bump, is $Re_h = 3600$, which lies in the range of values

TABLE III. Coordinates of the reference point and sampling probes.

Probe	x [m]	y [m]	ξ [m]
Reference	0.0	0.15	—
1	0.165	0.0417	0.06
2	0.225	0.0425	0.12
3	0.304	0.0348	0.2
4	0.2	0.05	—

TABLE IV. Summary of the cases simulated, including the definition of the inlet pressure condition, the characterization of time-averaged and phase-dependent values of the reference streamwise velocity, and the length of the recirculation region L_s .

Δt^*	CFL_{\max}	t_{data}^*	A_{in}	f_{in}^*	$\bar{u}_{\text{@Ref}}^*$	$\Delta u_{\text{@Ref}}^*$	L_s [m]	ΔL_s [m]
3×10^{-5}	0.9	72.3	—	—	1.11	—	0.2782	—
2×10^{-5}	0.7	50	0.01	0.5	1.1025	0.0293	0.2782	0
	0.7	50	0.01	1	1.1078	0.0301	0.2780	-0.0002
	0.7	50	0.01	2	1.1072	0.0282	0.2625	-0.0157
	0.7	50	0.05	0.5	1.1030	0.1467	0.2727	-0.0055
	0.7	50	0.05	1	1.1022	0.1467	0.2600	-0.0182
	0.7	50	0.05	2	1.1008	0.1401	0.1966	-0.0816
	0.8	50	0.1	0.5	1.0872	0.2929	0.2575	-0.0207
	0.8	50	0.1	1	1.0840	0.2961	0.2323	-0.0459
	0.8	50	0.1	2	1.0871	0.2772	0.1536	-0.1246

reported by Gaster [7] and other experiments cited by Diwan *et al.* [50]. The Reynolds number based on the boundary layer momentum thickness at the separation point is $\text{Re}_{\theta,s} = 27$. This number is significantly smaller than most of the values reported in the literature for laminar separation bubbles. However, this discrepancy is explained by the use of a wall-mounted bump, as opposed to flat plates under an adverse pressure gradient or airfoils with small to moderate camber. The strong acceleration of the flow in the upstream portion of the bump leads to a substantial reduction of the boundary layer thickness. The rather low value of $\text{Re}_{\theta,s}$ is comparable to that in geometry-induced separation bubbles like those reported by Aniffa *et al.* [51].

The harmonic change of the inlet total pressure leads to a periodic acceleration and deceleration of the bulk flow. Due to the relatively long upstream extension of the domain, the inflow changes reach the bump with a delay. A reference point located just upstream of the bump (see Fig. 5 and Table III) is used to characterize the bulk flow changes in the bump region. Figure 6 shows the total pressure and streamwise velocity evolution at the reference point for the case $A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$. The total pressure at this point is used to define the phase of oscillation ϕ in the analysis done in the rest of the paper. The phase $\phi = 0^\circ$ is chosen as the time in which the total pressure is at its mean value and has maximum positive derivative (i.e., maximum acceleration); $\phi = 90^\circ$ and 270° correspond respectively to the maximum and minimum values of the total pressure.

The streamwise velocity also exhibits a sinusoidal behavior, with the mean value u_{mean} remaining the same as for the steady inflow case. A small delay T_d^* exists between the maxima and minima of total pressure and streamwise velocity, stemming from flow inertia. Table IV shows the normalized amplitude of the streamwise velocity fluctuation, $\Delta u^* = (u_{\text{max}} - u_{\text{min}})/(2U_{\text{Ref}})$. This value is

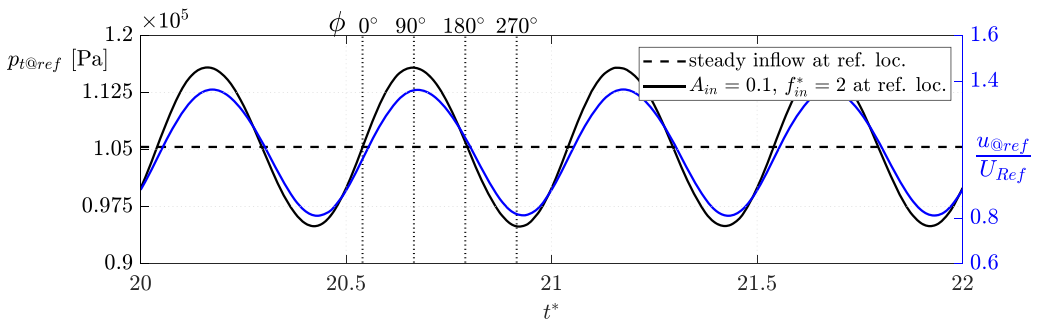


FIG. 6. Evolution of total pressure and streamwise velocity at the reference point.

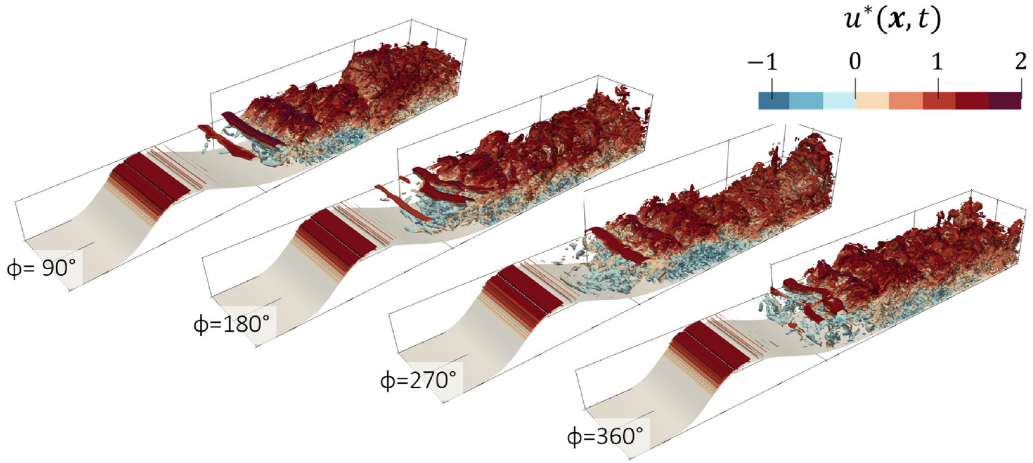


FIG. 7. $Q(+)$ isosurface colored by streamwise velocity. $A_{in} = 0.01$ and $f_{in}^* = 0.5$.

linearly proportional to A_{in} and the normalized streamwise velocity at the reference point can be approximated by

$$u^*(t) \approx \bar{u}^* + \Delta u^* \sin[2\pi f_{in}^*(t^* + T_d^*)]. \quad (12)$$

Table IV also shows the mean value of the streamwise length of the flow recirculation region L_s , and its relative change with respect to the steady inflow case, $\Delta L_s = L_s - L_{s,steady}$ for each one of the cases with harmonic oscillation inflow. As opposed to Δu^* , ΔL_s is not proportional to A_{in} , indicating that essentially nonlinear dynamics govern the separated flow and its reattachment. The nonlinearity is associated with the complex vortex dynamics originated in the separated flow region and the impact of the flow acceleration and deceleration upon them, which will be studied in the next sections.

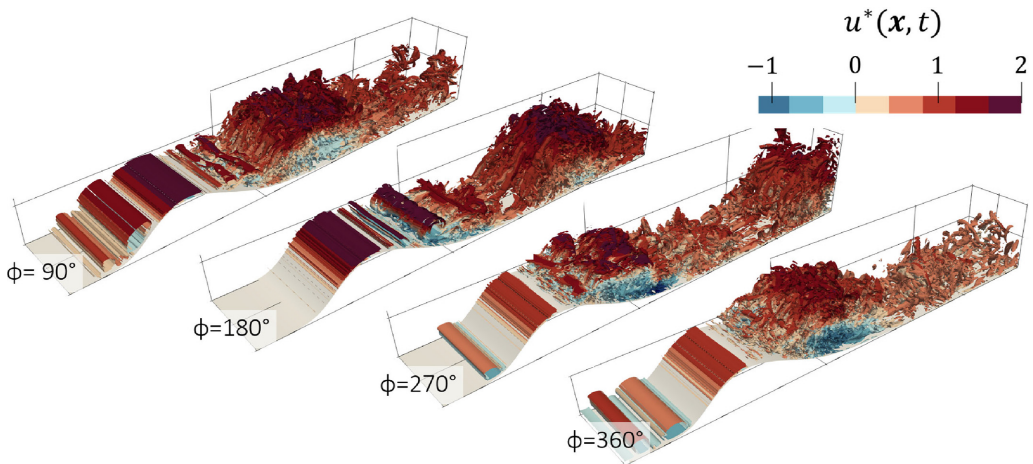
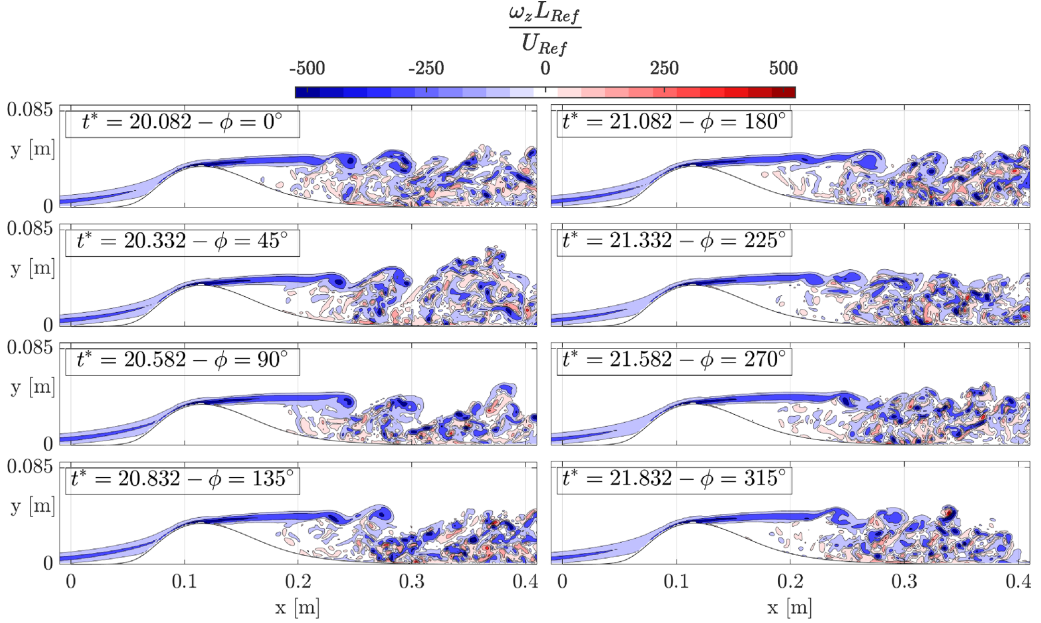


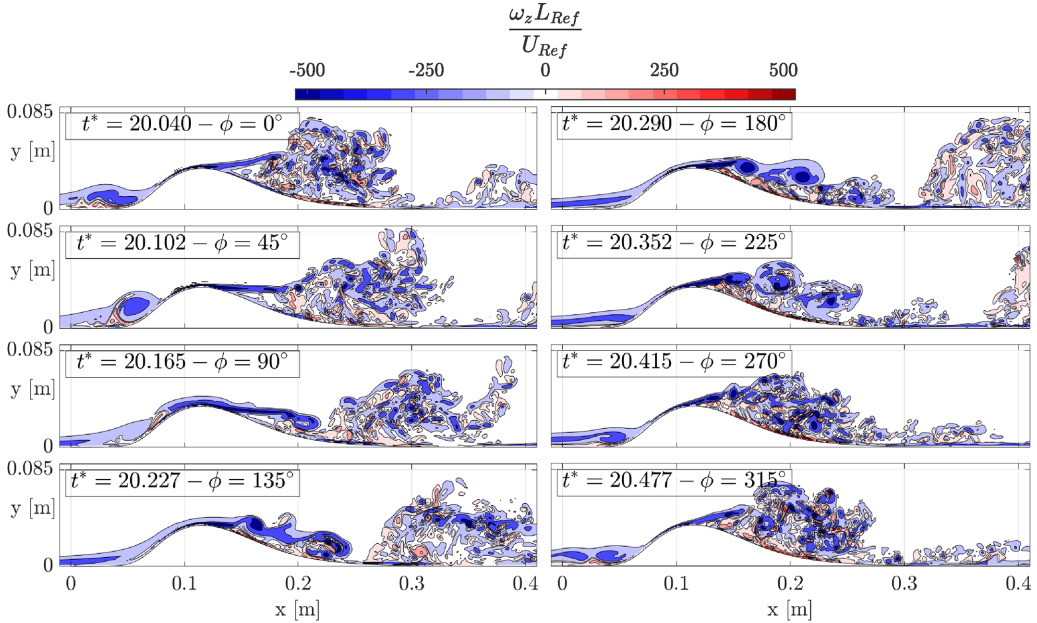
FIG. 8. $Q(+)$ isosurface colored by streamwise velocity. $A_{in} = 0.1$ and $f_{in}^* = 2$.


 FIG. 9. Instantaneous spanwise vorticity. $A_{in} = 0.01$ and $f_{in}^* = 0.5$.

A. Instantaneous flow fields

Figures 7 and 8 show the sequence of instantaneous three-dimensional visualizations of the vortical structures using the Q criterion [52] for two extreme cases, namely, ($A_{in} = 0.01$, $f_{in}^* = 0.5$) and ($A_{in} = 0.1$, $f_{in}^* = 2$) (see movies 1 and 2 in Supplemental Material [53]). The isosurfaces, corresponding to the arbitrary value $Q = 1000$, are colored using the streamwise velocity. Figures 9 and 10 show the respective spanwise vorticity fields at the midspanwise plane. In addition to the dimensionless time, each subfigure also indicates the corresponding phase ϕ . The three-dimensional visualizations show that the flow is fully two-dimensional upstream of the bump summit, even in the presence of vortices in the upstream part of the bump. This rules out the presence of Görtler [54] and centrifugal global instabilities [16,19] and ensures that the flow remains laminar and two-dimensional at separation in all cases.

The first case ($A_{in} = 0.01$, $f_{in}^* = 0.5$) corresponds to the weakest inflow oscillations, i.e., lowest inflow frequency and amplitude, and results in a mean recirculation length virtually identical to that of the steady case. For this case, Fig. 9 shows a continuous shedding of KH vortices from the separated shear layer followed by a fast transition to turbulence and recirculation of vortical structures, qualitatively identical to the steady case (cf. Fig. 1). However, careful observation of the separated shear layer upstream of the vortices shows some differences that gradually become more relevant for cases with increasing A_{in} and f_{in}^* . For $\phi = 0^\circ$ the reference total pressure starts to increase above its mean value, exerting an acceleration of the flow along the channel. Due to the increased mass and momentum fluxes, the separated shear layer is pushed towards the bump wall. As the flow is accelerated, the high-vorticity region associated with the laminar-turbulent transition is also pushed towards the wall. The maximum inflow pressure occurs at $\phi = 90^\circ$, followed closely by the inflow bulk velocity. For ϕ between 90° and 270° the reference pressure is reduced up to its minimum value, imposing a gradual deceleration of the bulk flow. The separated shear layer moves away from the wall; the high-vorticity region seems to detach from the wall and the apparent recirculation region extends farther in the streamwise direction. Finally, when $\phi > 270^\circ$ the flow gradually reaccelerates closing the period.


 FIG. 10. Instantaneous spanwise vorticity. $A_{\text{in}} = 0.1$ and $f_{\text{in}}^* = 2$.

The second case ($A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$) corresponds to the strongest inflow oscillation, i.e., the largest inflow oscillation frequency and amplitude, and results in a substantial reduction of the mean recirculation region ($\Delta L_s/L_{s,\text{steady}} \approx -44\%$). For this case, Fig. 10 does not show a periodic shedding of KH vortices from the separated shear layer, but rather the formation and release of a big vortex cluster following the harmonic change in the inflow conditions.

These observations suggest that two closely related but different physical mechanisms are at play when the inflow has a harmonic time dependence with relatively small A_{in} and f_{in}^* . First, the periodic acceleration-deceleration of the bulk flow modifies the angle of the separated shear layer through changes in the intensity of the streamwise pressure gradient. The location of the separation point is nearly unaffected by this. Second, the periodic vertical motion of the shear layer can impact its hydrodynamic instability properties and the ensuing dynamics of the KH vortices and the laminar-turbulent transition. As will be quantified later, the frequency of the KH vortex shedding is distinctly separated from the frequency of the inflow changes. On the other hand, for relatively large A_{in} and f_{in}^* , the organized shedding of KH vortices from the separated shear layer is replaced by a periodic formation and ejection of large vortex clusters that is driven by the bulk flow acceleration and deceleration. Intermediate cases are expected to show a transition from one behavior to the other, as will be detailed in the following sections.

With the aim of isolating the shedding of KH vortices from the bulk flow oscillations imposed by the inlet frequency, the triple decomposition proposed by Hussain and Reynolds [39] is applied. This decomposition takes the form

$$q(\mathbf{x}, t) = \bar{q}(\mathbf{x}) + \tilde{q}(\mathbf{x}, t) + q'(\mathbf{x}, t), \quad (13)$$

where \bar{q} stands for the mean (time-averaged) flow, \tilde{q} is the oscillatory component coherent with the inflow oscillation and q' is the incoherent component. The term ‘‘coherent’’ refers to flow fluctuations that occur in phase with the harmonic changes of the total pressure at the reference point. As such,

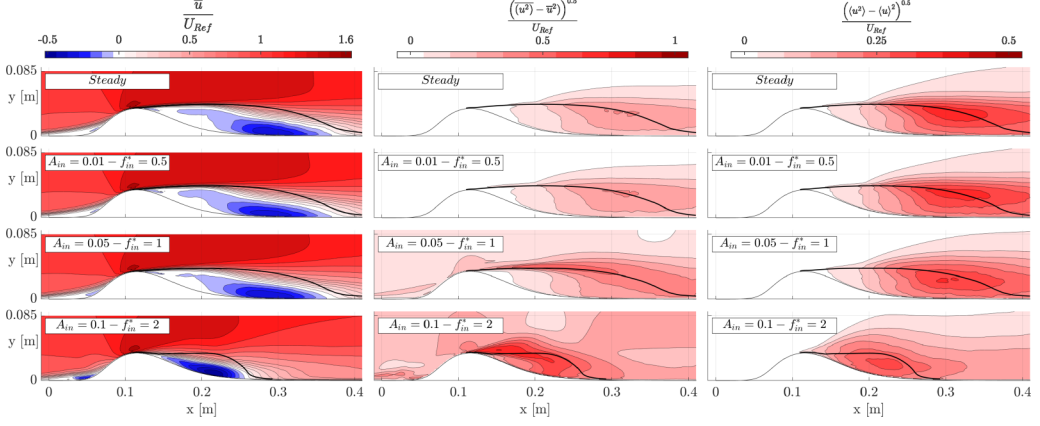


FIG. 11. Time-averaged (left) and root-mean-square (center) and root-mean-square of the incoherent component (right) of streamwise velocity field.

the mean plus coherent components are gathered together in the phase-averaged flow

$$\langle q(\mathbf{x}, \phi) \rangle = \frac{1}{N} \sum_{n=0}^N q\left(\mathbf{x}, t_\phi + \frac{n}{f_{\text{in}}^*}\right), \quad (14)$$

where t_ϕ is the time used as the phase reference, f_{in}^* is the inlet frequency and N is the number of periods used in the averaging. The incoherent part of the flow is computed as $q'(\mathbf{x}, t) = q(\mathbf{x}, t) - \langle q(\mathbf{x}, t) \rangle$. The same dimensionless time lapse was used in the averaging for the three frequencies f_{in}^* , resulting in $N = 25$ for $f_{\text{in}}^* = 0.5$, $N = 50$ for $f_{\text{in}}^* = 1$ and $N = 100$ for $f_{\text{in}}^* = 2$.

B. Mean flow fields

Figure 11 shows the mean (time-averaged) component of the streamwise velocity field \bar{u} for the steady inflow case, the former two extreme cases of inflow oscillations ($A_{\text{in}} = 0.01$, $f_{\text{in}}^* = 0.5$) and ($A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$), and the intermediate case ($A_{\text{in}} = 0.05$, $f_{\text{in}}^* = 1$). The thick black lines correspond to the time-averaged separation streamline. These results illustrate how the mean extent of the separated flow is only slightly reduced when the intensity of the inflow oscillations is mild, but reduced considerably for the strongest inflow oscillations. The peak reversed flow exceeds 20% of the reference velocity for the steady inflow case and is increased up to nearly 50% for ($A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$). Such intense reversed flow values suggest that, in all cases, a self-excited vortex shedding is possible stemming from an absolute KH instability [13,15]. Figure 11 also shows the root mean square of all the flow fluctuations (phase-averaged plus incoherent components, central column) and of the incoherent component alone (right column). The r.m.s. of the incoherent component shows intense fluctuations in the downstream part of the recirculation region, with its peak roughly coincident with the streamwise coordinate of the peak reversed flow and located on the separation streamline, which are typical features of laminar separation bubbles. For the case with strongest inflow oscillations, this peak is displaced inside of the separation streamline, indicating that the flow fluctuations that are uncorrelated to the inflow oscillations are contained in the recirculation region. This will be further discussed in Sec. III E. The r.m.s. of the complete fluctuations are identical to that for the incoherent component for the steady and weak inflow oscillation cases. A gradual departure appears as the amplitude of the inflow oscillation increases, as the relative intensity of the coherent component increase and becomes dominant. These fluctuations are then concentrated on the separation shear layer, upstream of the peak of the incoherent fluctuations, are

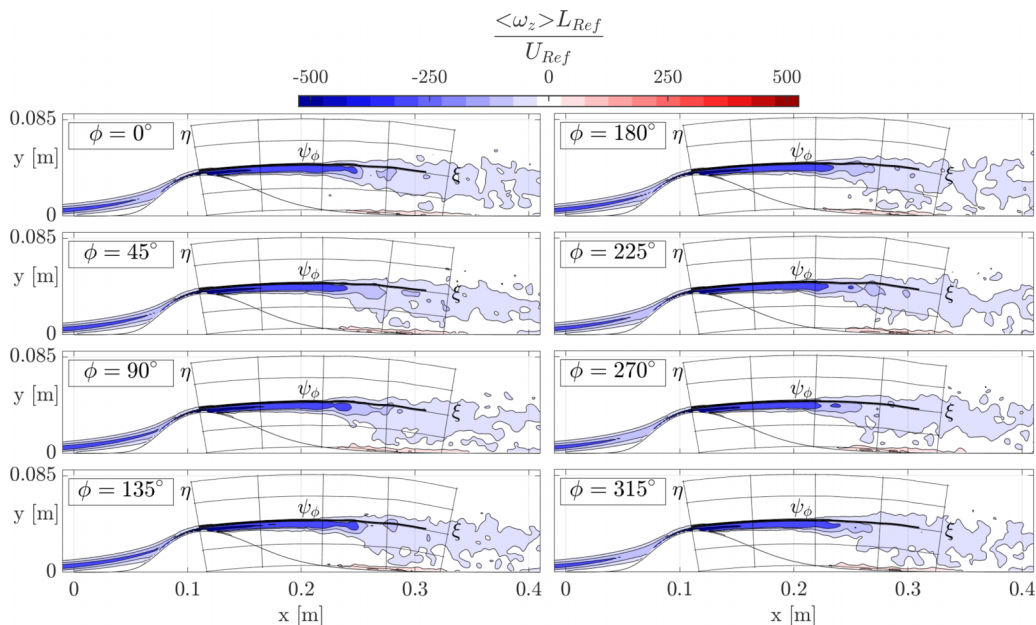


FIG. 12. Phase-averaged spanwise vorticity. $A_{in} = 0.01$ and $f_{in}^* = 0.5$.

associated with phase-averaged changes in the shape and size of the recirculation region. These changes are discussed in the next section.

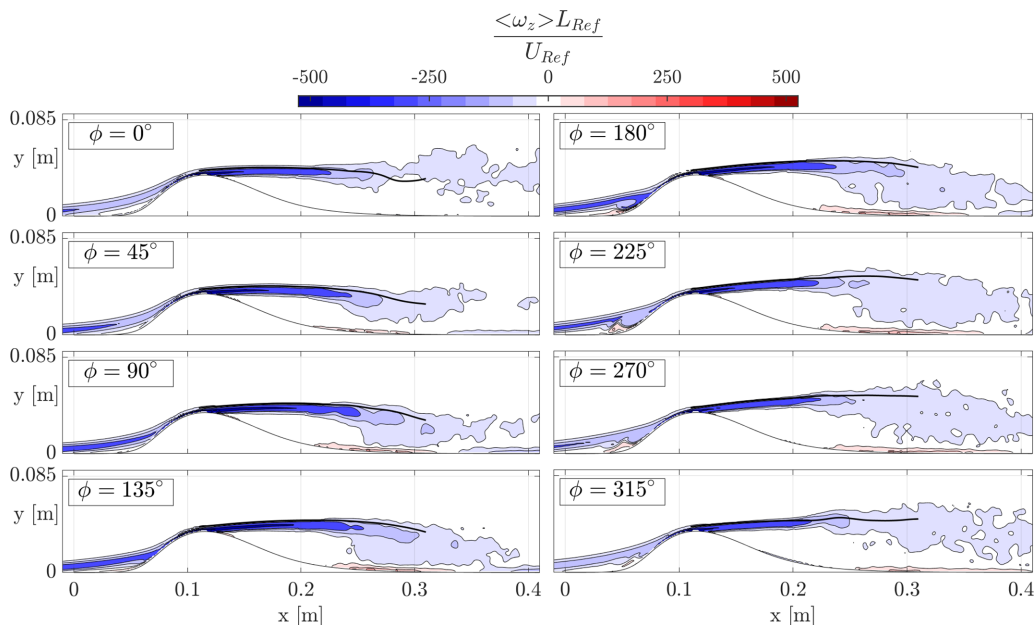
Besides the main recirculation region that is formed downstream of the bump, a smaller recirculation appears upstream of it, analogous to those appearing in forward-facing steps (e.g., [55]). While the appearance of this upstream recirculation is not caused by the inflow oscillations, vortices are periodically released for the cases with larger A_{in} that can interact with the main recirculation region. However, self-excited vortex shedding does not happen in the upstream recirculation, which also remains two-dimensional as shown in Figs. 7 and 8.

C. Phase-averaged flow fields

Figures 12–14 show the evolution of the phase-averaged spanwise vorticity fields at the midspan plane for the three representative unsteady inflow cases. The thick black lines in the figures approximate the separation streamline at each phase. This line is computed by neglecting the spanwise velocity component in the phase-averaged flow and integrating $dx/\langle u \rangle = dy/\langle v \rangle$ starting at the approximate location of the separation point.

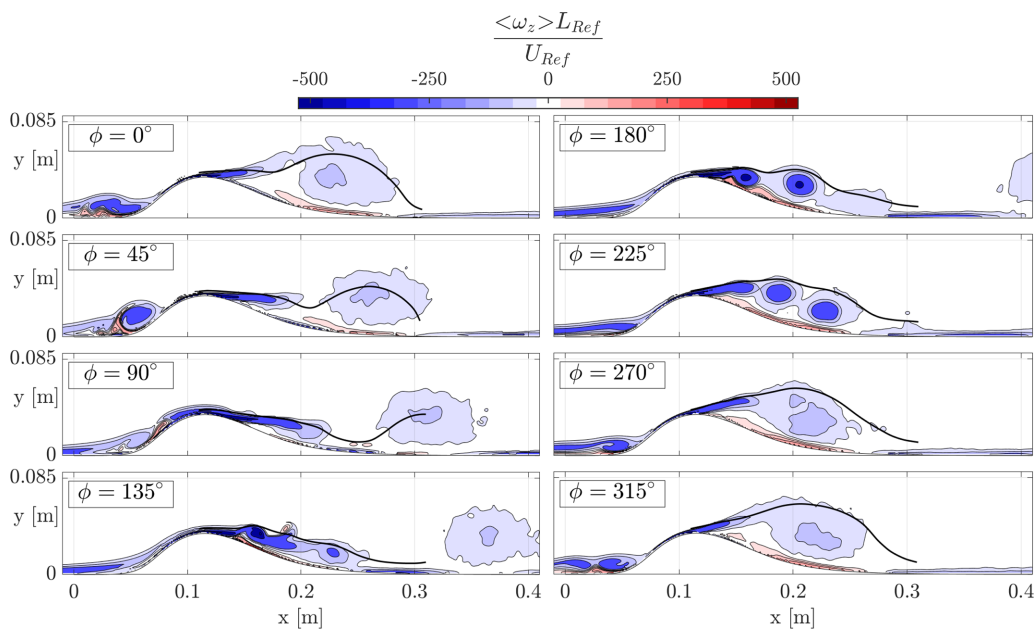
For the case with the weakest inflow oscillation ($A_{in} = 0.01$, $f_{in}^* = 0.5$; Fig. 12), phase averaging successfully isolates the shear layer motion that is coherent with the inflow changes from the KH vortex shedding: the periodic motion of the shear layer towards and apart from the wall is captured in the phase-averaged field (as will be further discussed in the next section), but no imprint of individual vortices or details of the subsequent transition are captured. Instead, the phase-averaged shear layer seems to diffuse as it evolves downstream, occupying the space where vortical structures are identified in the instantaneous flow. This region extends from the separated shear layer to the wall and presents a noticeable patch of positive vorticity adjacent to the wall for all phases; the latter is the imprint of vortical structures that are recirculated in the separation bubble.

The intermediate case ($A_{in} = 0.05$, $f_{in}^* = 1$; Fig. 13) shows more clearly the vertical flapping motion of the shear layer. As opposed to the previous case, the positive vorticity region apparently disappears for phase angles corresponding to bulk flow acceleration ($\phi \approx 0^\circ$ in the figure) and


 FIG. 13. Phase-averaged spanwise vorticity. $A_{in} = 0.05$ and $f_{in}^* = 1$.

becomes more intense around the peak deceleration ($\phi \approx 180^\circ$) where the wall-normal extension of the recirculation region is larger.

Finally, Fig. 14 shows the phase-averaged field vorticity for the case with strongest inflow oscillations ($A_{in} = 0.1$, $f_{in}^* = 2$). The phase-averaged field recovers the periodic formation and release of large


 FIG. 14. Phase-averaged spanwise vorticity. $A_{in} = 0.1$ and $f_{in}^* = 2$.

patches of spanwise vorticity, coherent with the harmonic change of the bulk velocity. Phase $\phi = 270^\circ$ corresponds approximately to the conditions of minimum bulk velocity; for this phase, a vortex of size comparable to the bump is clearly defined downstream of the bump summit. As the flow reaccelerates, the vortex is released ($\phi \approx 0^\circ$) and advected downstream pushing the separation shear layer towards the wall, sensibly reducing the length of the separated flow region. Concurrently with this, a smaller two-dimensional vortex is formed upstream of the bump for $270^\circ \leq \phi \leq 45^\circ$, which is shed at $\phi \approx 45^\circ$ and reaches the bump summit at $\phi \approx 90^\circ$. This vortex interacts with the separated shear layer giving rise to two coherent vortices ($\phi \approx 180^\circ$) that subsequently break down into smaller structures, as shown in the instantaneous flow visualization of Fig. 10. However, the phase-averaged field does not capture the evolution of these vortices after $\phi \approx 225^\circ$, indicating that their dynamics are chaotic and not reproduced from cycle to cycle. Comparing the instantaneous and phase-averaged fields (respectively, Figs. 10 and 14), it is observed that the vortical structures originated by the upstream vortex are completely entrapped in the recirculation region and contribute to its re-generation.

D. Impact of the inflow conditions on streamwise acceleration and length of the separated flow

The phase-averaged fields provide relevant information regarding the impact of the harmonic inflow changes on the streamwise flow acceleration and deceleration and the resulting length of the separated flow region.

In order to quantify the phase-dependent streamwise acceleration, the dimensionless parameter

$$\langle K(\mathbf{x}, \phi) \rangle = \left\langle \frac{v}{u^3(\mathbf{x}, \phi)} \left(\frac{\partial u(\mathbf{x}, \phi)}{\partial t} + u(\mathbf{x}, \phi) \frac{\partial u(\mathbf{x}, \phi)}{\partial x} \right) \right\rangle \quad (15)$$

is used, that can be further divided into two contributions: the contribution from the local flow acceleration

$$\langle K_{\partial u / \partial t}(\mathbf{x}, \phi) \rangle = \left\langle \frac{v}{u^3(\mathbf{x}, \phi)} \frac{\partial u(\mathbf{x}, \phi)}{\partial t} \right\rangle \quad (16)$$

and the convective acceleration

$$\langle K_{\partial u / \partial x}(\mathbf{x}, \phi) \rangle = \left\langle \frac{v}{u^2(\mathbf{x}, \phi)} \frac{\partial u(\mathbf{x}, \phi)}{\partial x} \right\rangle. \quad (17)$$

This parameter was introduced by Spalart [56] as a pressure gradient parameter in studies of flow relaminarization under favorable pressure gradients ($K > 0$) but it is also used to quantify the flow deceleration associated with an adverse pressure gradient ($K < 0$) [37,49,57,58] in boundary layer flows. In the ideal scenario of a boundary layer that is unbounded on the wall-normal direction, the free-stream value of the streamwise velocity would be used and Bernoulli's equation would relate it directly to the streamwise pressure gradient. In the geometry used herein, the upper wall of the channel prevents using this definition. As an approximation to the free-stream velocity, the streamwise velocity at the plane $y = 0.1$ m is used, which corresponds approximately to the midpoint between the bump summit and the upper wall. The streamwise velocity on this plane is expected to be less affected by the instantaneous vortical structures and viscous effects on the separated flow region, though the irrotational flow is strictly not attained. Figure 15 shows the spatio-temporal evolution of the phase-averaged acceleration parameter $\langle K \rangle$, for the three representative cases of harmonic inflow oscillation. The steady inflow case is also shown for comparison.

For the same cases, Fig. 16 shows the temporal evolution of the dimensionless streamwise skin friction at the midspan plane, defined as

$$Cf(\mathbf{x}, \phi) = \frac{\langle \tau_w(\mathbf{x}, \phi) \rangle}{p_t(\mathbf{x}_{\text{ref}}, \phi) - p(\mathbf{x}_{\text{ref}}, \phi)}, \quad (18)$$

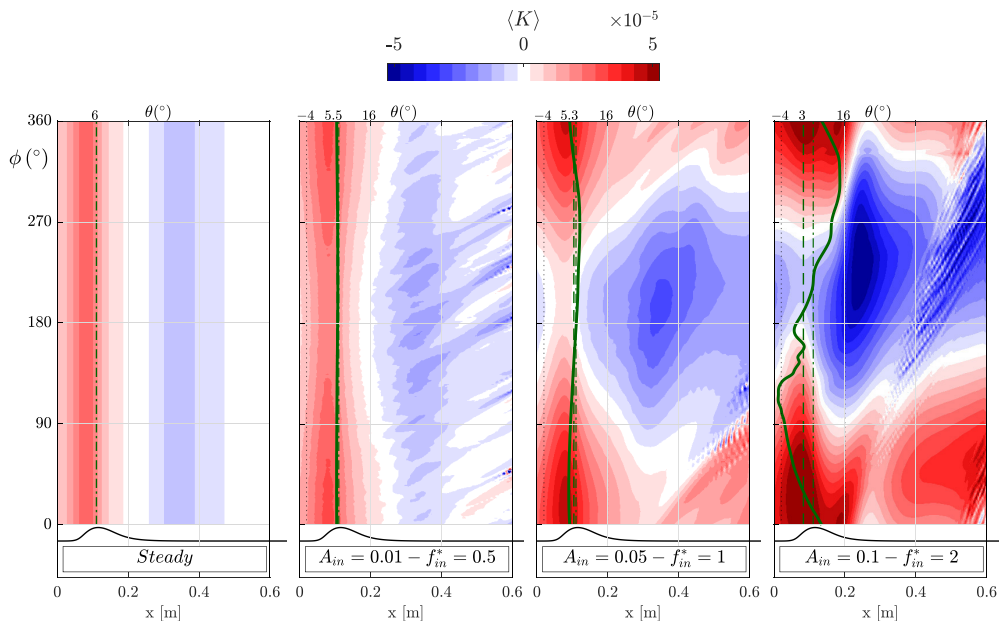


FIG. 15. Contours: Phase-averaged streamwise acceleration parameter at $y = 0.1$ m. Green lines: angle of the streamline at the separation point Γ corresponding to the phase-averaged component (solid line), the mean component (dashed) and the steady inflow case (dashed-dotted).

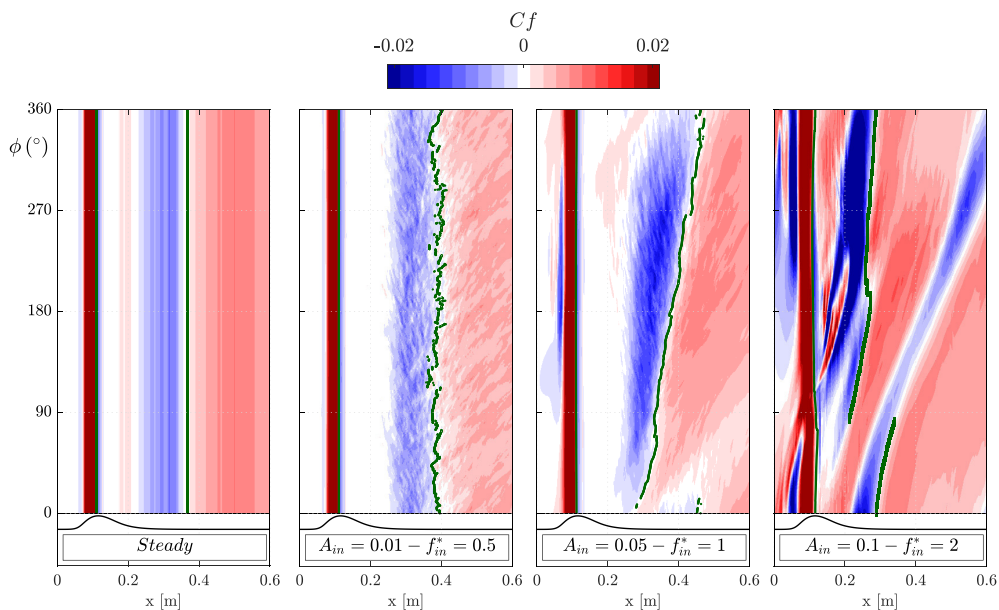


FIG. 16. Phase-averaged skin friction at the mid-span of the bump surface. Left: $A_{in} = 0.01$, $f_{in}^* = 0.5$. Center: $A_{in} = 0.05$, $f_{in}^* = 1$. Right: $A_{in} = 0.1$, $f_{in}^* = 2$. The location of the phase-averaged separation and reattachment points is shown as a green dot for each discrete phase value.

where \mathbf{x}_{ref} corresponds to the location of the reference point (cf. Table III). The phase-averaged location of the separation and reattachment point are also shown in the figure, illustrating the streamwise length of the main recirculation region.

The steady inflow case shows the flow acceleration-then-deceleration distribution generated by the bump. The minimum K value is obtained at $x \approx 0.35$ m, which is coincident with the location of minimum Cf . The time-averaged reattachment occurs a short distance downstream and can be identified as the coordinate where the skin friction changes from negative to positive. The streamwise acceleration parameter is still negative at reattachment, illustrating that the reattachment originated from unsteady flow entrainment rather than the action of a streamwise flow acceleration. Case ($A_{\text{in}} = 0.01$, $f_{\text{in}}^* = 0.5$) shows the same features as the steady inflow case, with a small amplitude modulation that follows the inlet frequency.

The cases with increasingly stronger/faster inflow oscillations exhibit a pattern of phase-dependent deceleration-acceleration localized in the reattachment region that is repeated with each period. For the intermediate case ($A_{\text{in}} = 0.05$, $f_{\text{in}}^* = 1$), the minimum value of $\langle K \rangle$ occurs for $\phi \approx 180^\circ$, coincident with the peak bulk flow deceleration. As the flow reaccelerates for $\phi > 270^\circ$, the region of negative $\langle K \rangle$ is displaced downstream and reduced in size. Then, for the peak acceleration phase $\phi = 0^\circ$, $\langle K \rangle$ is positive around the time-averaged reattachment point. This evolution of $\langle K \rangle$ is followed by the Cf distribution. At $\phi = 0^\circ$, the reattachment point is located around $x = 0.25$ m; reattachment moves downstream gradually for increasing ϕ , resulting in a longer recirculation bubble. During the flow reacceleration, a new region of positive skin friction is formed upstream of the region of minimum Cf , implying that a large coherent vortical structure has been released and a new one is being formed.

Finally, the case ($A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$) shows new distinct features in both the acceleration parameter and the skin friction. For relatively elevated values of A_{in} and f_{in}^* the local flow acceleration $\langle K_{\partial u/\partial t} \rangle$ becomes comparable to the convective one. In consequence, $\langle K \rangle$ is alternatively positive or negative during about half of the period. The peak deceleration is now displaced to the later phase $\phi \approx 275^\circ$. However, its peak magnitude is increased substantially with respect to the case ($A_{\text{in}} = 0.05$, $f_{\text{in}}^* = 1$), and as a result the same negative values of $\langle K \rangle$ are reached before in the period. For instance, case ($A_{\text{in}} = 0.05$, $f_{\text{in}}^* = 1$) presents the minimum $\langle K \rangle$ value -24.74×10^{-6} at ϕ slightly above 180° , while this value is attained in case ($A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$) already at $\phi = 135^\circ$. The intense periodic acceleration-deceleration influences the Cf distribution notably, involving the formation of multiple recirculation regions that are related to the advection of the coherent vortex clusters shown in Fig. 14. Cross-comparison of the streamwise acceleration and skin friction (cf. Fig. 15 and 16) suggests that the formation and release of the large vortical structures in the coherent flow component are associated to surpassing a threshold negative value of the acceleration parameter $\langle K \rangle$. The reasons for this will be further discussed in later sections.

Figure 15 also shows the angle of the separation streamline with the x direction at the separation point corresponding to the phase-averaged flow, the mean flow and the steady inflow case. Under steady flow conditions, a direct relation exists between the streamwise pressure gradient (K), the gradient of the wall shear ($\partial Cf/\partial x$) and the separation angle Γ [59]:

$$\tan \Gamma = -B \left(\frac{\partial Cf}{\partial x} / K \right)_s, \quad (19)$$

where B is a dimensionless constant and the subscript s refers to the separation point. This dependence of Γ on K and Cf together with the complex dependence of the latter on the harmonic inflow oscillations anticipates that the separation angle has a strongly nonlinear behavior. The steady inflow case presents a mean angle $\Gamma \approx 6$ deg. Following the periodic changes in $\langle K \rangle$, the shear layer moves towards and apart the wall which translates in a reduction or increase of the streamline angle at separation. For increasingly stronger inflow oscillations, the mean separation angle is reduced and its oscillation amplitude increased. In line with the phase-averaged oscillations of the separated shear layer already discussed, the separation angle oscillations become substantial for the strongest

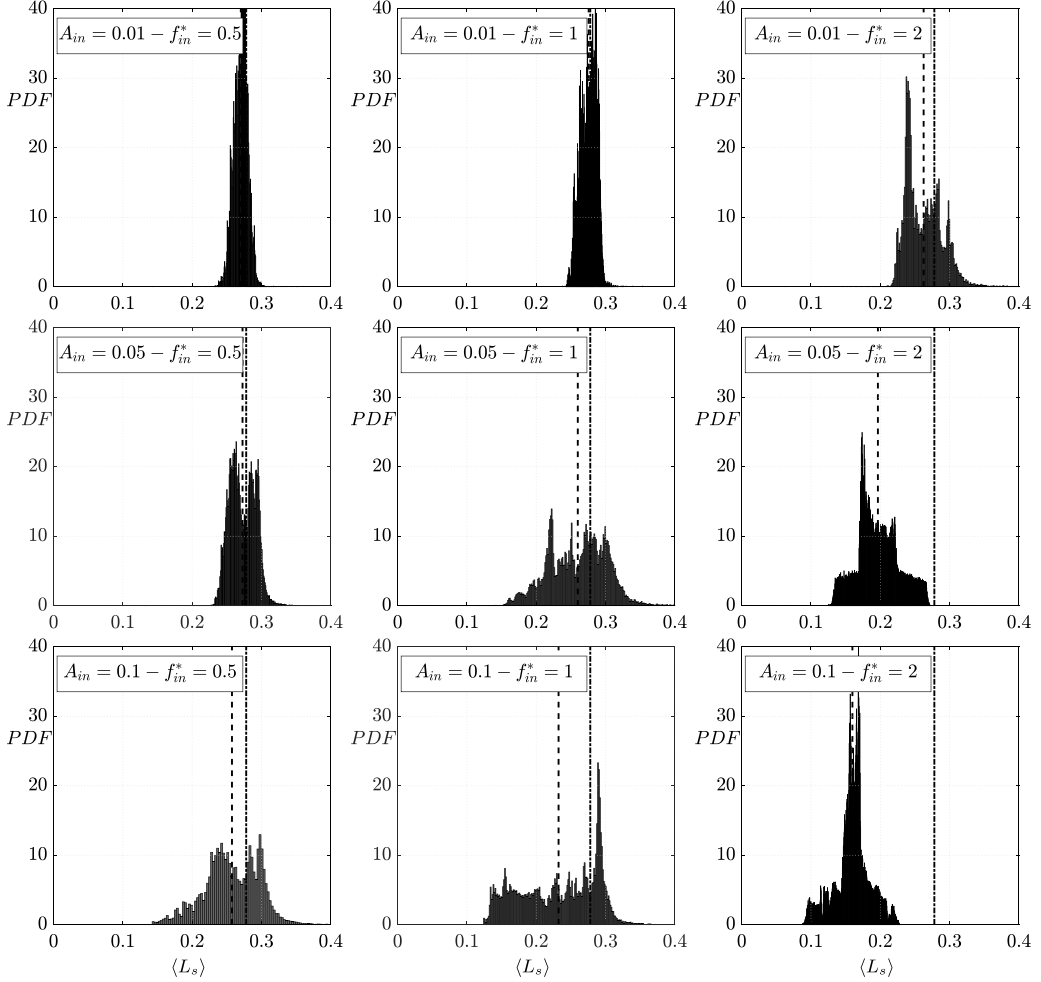


FIG. 17. Probability density function of the phase-averaged recirculation bubble length $\langle L_s \rangle$. (- -) time-averaged value; (- · -) time-averaged for the steady inflow case, $L_{s,steady}$.

inflow oscillation case ($A_{in} = 0.1$, $f_{in}^* = 2$) and a phase-delay appears between the extreme values of $\langle K \rangle$ (positive or negative) and the angle.

The streamwise length of the recirculation bubble L_s is computed at each phase ϕ as the distance between the separation point near the bump summit and the first reattachment point downstream, that are highlighted as green dots in Fig. 16. Due to the fine discretization of ϕ , the black dots visually form nearly continuous lines. Figure 17 shows the probability density function (PDF) of $\langle L_s \rangle$ for all the simulated cases. The time-averaged length for each case and the time-averaged length for the steady inflow case are also shown. Table IV tabulates the numeric values. Cases with relatively low values of A_{in} and f_{in}^* (towards the upper left panels of the figure) show that the recirculation length remains close to that of the steady inflow case with a very narrow distribution range. As A_{in} or f_{in}^* are increased individually (e.g., $A_{in} = 0.05$, $f_{in}^* = 1$), the PDF becomes significantly broader, indicating large changes in $\langle L_s \rangle$ over the period. The time-averaged recirculation length is reduced in all cases, but the distribution is not centered around it and the length at some phases can be substantially longer or shorter than $L_{s,steady}$. This intense temporal variation of $\langle L_s \rangle$ is expected to be associated with periodic changes in the aerodynamic forces exerted on the bump, which may be

highly undesirable in the practical scenario of a low-pressure turbine [10]. Finally, the cases with larger values of A_{in} and f_{in}^* (towards the bottom right panels of Fig. 17, and particularly $A_{in} = 0.1$, $f_{in}^* = 2$) present a PDF which is again centered on the mean L_s value. This value is remarkably reduced with respect to $L_{s,steady}$. Notably, the PDF tail falls to zero for recirculation lengths below $L_{s,steady}$, implying that the separation length is reduced for all phases, including those in which the bulk flow is decelerated. This is also relevant in practical scenarios, as the detrimental effects of flow separation would be consistently reduced with respect to the steady inflow case.

E. Incoherent vorticity and vortex dynamics

The phase-averaged results in the previous section illustrate the overall dynamics of the shear layer and the behavior of the separated flow on account of the harmonic inflow changes. The length of the recirculation region and the impact of the inflow oscillation frequency and amplitude are, in turn, a consequence of changes in the vortex dynamics induced by the transient changes in the streamwise acceleration. For comparatively low values of A_{in} and f_{in}^* , vortex rolls are formed at the separated shear layer resulting from the KH instability, which initiates the laminar-to-turbulent transition. The spread of the shear layer, entrainment of fluid into the recirculation region and ultimately flow reattachment are governed by the complex dynamics of these vortices. These processes are not captured by the phase-averaged fields, which only recover their consequences.

For low f_{in}^* , the characteristic period in which the shear-layer motion occurs is much longer than the characteristic time for the formation and advection of the vortices, as will be shown below. In consequence, the vortex dynamics are completely recovered in the incoherent flow component q' of the triple decomposition. The vortices are formed and initially move along the separated shear layer, whose motion is captured by the phase-averaged component. To separate the vortex dynamics from the shear-layer motion, the following procedure is adopted. First, an orthogonal curvilinear coordinate system (ξ, η) is defined, as shown in Fig. 12. The line $\eta = 0$ corresponds to the phase-averaged separation streamline and ξ is the curvilinear coordinate measured along it. Then, an inverse transformation is performed to map the curvilinear-coordinates grid points to cartesian coordinates with the streamline as the centerline axis. Details of the inverse transformation process are described by Legleiter and Kyriakidis [60].

This shear-layer fitted system of coordinates is inspired by the one used by Himeno *et al.* [61] to study the vortex dynamics along the steady shear layer formed within a slat cove. In the present case, the curvilinear coordinates depend on the phase angle, following the motion of the phase-averaged separation streamline. Figure 18 shows the total (left column), the phase-averaged (middle column) and the incoherent (right column) components of the spanwise vorticity in the curvilinear mesh, for the case representative of the weaker inflow oscillations ($A_{in} = 0.01$, $f_{in}^* = 0.5$). Note that the phase-averaged separation streamline corresponds to the horizontal line $\eta = 0$, and the bump wall is mapped to a curve line with $\eta < 0$ whose geometry changes with the phase.

The incoherent component portrays a complex arrangement of vortices typical of a transitional shear layer [9,11,12,35]. The process is initiated with the formation of organized pockets of spanwise vorticity in the initial part of the shear layer. These vortices grow in amplitude as they travel downstream along $\eta = 0$, soon reaching non-linear amplitudes and then interacting and merging with the recirculating vortical structures adjacent to the wall.

To visualize the spatiotemporal dynamics of these vortices, the incoherent spanwise vorticity ω'_z at $\eta = 0$ is plotted in the (ξ, t^*) plane for the steady inflow case and the three cases representative of harmonic inflow in Fig. 19. Note that ω'_z is a disturbance superimposed upon the phase-averaged flow and consequently its positive and negative values do not directly imply vortical structures rotating both in clockwise and counterclockwise directions.

The steady inflow case shows the continuous formation of vortical structures of similar amplitude and apparent shedding frequency that propagate downstream at a nearly constant speed along the mean shear layer. With the choice of contour levels used, they become observable around $\xi = 0.1$ m, which for the steady case corresponds to $x \approx 0.2$ m. Case ($A_{in} = 0.01$, $f_{in}^* = 0.5$) presents a very

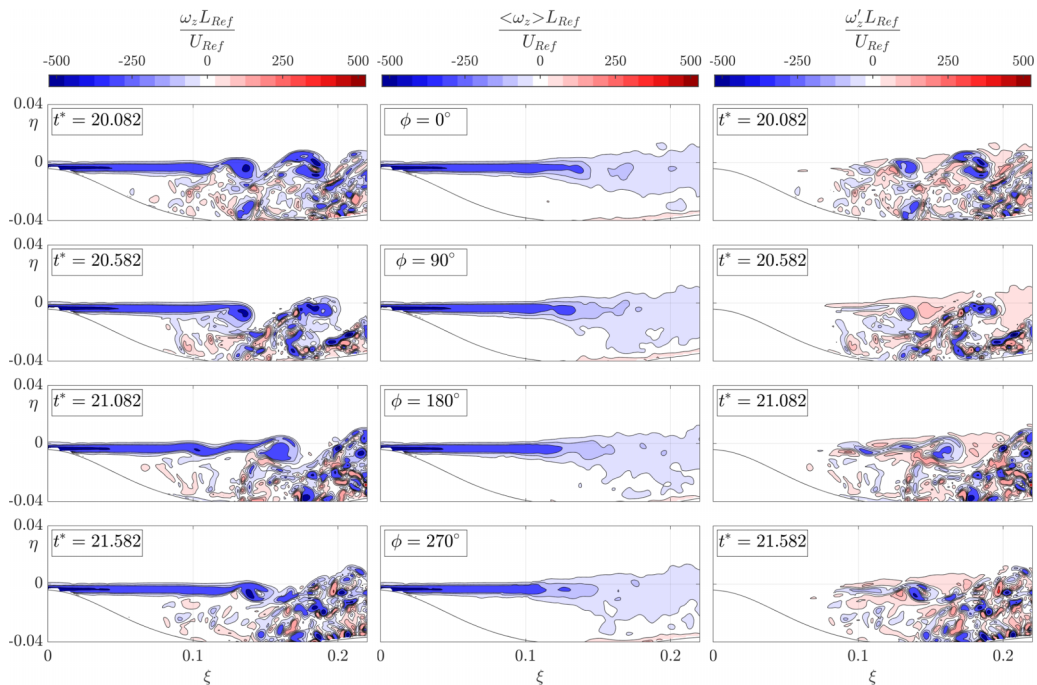


FIG. 18. Triple decomposition of spanwise vorticity. $A_{in} = 0.01$ and $f_{in}^* = 0.5$.

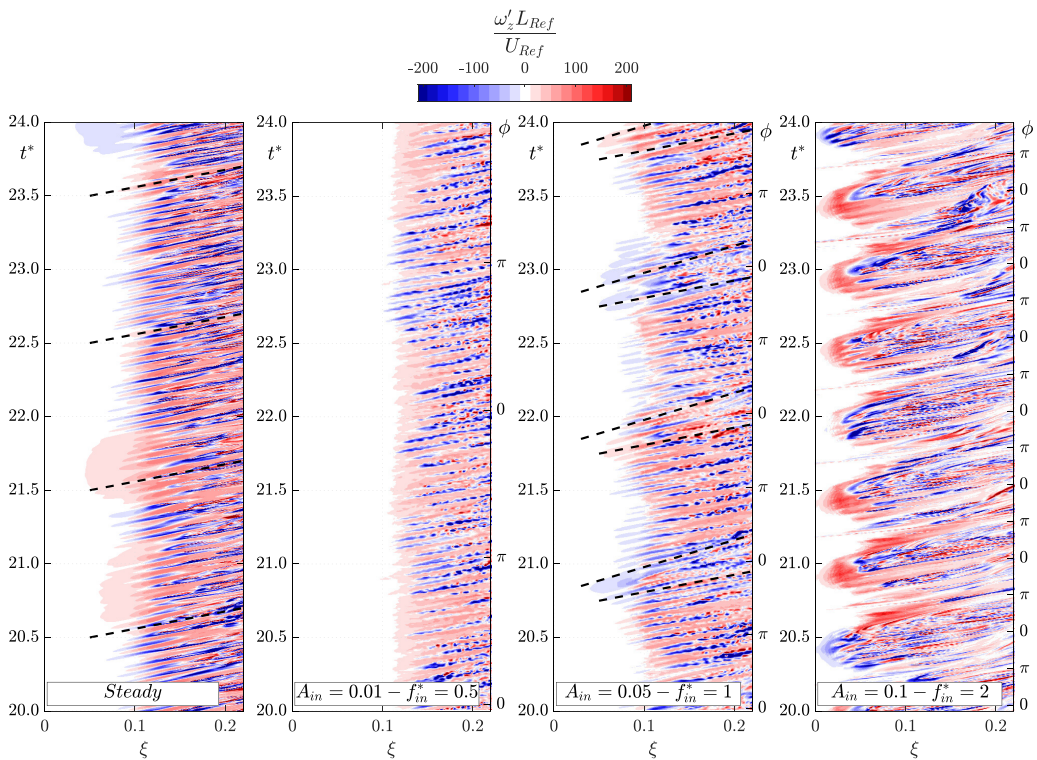


FIG. 19. Incoherent spanwise vorticity at $\eta = 0$ for the steady inflow case and the three representative cases with harmonic inflow.

similar picture, but the location where the vortical structures are first seen now oscillates very mildly following the inflow phase; bulk flow deceleration displaces the observable incoherent vorticity upstream and *vice versa*. The modulation of the vortex shedding location is increased by increasing either A_{in} or f_{in}^* .

The intermediate case ($A_{\text{in}} = 0.05$, $f_{\text{in}}^* = 1$) presents an additional feature for certain regions of the (ξ, ϕ) plane. As in the previous cases, regular periodic positive and negative streaks of vorticity are visible, with their initial ξ location following the inflow fluctuation phase. However, for a certain phase range (between $\phi = 3\pi/2 = 270^\circ$ and $\phi = 2\pi = 360^\circ$) this pattern is replaced by a “wedge” of distinct behavior, as schematized in the corresponding panel of Fig. 19. The wedge originates at the phase $\phi \approx 3\pi/2$, for which ω'_z reaches observable amplitudes sensibly upstream than for the preceding phases. From this point, two rays depart at different downstream speeds that enclose a region where the vorticity presents a disorganized behavior.

The wedge’s origin is coincident with the phase of minimum bulk velocity. This is interpreted in the following manner: during most of the inflow period, the vortex dynamics follow the same qualitative picture as for the lower A_{in} and f_{in}^* cases, characterized by a regular formation of KH vortices for which the shedding location follows the inflow phase. At some instant during the phase-average deceleration ($90^\circ < \phi < 270^\circ$), the streamwise deceleration parameter (K) surpasses a threshold value in the region neighboring the reattachment point. As a result, the KH vortices and other shear layer eddies are not released from the rear part of the reversed flow region but are entrapped in the recirculation region and initiate the formation of a large cluster of vortical structures. Immediately following the beginning of the acceleration phase, the large vortex cluster is released, similar to the shedding of leading-edge vortices in oscillating airfoils (e.g., Lind and Jones [62]).

The vortex cluster is advected downstream during the initial part of the acceleration phase and subsequently, the regular shedding of KH vortices resumes. In the visualization of the phase-average flow for this case (Fig. 13) a large vortex cannot be clearly discerned, but the presence of the vortex lump can be inferred by the changes in the streamline curvature between $\phi = 315^\circ$ and 360° and by the emergence of a region with positive C_f within the main recirculation, prior to its release, in Fig. 16. The formation and release of a large vortex cluster is more easily observed by visualizing the complete flow field (without separating phase-averaged and incoherent components), as done in Fig. 20.

Finally, the incoherent vorticity for the case ($A_{\text{in}} = 0.1$, $f_{\text{in}}^* = 2$) does not present a regular shedding of KH vortices akin to the steady or low inflow frequency cases. Instead, it is characterized by the continuous appearance of wedges, corresponding to the periodic formation and shedding of large vortex clusters following the inflow changes. With the large vortex cluster being recovered in the phase-averaged flow (Fig. 14), the incoherent component consists of smaller size eddies that are trapped and evolve inside the recirculation region during the deceleration part of the period and are convected downstream when the vortex cluster is released. Owing to their chaotic nature, these structures are not repeated from cycle to cycle and hence are not coherent with the inflow changes, but their presence in the ω'_z field allows the location and tracking of the coherent vortex clusters.

E. Frequency spectra

Frequency spectra are studied in this section to further the characterization of the impact of the inflow oscillations on the separated flow dynamics. The procedure for the calculation of the power spectral densities (PSDs) for the steady inflow case is described in Sec. IID. For the harmonic inflow cases, the procedure is adapted to account for the periodic nature of the data. The sampling parameters are summarized in Table V. To allow for comparisons, the same averaging period is used for all cases, corresponding to the lowest of the inflow frequencies, $\Delta f^* = 0.5$. Thus, the averaging is done over segments comprising one inflow period for $f_{\text{in}}^* = 0.5$, two periods for $f_{\text{in}}^* = 1$ and four periods for $f_{\text{in}}^* = 2$. For all cases, the PSD is averaged over 25 segments of length $1/\Delta f^* = 1/0.5 = 2$.

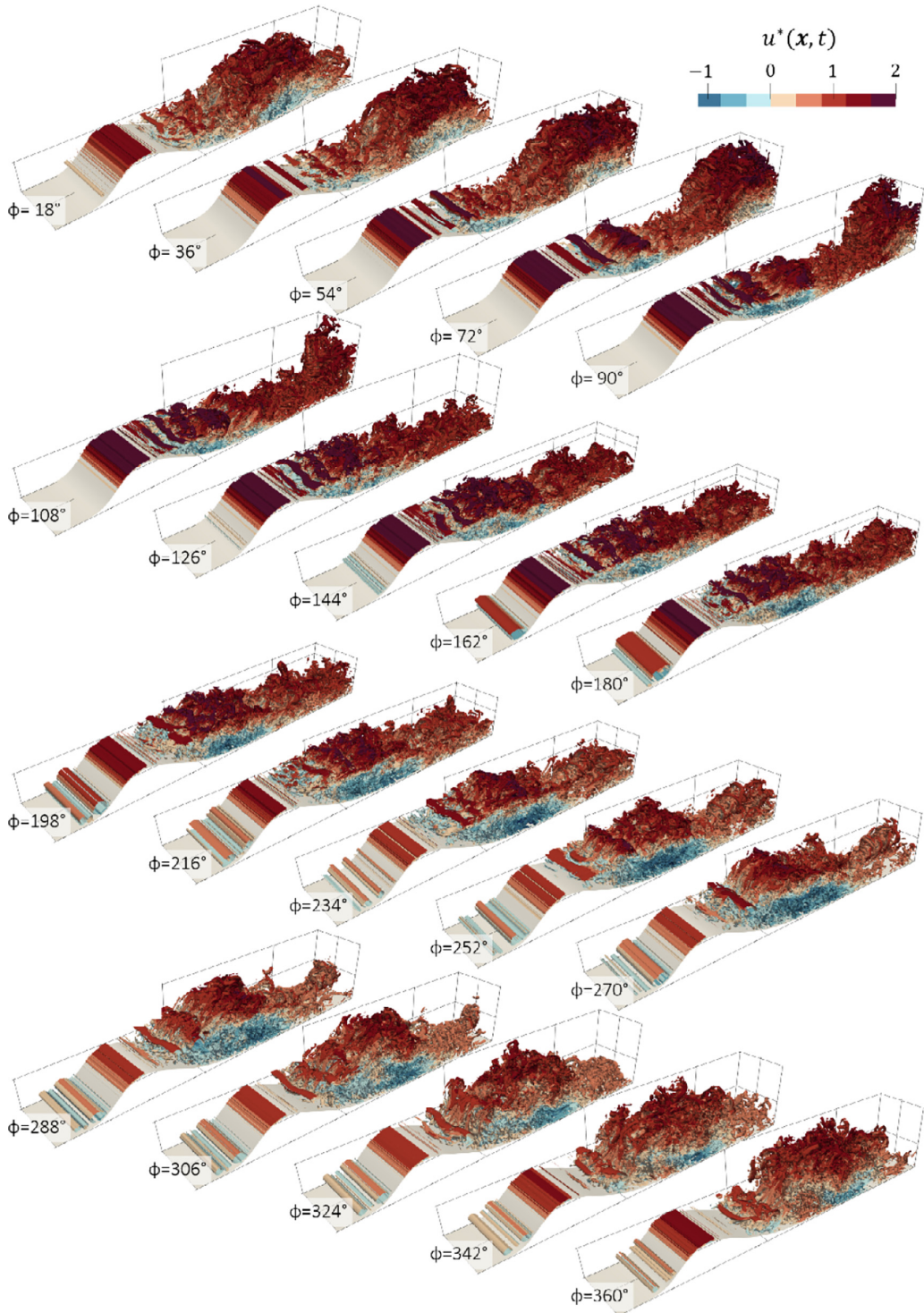


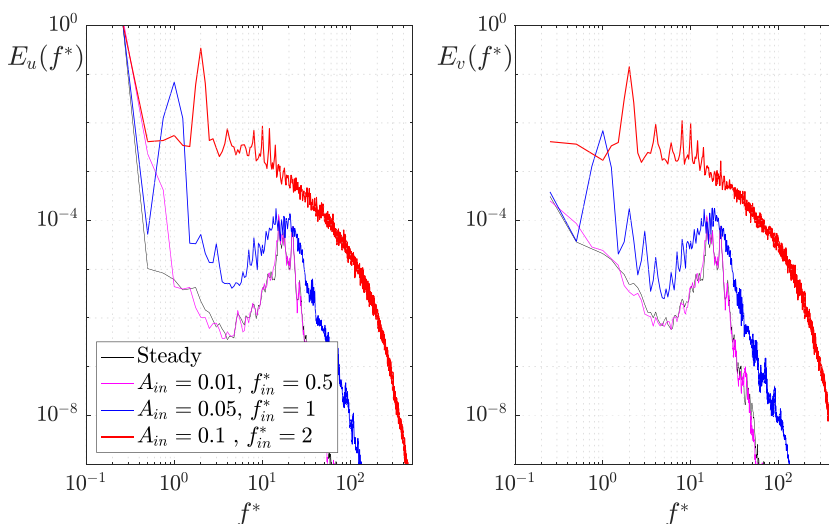
FIG. 20. $Q(+)$ isosurface colored by streamwise velocity. $A_{in} = 0.1$ and $f_{in}^* = 1$.

TABLE V. Temporal sampling parameters: N_s : total number of snapshots; Δt_s^* : time step between snapshots used in the analysis; N_{ss} : number of snapshots per segment.

f_{in}^*	N_s	Δt_s^*	N_{ss}	Overlap
Steady	2 410 000	3×10^{-5}	66 666	50%
0.5	50 000	1×10^{-3}	2000	0%
1	50 000	1×10^{-3}	2000	0%
2	100 000	5×10^{-4}	4000	0%

Figure 21 shows the spectra of the streamwise and wall-normal velocity components at Probe 4 (see Fig. 5 and Table III). For the steady inflow case, this probe is located just outside of the separated shear layer at the streamwise location where the first KH vortices are formed. The spectra for the steady inflow and the weak inflow oscillation ($A_{in} = 0.01$, $f_{in}^* = 0.5$) cases are identical, presenting a narrowband peak at frequencies $f^* = 15 - 21$ with a maximum at $f^* \approx 18$. As shown in Appendix A, this frequency corresponds to KH instability and follows accurately the scaling proposed by Diwan and Ramesh [11]. The intermediate case ($A_{in} = 0.05$, $f_{in}^* = 1$) also exhibits the narrowband peak corresponding to the KH vortices; the peak amplitude and frequency remain comparable to that of the steady forcing case, but the sidebands are broader. However, new peaks appear for the inflow frequency and its harmonics. The f_{in}^* peak has an amplitude that is two orders of magnitude larger than the peak KH frequency. The spectra for the case with the strongest inflow oscillation ($A_{in} = 0.1$, $f_{in}^* = 2$) also contain the peaks corresponding to the inflow frequency and its harmonics. However, the narrowband peak corresponding to KH vortices is not observable in this case. The amplitude for all frequencies is increased above those corresponding to KH vortices. In consequence, KH vortices could still exist but be shadowed by other, more energetic, fluctuations.

To shed light on the last point, the PSD of the incoherent spanwise vorticity ω_z' is calculated at a location at the shear layer corresponding to $\xi = 0.12$ for all the cases simulated and shown in Fig. 22. Left, center and right subfigures correspond to the lower, intermediate and higher amplitude of the inlet oscillation A_{in} , respectively. For each of them, the three frequencies are shown. The spectra for the steady inflow case are also shown in the three figures for reference. The spectra for the lower A_{in} cases (Fig. 22, left) are qualitatively identical, being dominated by KH vortices.


 FIG. 21. Power spectral density of the streamwise and wall normal velocity at Probe 4, $(x, y) = (0.2, 0.05)$.

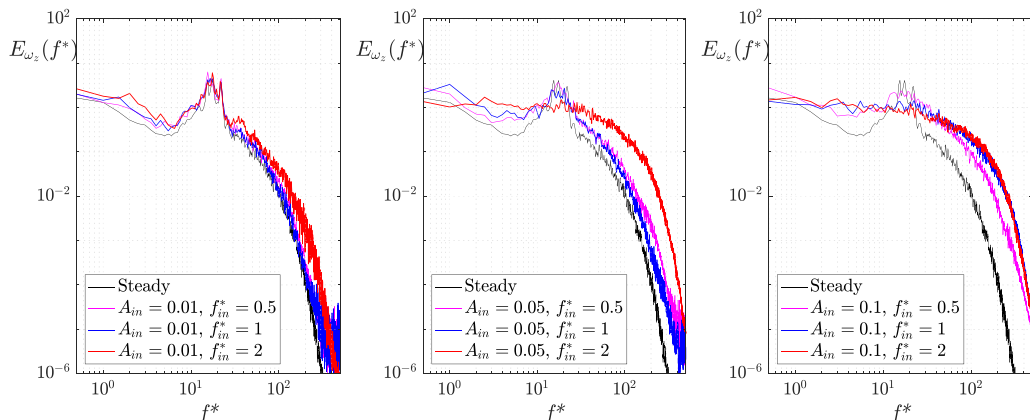


FIG. 22. Power spectral density of the incoherent spanwise vorticity ω'_z at the shear layer, $\xi = 0.12$.

Roughly the same amplitudes are obtained for the frequencies associated with the KH vortices for the three values of f_{in}^* , while the higher frequency range, corresponding to the turbulent cascade, is found to be slightly more energetic with increasing f_{in}^* . Conversely, the spectral for the intermediate A_{in} (Fig. 22, middle) shows qualitative changes that occur gradually as the inflow frequency f_{in}^* is increased. The amplitude of the KH narrowband peak is reduced while it is increased for all the other frequencies. For the largest inflow frequency, $f_{in}^* = 2$, the KH peak is not present anymore and the amplitude at the corresponding frequency is lower than for the cases with KH vortices. This shows that the regular KH vortex shedding is not shadowed by more energetic fluctuations, but rather eliminated. Individual KH vortices are still formed in the initial part of the separated shear layer, but in an irregular manner, and they are not released from the recirculation bubble but trapped within it and quickly distorted by their interaction with the fine-grained turbulence. Finally, the spectral for the largest A_{in} (Fig. 22, right) follows the same trend as the intermediate A_{in} ones.

The changes observed in the PSD with increasing A_{in} and f_{in}^* are consistent with the vortex dynamics discussed in Sec. III E. As the large vortex cluster is formed, most of the vortical structures present in the separated shear layer get trapped in the recirculation region instead of being shed and advected downstream. Nonlinear interactions between the recirculating structures lead to their merging and progressive breakdown into smaller structures, which leads to a more energetic and flatter spectral. In turn, the recirculation of random eddies of diverse scales prevents the formation of well-defined KH vortices in the separated shear layer.

IV. DISCUSSION AND CONCLUSIONS

This paper studies the impact of harmonic oscillations of the inflow velocity, imposed via the total pressure, on the flow over a wall-mounted bump geometry. This geometry gives rise to a streamwise pressure gradient distribution with similar features to those encountered in LPT blades. The harmonic inflow oscillation roughly models the effect of the passage of the wake due to the previous stage of blades, periodically creating a velocity deficit followed by acceleration, and consequently a periodic modification of streamwise velocity gradients.

Under steady inflow conditions, the laminar-to-turbulent transition is initiated by a self-excited KH instability with a well-defined vortex-shedding frequency ($f_{KH}^* \approx 18$). These vortices are subject to secondary instabilities and interactions with turbulent structures that are recirculated within the separated flow and progressively break down into smaller eddies as they travel downstream. The associated entrainment leads to the reattachment of the mean flow.

The impact of the inflow oscillations on the flow dynamics is strongly dependent on the frequency and amplitude of the oscillations. The cases studied involve oscillations of the inflow total pressure

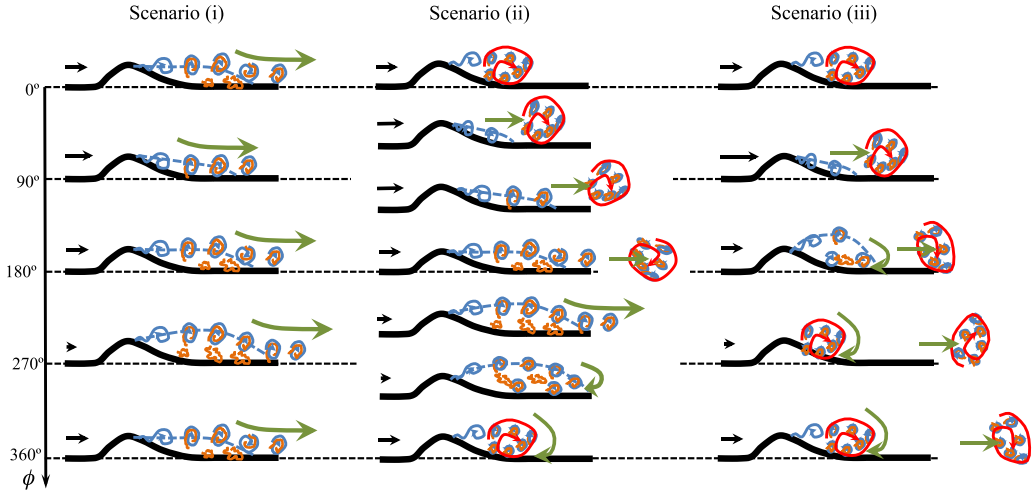


FIG. 23. Descriptive illustrations of the three scenarios. Dashed blue lines: phase-averaged separation streamline; solid blue: KH vortices; orange: small-grained vorticity; red: vortex cluster; green arrows show the instantaneous trajectory of the flow structures.

A_{in} between 1% and 10% of the mean value, and frequencies $f_{in}^* = 0.5-2$, substantially lower than f_{KH}^* but comparable to the wake-passing frequency in LPT turbines. If the reference velocity U_{Ref} and the mean recirculation length for the steady inflow case $L_s = 0.2782$ m are used to calculate the reduced frequency F defined in Sec I, the inlet frequencies f_{in}^* take the values $F = 0.1391, 0.2782$, and 0.5564 , which lie between the $F \ll 1$ and $F \sim 1$ ranges. Three different scenarios have been identified. Appendix B shows the classification of the cases, and Fig. 23 depicts them schematically:

Scenario (i): *Inflow-modulated Kelvin-Helmholtz vortex shedding*, corresponding to comparatively weak inflow oscillations (lowest values of A_{in} and f_{in}^* ; cf. Figs. 7 and 23 left). The transition process is qualitatively the same as for the steady inflow case. However, the harmonic changes of the bulk velocity lead to a periodic vertical displacement of the separated shear layer, phase-locked to the inflow oscillation. While the self-sustained vortex shedding originated by KH instability remains, its spatial amplification is modulated by the changes in the shear layer properties: vortex shedding takes place upstream for the part of the period in which the bulk flow is decelerated and downstream for the part in which it is accelerated. Consequently, the phase-averaged length of the recirculation region L_s changes gradually over the period, but the mean length (i.e., averaged over the period) is approximately the same as for the steady case. This scenario is consistent with the experiments by Lou and Hourmouziadis [30] that imposed inflow oscillations with $\Delta u^* = 0.13$ and $F \approx 0.06$ (obtained by scaling their reported dimensionless frequency with the mean length of their separation bubble).

Scenario (ii): *Alternation between KH vortex shedding and formation/release of a large vortex cluster*, corresponding to intermediate inflow oscillations (see Figs. 20 and 23 center, and movies 3 and 4 in Supplemental Material [53]). For a portion of the period, KH-initiated vortex shedding is dominant, identical to scenario (i). However, as the bulk flow decelerates, the phase-averaged streamwise acceleration parameter $\langle K \rangle$ transiently surpasses a threshold negative value in the region neighboring flow reattachment, giving rise to new dynamics: the vortical structures formed in the shear layer are not shed and advected downstream. Instead, they are entrapped in the recirculation region, which accordingly grows in size forming a large vortex cluster formed by eddies of a wide range of scales. When the bulk flow is reaccelerated, instead of reducing the recirculation region gradually through the shedding of KH vortices [as in scenario (i)], the large vortex cluster is released and advected downstream. The advection of the vortex cluster pulls the stagnant fluid,

transiently reducing the separated flow extent. Subsequently, the recirculation region regenerates and the KH vortex shedding restarts. The periodic formation and release of the large vortex cluster impact the phase-averaged length of the reversed flow region drastically: the time-averaged L_s is reduced with respect to the steady inflow case, but the deviations from the mean value (see the PDFs in Fig. 17) become considerably broader including lengths longer than those for the steady inflow. Important hysteresis effects appear between the accelerating and decelerating parts of the period. This scenario would translate into undesirable conditions regarding practical LPT blades, involving strong oscillatory loads without a substantial reduction of the mean separation length.

Scenario (iii): *Phase-locked formation and release of large vortex clusters*, corresponding to strong inflow oscillations (the largest values of A_{in} and f_{in}^* ; cf. Figs. 8 and 23 right). The dynamics of the formation and release of the large vortex cluster following the inflow oscillations occupy all periods: before one vortex cluster has time to travel downstream a distance sufficient for the recirculation region to reinitiate the KH vortex shedding, the threshold value of the streamwise acceleration parameter is reached and a new vortex cluster is being formed. The deviation of the separated flow length over the period is larger than that of scenario (i) but less than that of scenario (ii). More importantly, the phase-averaged L_s is smaller than the mean length for the steady inflow case for all the phases and the time-averaged L_s is remarkably reduced, above a 40% for the case ($A_{in} = 0.1$, $f_{in}^* = 2$). Regarding a practical LPT application, this scenario would be preferable over the other two. This scenario is the same recovered in the simulations by Wissink and Rodi [31] and Wissink [32], who considered different inflow frequencies between $F \approx 1$ and $F \approx 6$ and amplitudes Δu^* between 0.05 and 0.2.

In order to clarify if our conclusions regarding the three different scenarios are general or particular to wall-mounted bump geometry considered so far, Appendix C briefly presents an analogous study considering the related setup of the NASA hump. The same three scenarios are recovered, while the combination of values of A_{in} and f_{in}^* for each of them is changed.

It is to be noted that the impact of the inflow oscillations is not related to the individual parameters A_{in} or f_{in}^* but to a combination of both. In all the cases A_{in} is too large to be considered a linear flow disturbance, and f_{in}^* is an order of magnitude lower than the natural frequency of the Kelvin-Helmholtz instability, f_{KH}^* . The transition between the different scenarios is thus not associated with the excitation of the KH instability. The transition between scenarios (i) and (ii) is related to the existence of a threshold value of the acceleration parameter $\langle K \rangle$. When this value is exceeded transiently in the region towards the end of the bump, a large vortex cluster is formed and eventually released. According to the cases simulated, the threshold value is bounded between $\langle K \rangle = -16.71 \times 10^{-6}$ and -22.43×10^{-6} (see Appendix B). However, the numerical value of this parameter is particular to the definition of $\langle K \rangle$ used, which is based on an arbitrary y coordinate. On the other hand, the transition between scenarios (ii) and (iii) is related to the ratio between the inlet oscillation period and the time required for the recirculation region to regenerate and restart the KH vortex shedding after the release of one vortex cluster.

Our results cannot ascertain if a further increase in the inflow frequency would lead to further reductions in the time-averaged separated flow length. Within scenario (iii), the optimal time-averaged L_s reduction would be achieved by a balance between increasing the amount of recirculating fluid advected with one vortex cluster (i.e., the size of the phase-averaged vortex) and increasing the frequency of release of such clusters. However, the impact of inflow oscillations on the dynamics and time-averaged length of separated flows described in this work presents similarities with studies of active flow control via harmonic suction/blowing or moving parts reviewed by Greenblatt and Wygnanski [5] and with the experiments by Hasan [63], Sigurdson [64]. These works report the existence of an optimal forcing frequency that scales with the global size of the recirculating flow region and the free-stream inflow velocity. Forcing at this frequency range promotes the phase-locked formation and shedding of large vortices similar to present scenario (iii), altering the dynamics from those of the KH-initiated transition [scenario (i)]. Greenblatt and Wygnanski [5] concluded that the optimal forcing frequency scales with the streamwise length of the unforced recirculating flow region, L_s . Conversely, Hasan [63], Sigurdson [64] argue that the natural frequency for the “shedding of the

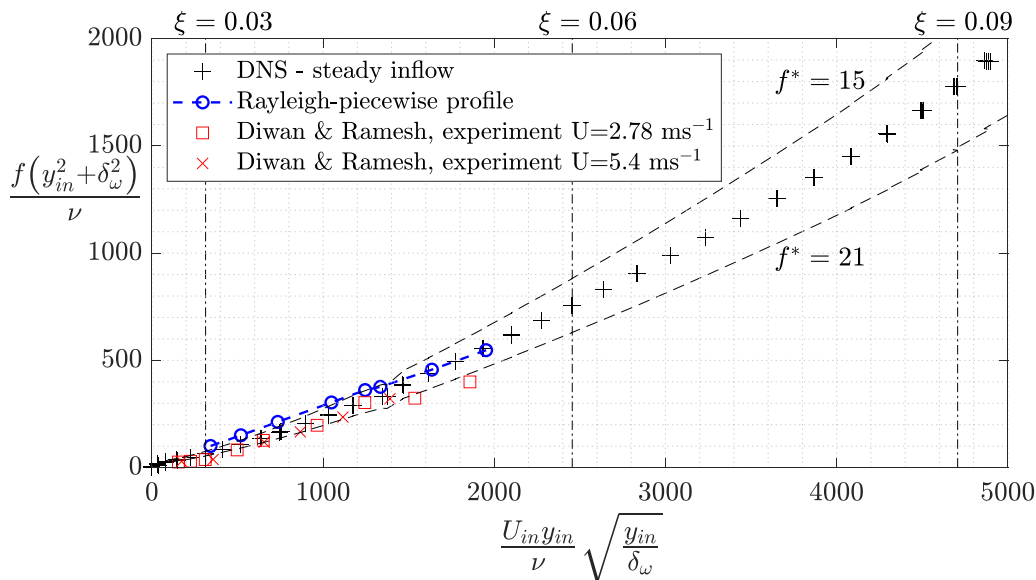


FIG. 24. Scaling of the most amplified frequency along the separated shear layer.

entire bubble” (i.e., release of the large coherent vortex cluster) scales with its wall-normal height h , in analogy to the von Karman vortex street, and the optimum forcing would act at this frequency. Sigurdson [64] proposed a dimensionless frequency $St_{\text{shedding}} = fh/U_s = 0.07\text{--}0.08$ (where U_s is the free-stream velocity at the separation point) and demonstrated that it correlates well with a variety of geometry-induced separation bubbles. Translated to the dimensionless form used herein, the global shedding frequency results $f_{\text{shedding}}^* = 2.9\text{--}3.3$, which is only slightly higher than the largest f_{in}^* considered.

On the other hand, forcing with higher frequencies, aimed at exciting the -local- KH instability, may lead to a faster transition via scenario (i) [47] or by inducing pairing of KH-vortices [12,48]. This certainly leads to reductions in the time-averaged separation length, but in view of present results, these reductions are expected to be smaller than the ones achieved by exciting the global dynamics of the separated flow.

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APPENDIX A: FREQUENCY OF THE KELVIN-HELMHOLTZ INSTABILITY

Diwan and Ramesh [11] proposed a scaling of the frequency corresponding to the most amplified KH instability based on the linear instability analysis of piecewise linear velocity profiles resembling separated shear layers in the presence of a wall. This scaling combines the local (i.e., at individual streamwise locations) properties of the flow: the wall-normal coordinate and streamwise velocity of the inflection point (y_{in} and U_{in}), the vorticity thickness δ_ω , and the most amplified frequency f , into two dimensionless parameters: the dimensionless frequency $f(y_{\text{in}}^2 + \delta_\omega^2)/\nu$ and a modified Reynolds

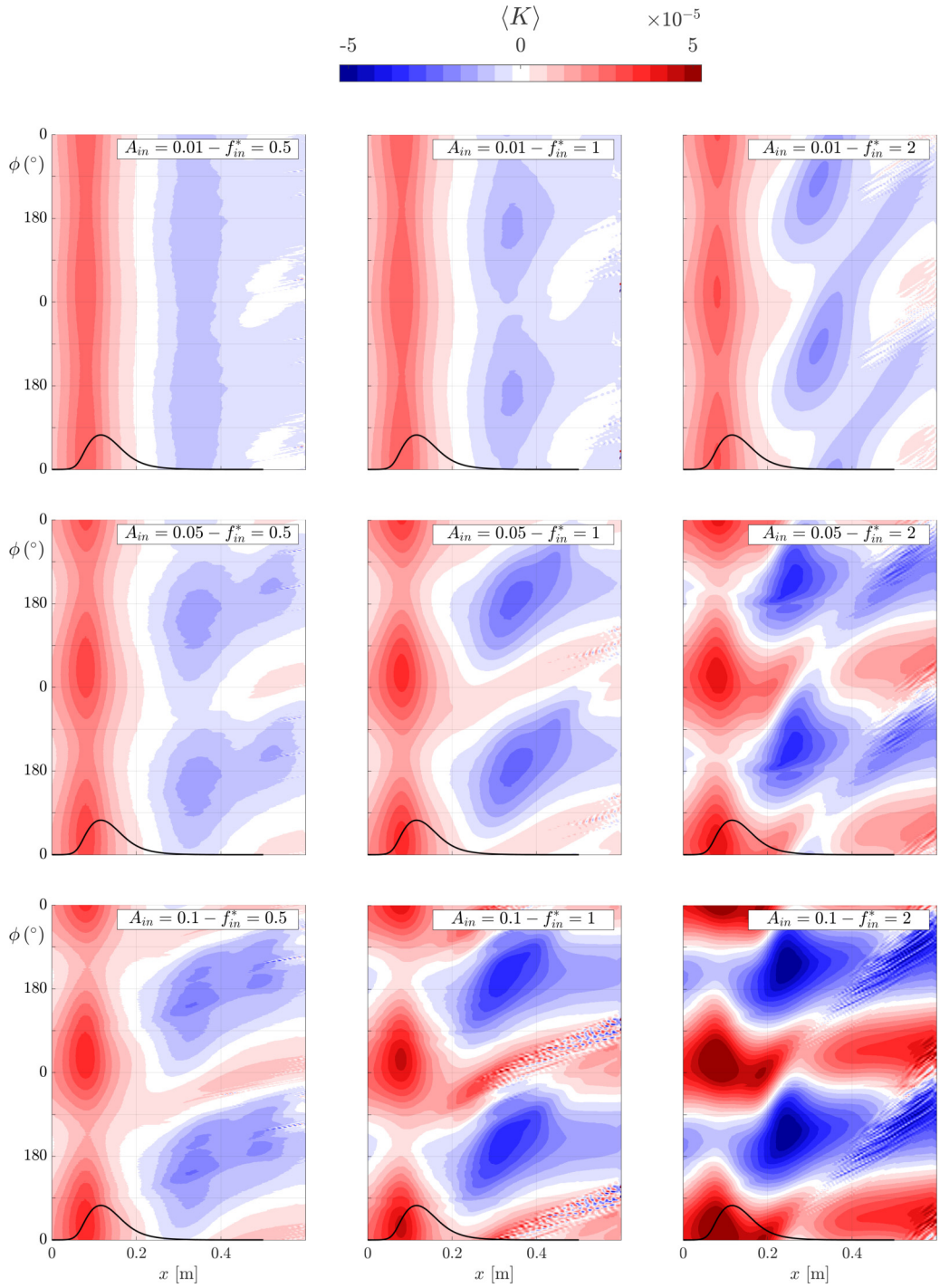


FIG. 25. Phase-averaged streamwise acceleration parameter.

TABLE VI. Classification of the cases. Minimum values of the time-averaged and phase-averaged streamwise acceleration parameter and reduction of the time-averaged length of the separated flow region. The first row corresponds to the steady inflow case.

A_{in}	f_{in}^*	$A_{in}f_{in}^*$	\bar{K}_{min} $\times 10^{-6}$	$\langle K \rangle_{min}$ $\times 10^{-6}$	$\langle K_{\partial u/\partial x} \rangle_{min}$ $\times 10^{-6}$	$\langle K_{\partial u/\partial t} \rangle_{min}$ $\times 10^{-6}$	$\Delta L_s/L_{s,steady}$	Scenario
—	—	—	-9.76	—	—	—	—	(i)
0.01	0.5	0.005	-10.62	-12.44	-13.22	-2.37	0	(i)
0.01	1	0.01	-10.54	-13.60	-13.96	-2.44	-0.0007	(i)
0.01	2	0.02	-11.45	-22.43	-24.45	-4.51	-0.0564	(ii)
0.05	0.5	0.025	-10.24	-16.71	-16.04	-6.46	-0.0198	(i)
0.05	1	0.05	-10.48	-24.74	-22.20	-11.53	-0.0654	(ii)
0.05	2	0.1	-14.10	-36.12	-39.93	-22.87	-0.2933	(iii)
0.1	0.5	0.05	-9.11	-23.34	-22.75	-12.82	-0.0744	(ii)
0.1	1	0.1	-12.16	-36.44	-36.14	-22.61	-0.1650	(ii)
0.1	2	0.2	-17.86	-53.37	-50.99	-40.87	-0.4490	(iii)

number $\bar{R} = (U_{in}y_{in}/\nu)\sqrt{y_{in}/\delta_{\omega}}$. A linear dependence between the two numbers is postulated. In a practical flow field, the separated shear layer evolves downstream, which translates into increasing values of \bar{R} . As the scaling stems from inviscid linear instability, its predictions are expected to hold up to limited \bar{R} values. Diwan and Ramesh [11] demonstrated that the scaling correctly predicts the KH frequency of several different experiments, up to $\bar{R} = 1400$.

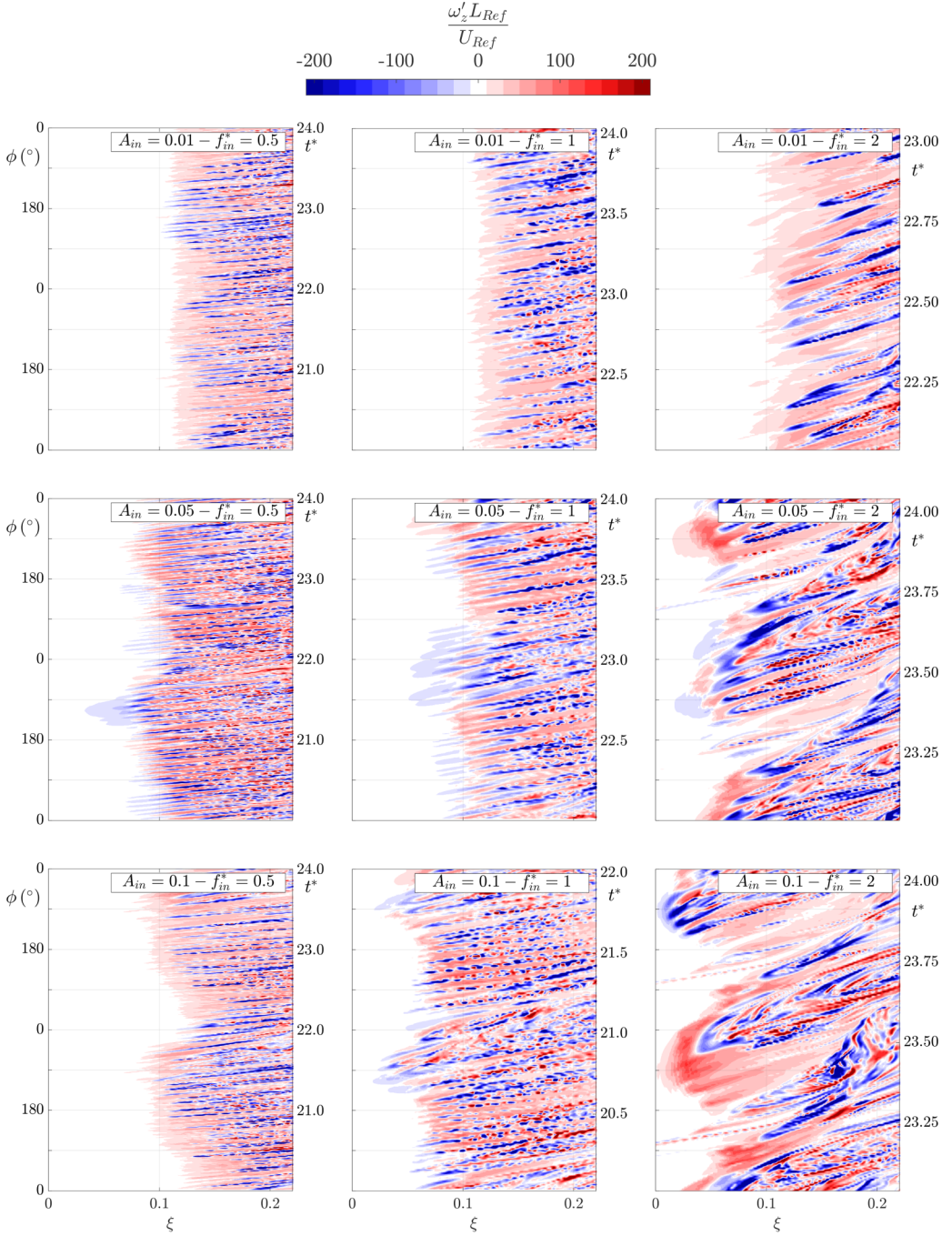
To ascertain that the narrowband frequency peak of $f^* \approx 18$ identified in the PSD for the steady inflow case corresponds to KH instability, the scaling is applied to our simulation data and shown in Fig. 24. The theoretical and experimental results of Diwan and Ramesh [11] are also shown for comparison. This comparison concludes that $f^* = 18$ indeed corresponds to the KH instability.

APPENDIX B: CLASSIFICATION OF THE HARMONIC INFLOW CASES

Table VI shows the correspondence between the simulated cases and the scenario observed for the vortex dynamics, as described in Sec. IV. This classification is based on cross-comparing the time histories of the phase-averaged streamwise acceleration parameter (Fig. 25), the incoherent spanwise vorticity along the phase-averaged shear layer (Fig. 26), the PDFs of the phase-averaged length of the separated flow region (Fig. 17) and the frequency spectra 22. For reference, the evolution of the three-dimensional flow represented as in Fig. 20 is also checked. Animations showing the flow field evolution for representative cases are provided as Supplemental Material [53].

 TABLE VII. Summary of the cases simulated for the NASA hump, including the definition of the inlet pressure condition and the characterization of time-averaged and phase-dependent values of the reference streamwise velocity and the length of the recirculation region L_s .

t_{data}^*	A_{in}	f_{in}^*	$\bar{u}_{@Ref}^*$	$\Delta u_{@Ref}^*$	L_s [m]	ΔL_s [m]
27	—	—	1.0958	—	1.2460	—
16	0.005	0.25	1.0974	0.0339	1.2284	-0.0176
16	0.005	0.5	1.0967	0.0348	1.0012	-0.2448
16	0.025	0.25	1.0953	0.1699	0.8610	-0.3850
12	0.025	0.5	1.0932	0.1744	0.4531	-0.7929
12	0.05	0.5	1.0887	0.3498	0.4079	-0.8381


 FIG. 26. Incoherent spanwise vorticity at $\eta = 0$ for the all nine cases with harmonic inflow.

The table shows, for each simulated case, the minimum values of the acceleration parameter for the corresponding time-averaged flow \bar{K} , the phase-averaged flow $\langle K \rangle$ and its two components. The time-averaged values are considerably lower than the phase-averaged ones and are not useful in the classification of the cases. The minimum of $\langle K \rangle$ decreases independently with the amplitude

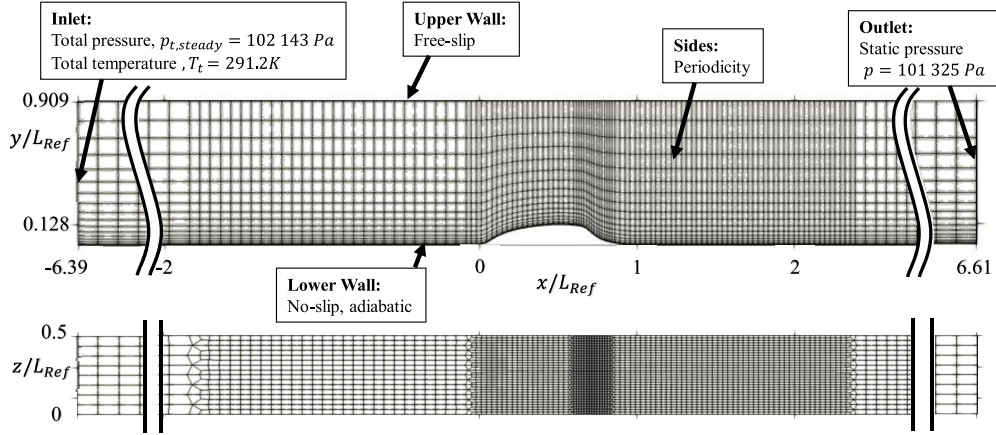


FIG. 27. Computational domain, representative mesh, and boundary conditions for NASA Hump cases.

and frequency of the inlet oscillations. The convective acceleration $\langle K_{\partial u/\partial x} \rangle$ dominates for low A_{in} and/or f_{in}^* values. As a rough approximation, the temporal acceleration $\langle K_{\partial u/\partial t} \rangle \sim A_{in} f_{in}^*$, and this component becomes comparable to the convective acceleration for the largest values of the product $A_{in} f_{in}^*$.

APPENDIX C: SIMULATIONS CONSIDERING THE NASA HUMP

To demonstrate the generality of the physical phenomena with respect to other wall-bounded bump geometries, an analogous study is conducted on a wall-mounted modified Glauert hump model [33,40], often referred to in the literature as the NASA hump. The setup of the simulation is shown in Fig. 27. The domain of the simulation is $(13 \times 0.909 \times 0.5)$ m in the longitudinal,

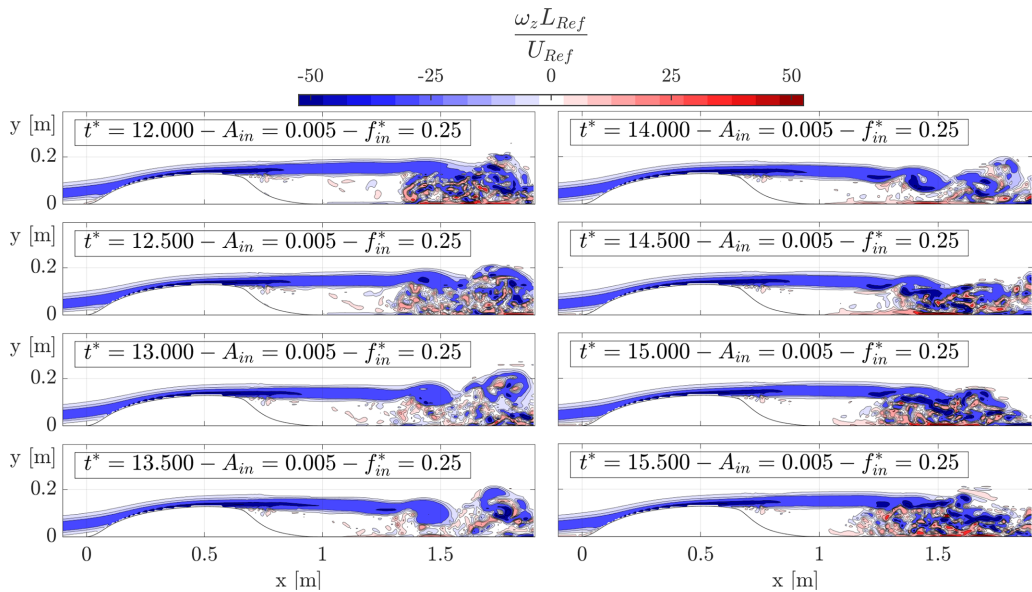


FIG. 28. Instantaneous spanwise vorticity for the NASA hump. $A_{in} = 0.005$ and $f_{in}^* = 0.25$.

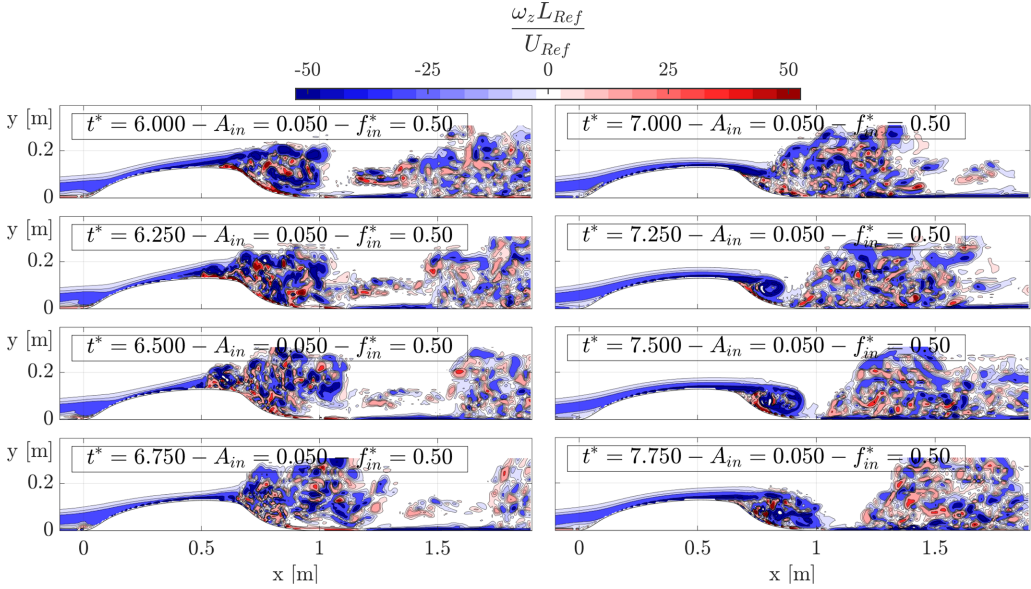


FIG. 29. Instantaneous spanwise vorticity for the NASA hump. $A_{in} = 0.05$ and $f_{in}^* = 0.5$.

normal, and spanwise directions, respectively. A scaling of the hump geometry allows the reference chord length of the model, L_{Ref} to be unity. The ratio of the hump height to the channel height is identical to Greenblatt *et al.* [40], which is 0.1407. The domain is discretized into 77 616 high-order elements and simulated with third-order polynomial (4 967 424 DOFs). The Fringe region starts from $x_{start} = 3$ m with $\Delta_{rise} = 2$ m and $\bar{\lambda} = 400$.

The boundary conditions of the domain are nearly identical to the aforementioned study but a free-slip condition is imposed at the upper boundary. In the harmonic inflow cases, the total pressure variation with time follows Eq. (1), with $p_{t,steady} = 102\,143$ Pa. Simulations are conducted with the inflow air density $\rho = 1.184$ kg/m³, and the dynamic viscosity $\mu = 1.366 \times 10^{-3}$ Pa s. In these conditions, the flow has a characteristic Reynolds number $Re = 30\,000$ and Mach number 0.1 at the inlet for the steady inflow. The Reynolds number based on the maximum height of the recirculation region and respectively the length of the separated region is $Re_h \approx 3840$ and $Re_{L_s} = 37380$. These numbers are comparable with those of the simulations presented in the paper. The Reynolds number based on the boundary layer momentum thickness at the separation point is $Re_{\theta_s} = 171$. Based on the proposed scaling of Hasan [63] and Sigurdson [64], the natural frequency of shedding of the KH vortices is $f_{KH}^* \approx 2.1$, and the vortex cluster shedding frequency is $f_{shed}^* \approx 0.6-0.7$.

Table VII summarizes the cases simulated, showing the harmonic inflow parameters, the mean and oscillation amplitude of the streamwise velocity at a reference point, $(x, y, z) = (0, 0.9, 0.25)$ m, the time-averaged length of the separated flow region and its reduction with respect to the steady inflow. The instantaneous spanwise vorticity for the harmonic inflow cases ($A_{in} = 0.005$, $f_{in}^* = 0.25$) and ($A_{in} = 0.05$, $f_{in}^* = 0.5$) is presented in Figs. 28 and 29, respectively. The first case recovers scenario (i), dominated by KH vortices, while the second one shows the phased-locked formation and release of the vortex cluster, characteristic of scenario (iii). The classification of the cases for the NASA hump geometry is given in Table VIII along with the minimum value of the acceleration parameter components computed at the wall-normal coordinate $y = 2.778h$. This coordinate is the same used for the computation of K in Sec. III.

TABLE VIII. Classification of the cases for the NASA hump geometry. Minimum values of the time-averaged and phase-averaged streamwise acceleration parameter and reduction of the time-averaged length of the separated flow region. The first row corresponds to the steady inflow case.

A_{in}	f_{in}^*	$A_{in}f_{in}^*$	\bar{K}_{min} $\times 10^{-6}$	$\langle K \rangle_{min}$ $\times 10^{-6}$	$\langle K_{\partial u/\partial x} \rangle_{min}$ $\times 10^{-6}$	$\langle K_{\partial u/\partial t} \rangle_{min}$ $\times 10^{-6}$	$\Delta L_s/L_{s,steady}$	Scenario
—	—	—	-6.05	-6.05	-6.05	—	—	(i)
0.005	0.25	0.00125	-6.78	-15.24	-15.28	-0.18	-0.0141	(i)
0.005	0.5	0.0025	-8.51	-19.61	-19.85	-0.27	-0.1965	(ii)
0.025	0.25	0.00625	-9.15	-20.53	-20.40	-0.28	-0.3090	(ii)
0.025	0.5	0.0125	-10.02	-24.48	-25.29	-3.66	-0.6364	(ii)
0.05	0.5	0.025	-16.70	-42.77	-43.74	-8.33	-0.6726	(iii)

- [1] R. E. Mayle, The 1991 IGTI Scholar Lecture: The role of Laminar-Turbulent transition in gas turbine engines, *J. Turbomach.* **113**, 509 (1991).
- [2] J. P. Bons, R. Sondergaard, and R. B. Rivir, The fluid dynamics of LPT blade separation control using pulsed jets, *J. Turbomach.* **124**, 77 (2002).
- [3] J. D. Coull and H. P. Hodson, Unsteady boundary-layer transition in low-pressure turbines, *J. Fluid Mech.* **681**, 370 (2011).
- [4] M. V. Ol, Unsteady Low Reynolds number aerodynamics for micro air vehicles (MAVs), Tech. Rep. AFRL-RB-WP-TR-2010-3013 (Air Force Research Laboratory, Ohio, 2010), <https://apps.dtic.mil/sti/tr/pdf/ADA522060.pdf>.
- [5] D. Greenblatt and I. J. Wagnanski, The control of flow separation by periodic excitation, *Prog. Aerosp. Sci.* **36**, 487 (2000).
- [6] R. J. Volino, O. Kartuzova, and M. B. Ibrahim, Separation control on a very high lift low pressure turbine airfoil using pulsed vortex generator jets, *J. Turbomach.* **133**, 041021 (2011).
- [7] M. Gaster, The structure and behaviour of separation bubbles, Reports and Memoranda 3595 (Queen Mary College, London, 1967), <https://reports.aerade.cranfield.ac.uk/bitstream/handle/1826.2/2862/arc-rm-3595.pdf?sequence=1&isAllowed=y>.
- [8] A. Dovgal, V. Kozlov, and A. Michalke, Laminar boundary layer separation: Instability and associated phenomena, *Prog. Aerosp. Sci.* **30**, 61 (1994).
- [9] B. R. McAuliffe and M. I. Yaras, Transition mechanisms in separation bubbles under low- and elevated-freestream turbulence, *J. Turbomach.* **132**, 011004 (2010).
- [10] E. M. Curtis, H. P. Hodson, M. R. Baniaghbal, J. D. Denton, R. J. Howell, and N. W. Harvey, Development of blade profiles for low-pressure turbine applications, *J. Turbomach.* **119**, 531 (1997).
- [11] S. S. Diwan and O. N. Ramesh, On the origin of the inflectional instability of a laminar separation bubble, *J. Fluid Mech.* **629**, 263 (2009).
- [12] O. Marxen, M. Lang, and U. Rist, Vortex formation and vortex breakup in laminar separation bubbles, *J. Fluid Mech.* **728**, 58 (2013).
- [13] D. A. Hammond and L. G. Redekopp, Local and global instability properties of separation bubbles, *Eur. J. Mech. B Fluids* **17**, 145 (1998).
- [14] U. Rist and U. Maucher, Investigations of time-growing instabilities in laminar separation bubbles, *Eur. J. Mech. B Fluids* **21**, 495 (2002).
- [15] M. P. Avanci, D. Rodríguez, and L. S. B. Alves, A geometrical criterion for absolute instability in separated boundary layers, *Phys. Fluids* **31**, 014103 (2019).
- [16] V. Theofilis, S. Hein, and U. Dallmann, On the origin of unsteadiness and three-dimensionality in a laminar separation bubble, *Philos. Trans. R. Soc. London A* **358**, 3229 (2000).
- [17] F. Gallaire, M. Marquillie, and U. Ehrenstein, Three-dimensional transverse instabilities in detached boundary layers, *J. Fluid Mech.* **571**, 221 (2007).

- [18] D. Rodríguez and V. Theofilis, Structural changes of laminar separation bubbles induced by global linear instability, *J. Fluid Mech.* **655**, 280 (2010).
- [19] D. Rodríguez, E. M. Gennaro, and M. P. Juniper, The two classes of primary modal instability in laminar separation bubbles, *J. Fluid Mech.* **734**, R4 (2013).
- [20] P.-Y. Passaggia, T. Leweke, and U. Ehrenstein, Transverse instability and low-frequency flapping in incompressible separated boundary layer flows: an experimental study, *J. Fluid Mech.* **703**, 363 (2012).
- [21] D. Rodríguez and E. M. Gennaro, Enhancement of disturbance wave amplification due to the intrinsic three-dimensionalisation of laminar separation bubbles, *Aeronaut. J.* **123**, 1492 (2019).
- [22] D. Rodríguez, E. M. Gennaro, and L. F. Souza, Self-excited primary and secondary instability of laminar separation bubbles, *J. Fluid Mech.* **906**, A13 (2021).
- [23] R. G. Jacobs and P. A. Durbin, Simulations of bypass transition, *J. Fluid Mech.* **428**, 185 (2001).
- [24] D. Simoni, D. Lengani, M. Ubaldi, P. Zunino, and M. Dellacasagrande, Inspection of the dynamic properties of laminar separation bubbles: Free-stream turbulence intensity effects for different Reynolds numbers, *Exp. Fluids* **58**, 66 (2017).
- [25] S. Hosseinverdi and H. Fasel, Numerical investigation of laminar-turbulent transition in laminar separation bubbles: The effect of free-stream turbulence, *J. Fluid Mech.* **858**, 714 (2019).
- [26] H. P. Hodson and R. J. Howell, Bladerow interactions, transition and high-lift aerofoils in low-pressure turbine, *Annu. Rev. Fluid Mech.* **37**, 71 (2005).
- [27] R. J. Volino, Effect of unsteady wakes on boundary layer separation on a very high lift low pressure turbine flow, *J. Turbomach.* **134**, 011011 (2012).
- [28] A. G. Gungor, M. P. Simens, and J. Jiménez, Direct numerical simulations of wake-perturbed separated boundary layers, *J. Turbomach.* **134**, 061024 (2012).
- [29] S. Karaca and A. Gungor, DNS of unsteady effects on the control of laminar separated boundary layers, *Eur. J. Mech. B Fluids* **56**, 71 (2016).
- [30] W. Lou and J. Hourmouziadis, Separation bubbles under steady and periodic-unsteady main flow conditions, *J. Turbomach.* **122**, 634 (2000).
- [31] J. G. Wissink and W. Rodi, DNS of a laminar separation bubble in the presence of oscillating flow, *Flow, Turbul. Combust.* **71**, 311 (2003).
- [32] J. G. Wissink, Separating, transitional flow affected by various inflow oscillation regimes, *Appl. Math. Modell.* **30**, 1134 (2006).
- [33] A. Seifert and L. G. Pack, Active flow separation control on wall-mounted hump at high Reynolds numbers, *AIAA J.* **40**, 1363 (2002).
- [34] A. Bernard, J. M. Foucaut, P. Dupont, and M. Stanislas, Decelerating boundary layers, a new scaling and mixing length model, *AIAA J.* **41**, 248 (2003).
- [35] M. Marquillie and U. Ehrenstein, On the onset of nonlinear oscillations in a separating boundary-layer flow, *J. Fluid Mech.* **490**, 169 (2003).
- [36] E. Pescini, F. Marra, M. D. Giorgi, and A. Ficarella, Investigation of the boundary layer characteristics for assessing the DBD plasma actuator control of the separated flow at low Reynolds numbers, *Exp. Therm. Fluid Sci.* **81**, 482 (2017).
- [37] J. Saavedra and G. Paniagua, Transient performance of separated flows: Characterization and active flow control, *J. Eng. Gas Turbines Power* **141**, 011002 (2018).
- [38] J. Saavedra and G. Paniagua, Experimental analysis of reynolds effect on flow detachment and sudden flow release on a wall-mounted hump, *Exp. Therm Fluid Sci.* **126**, 110398 (2021).
- [39] A. K. M. F. Hussain and W. C. Reynolds, The mechanics of an organized wave in turbulent shear flow, *J. Fluid Mech.* **41**, 241 (1970).
- [40] D. Greenblatt, K. B. Paschal, C.-S.-Yao, J. Harris, N. W. Schaeffer, and A. E. Washburn, Experimental investigation of separation control Part 1: Baseline and steady suction, *AIAA J.* **44**, 2820 (2006).
- [41] D. A. Kopriva, HOHQMesh, the high order hex-quad mesher, <https://trixi-framework.github.io/HOHQMesh/> (2010).
- [42] P. Spalart, Direct numerical study of leading-edge contamination, in *Fluid Dynamics of 3D Turbulent Shear Flows and Transition*, AGARD Conf. Proc. No. 438 (AGARD, Neuilly-sur-Seine, 1989), pp. 5.1–5.13.

- [43] P. R. Spalart and J. H. Watmuff, Experimental and numerical study of a turbulent boundary layer with pressure gradients, *J. Fluid Mech.* **249**, 337 (1993).
- [44] J. Nordström, N. Nordin, and D. Henningson, The fringe region technique and the Fourier method used in the direct numerical simulation of spatially evolving viscous flows, *SIAM J. Sci. Comput.* **20**, 1365 (1999).
- [45] E. Ferrer, G. Rubio, G. Ntoukas, W. Laskowski, O. Mariño, S. Colombo, A. Mateo-Gabín, H. Marbona, F. Manrique de Lara, D. Huergo *et al.*, HORSES3D: A high-order discontinuous Galerkin solver for flow simulations and multi-physics applications, *Comput. Phys. Commun.* **287**, 108700 (2023).
- [46] P. D. Welch, The use of Fast Fourier Transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms, *IEEE Trans. Audio Electroacoust.* **15**, 70 (1967).
- [47] M. Embacher and H. F. Fasel, Direct numerical simulations of laminar separation bubbles: Investigation of absolute instability and active flow control of transition to turbulence, *J. Fluid Mech.* **747**, 141 (2014).
- [48] J. W. Kurelek, M. Kotsonis, and S. Yarusevych, Vortex merging in a laminar separation bubble under natural and forced conditions, *Phys. Rev. Fluids* **4**, 063903 (2019).
- [49] M. Dellacasagrande, D. Basi, D. Lengani, D. Simoni, and J. Verdoya, Response of a flat plate laminar separation bubble to Reynolds number, free-stream turbulence and adverse pressure gradient variation, *Exp. Fluids* **61**, 128 (2020).
- [50] S. S. Diwan, S. J. Chetan, and O. N. Ramesh, On the bursting criterion for laminar separation bubbles, in *IUTAM Symposium on Laminar-Turbulent Transition*, edited by R. Govindarajan (Springer Netherlands, Dordrecht, 2006), pp. 401–407.
- [51] S. M. Aniffa, V. Caesar, V. Dabaria, and A. Mandal, Characteristics of geometry- and pressure-induced laminar separation bubbles at an enhanced level of free-stream turbulence, *J. Fluid Mech.* **957**, A19 (2023).
- [52] J. Jeong and F. Hussain, On the identification of a vortex, *J. Fluid Mech.* **285**, 69 (1995).
- [53] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevFluids.9.053901> for movies depicting the $Q(+)$ isosurface for cases involving scenarios 1, 2, and 3.
- [54] O. Marxen, M. Lang, U. Rist, O. Levin, and D. Henningson, Mechanisms for spatial steady three-dimensional disturbance growth in a non-parallel and separating boundary layer, *J. Fluid Mech.* **634**, 165 (2009).
- [55] A. J. Bowen and D. Lindley, A wind-tunnel investigation of the wind speed and turbulence characteristics close to the ground over various escarpment shapes, *Boundary Layer Meteorol.* **12**, 259 (1977).
- [56] P. R. Spalart, Numerical simulation of boundary layers: Part 3. Turbulence and relaminarization in sink flows, NASA Technical Memorandum 88220, NASA Ames Research Center, California, 1986.
- [57] Y. B. Suzen, P. Huang, L. S. Hultgren, and D. E. Ashpis, Predictions of separated and transitional boundary layers under low-pressure turbine airfoil conditions using an intermittency transport equation, *J. Turbomach.* **125**, 455 (2003).
- [58] F. Ambrogi, U. Piomelli, and D. Rival, Characterization of unsteady separation in a turbulent boundary layer: Mean and phase-averaged flow, *J. Fluid Mech.* **945**, A10 (2022).
- [59] M. J. Lighthill, Introduction. Boundary layer theory, in *Laminar Boundary Layers*, edited by L. Rosenhead (Clarendon Press, Oxford, 1963), Chap. II, pp. 46–113.
- [60] C. Legleiter and P. Kyriakidis, Forward and inverse transformations between cartesian and channel-fitted coordinate systems for meandering rivers, *Math. Geol.* **38**, 927 (2006).
- [61] F. H. Himeno, D. S. Souza, F. R. Amaral, D. Rodríguez, and M. Medeiros, SPOD analysis of noise-generating Rossiter modes in a slat with and without a bulb seal, *J. Fluid Mech.* **915**, A67 (2021).
- [62] A. H. Lind and A. R. Jones, Unsteady aerodynamics of reverse flow dynamic stall on an oscillating blade section, *Phys. Fluids* **28**, 077102 (2016).
- [63] M. A. Z. Hasan, The flow over a backward-facing step under controlled perturbation: Laminar separation, *J. Fluid Mech.* **238**, 73 (1992).
- [64] L. W. Sigurdson, The structure and control of a turbulent reattaching flow, *J. Fluid Mech.* **298**, 139 (1995).
- [65] www.upm.es.