

Lagrangian acceleration and its Eulerian decompositions in fully developed turbulence

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We study the properties of various Eulerian contributions to fluid particle acceleration by using well-resolved direct numerical simulations of isotropic turbulence, with the Taylor-scale Reynolds number R_λ in the range 140–1300. The variance of convective acceleration, when normalized by Kolmogorov scales, increases as R_λ , consistent with simple theoretical arguments, but differing from classical Kolmogorov’s phenomenology, as well as Lagrangian extension of Eulerian multifractal models. The scaling of the local acceleration is also linear in R_λ to the leading order, but more complex in detail. The strong cancellation between the local and convective acceleration—faithful to the random sweeping hypothesis—results in the variance of the Lagrangian acceleration increasing only as $R_\lambda^{0.25}$, as recently shown by Buaria and Sreenivasan [*Phys. Rev. Lett.* **128**, 234502 (2022)]. The acceleration variance is dominated by the irrotational pressure gradient contribution, whose variance essentially follows the $R_\lambda^{0.25}$ scaling; the solenoidal viscous contributions are comparatively small and follow $R_\lambda^{0.13}$, which is the only acceleration component consistent with multifractal prediction.

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I. INTRODUCTION

In classical mechanics, the dynamics of particle motion is characterized by the acceleration \mathbf{a} , defined by the rate of change of the particle velocity \mathbf{u} in a Lagrangian frame. Given its fundamental role, the statistics of acceleration are of obvious interest in the study of turbulent fluid flows [1–5] and also for stochastic modeling of associated transport phenomena [6–9]. Of particular interest is the scaling of acceleration variance $\langle |\mathbf{a}|^2 \rangle$ which, according to Kolmogorov’s 1941 mean-field phenomenology [10] (henceforth K41), can be written as $\langle |\mathbf{a}|^2 \rangle = a_0 \langle \epsilon \rangle^{3/2} \nu^{-1/2}$ [11], where $\langle \epsilon \rangle$ is the mean dissipation rate, ν is the kinematic viscosity, and a_0 is a universal constant. However, it is widely known that, owing to small-scale intermittency, a_0 is instead a variable that depends on the Reynolds number. Following a few decades of investigations [12–20], recent data from direct numerical simulations (DNS) of isotropic turbulence at high Reynolds numbers have established that $a_0 \sim R_\lambda^\chi$ with $\chi \approx 0.25$ [21], where R_λ is the Taylor-scale Reynolds number. This result is obviously at odds with K41 but, as demonstrated in [21], it is also at odds with the prediction

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$\chi \approx 0.135$ from Lagrangian extensions of Eulerian multifractal models [22,23]. Our goal here is to further analyze the scaling of acceleration variance by considering the underlying contributing parts to the acceleration (as described next).

While particle acceleration is inherently Lagrangian, an Eulerian viewpoint is often more convenient to study fluid flows, whereby the Lagrangian or material derivative is given as

$$\mathbf{a} = D\mathbf{u}/Dt = \partial\mathbf{u}/\partial t + \mathbf{u} \cdot \nabla\mathbf{u}, \quad (1)$$

where

$$\mathbf{a}_L \equiv \partial\mathbf{u}/\partial t, \quad \mathbf{a}_C \equiv \mathbf{u} \cdot \nabla\mathbf{u} \quad (2)$$

are, respectively, the local and convective components, such that \mathbf{a}_L captures the unsteady rate of change at a fixed spatial position and \mathbf{a}_C captures the rate of change due to spatial variations. Evidently, $\mathbf{a} = \mathbf{a}_L + \mathbf{a}_C$ (note that the subscripts L and C denote local and convective parts, respectively) [24,25]. In addition, the dynamics of fluid motion in incompressible turbulent flows is governed by the Navier-Stokes equations

$$\mathbf{a} = -\nabla P + \nu\nabla^2\mathbf{u}, \quad (3)$$

where P is the kinematic pressure. Since $\nabla \cdot \mathbf{u} = 0$ from incompressibility, the viscous term is solenoidal as well, whereas the pressure gradient term is irrotational, i.e., its curl is zero. Thus the acceleration can also be written as $\mathbf{a} = \mathbf{a}_I + \mathbf{a}_S$ (with subscripts I and S representing irrotational and solenoidal parts, respectively) [13,24], with

$$\mathbf{a}_I \equiv -\nabla P, \quad \mathbf{a}_S \equiv \nu\nabla^2\mathbf{u}. \quad (4)$$

In this Letter, we shall consider both methods of decomposition and study how the Eulerian components contribute to the observed scaling of acceleration variance. It is worth noting that intermittency theories such as multifractals do not inherently differentiate between various components and naively predict the same scaling result for all of them, i.e., their variance when scaled with Kolmogorov variables would scale as R_λ^χ , with $\chi \approx 0.135$. However, we consider each component individually and demonstrate that this is not the case, partly because the components are nontrivially correlated. Utilizing data from the state-of-the-art DNS of isotropic turbulence, we show that the variance of convective acceleration varies as R_λ , which follows from very simple theoretical arguments, but differs from multifractal predictions. The variance of local component also varies as R_λ to the leading order (with weaker second order dependencies), while always remaining slightly smaller than \mathbf{a}_C^2 . The Lagrangian acceleration results from strong cancellation between these two large quantities, varying as $R_\lambda^{0.25}$. We additionally explore how the properties \mathbf{a}_I and \mathbf{a}_S relate to local and convective accelerations.

II. NUMERICAL APPROACH AND DATABASE

The DNS data utilized here are obtained by solving the incompressible Navier-Stokes equations, corresponding to the canonical setup of forced stationary isotropic turbulence in a periodic domain [26,27]. Highly accurate Fourier pseudospectral methods are utilized for spatial calculations, with aliasing errors controlled using a combination of grid shifting and truncation [28]. An explicit second-order Runge-Kutta scheme is used for time integration. The database for the present work is the same as that of our recent study on acceleration [21] and several other recent works [29–34]. The grid resolution is as high as 12288^3 and the Taylor-scale Reynolds number R_λ lies in the range 140–1300. Convergence with respect to small-scale resolution and statistical sampling has been assessed in these previous studies.

As in [21], we have also calculated the relevant statistics using Lagrangian fluid particle trajectories in the same range of R_λ , albeit with lower small-scale resolution [35–37]. At the level of second order moments reported in this work, the statistics are essentially identical from both Eulerian and Lagrangian data. However, the Lagrangian particle data are not suitable for studying

higher order moments, due both to the lack of resolution and accumulated numerical errors resulting from interpolation of particle velocities [12].

III. RESULTS

A. Theoretical analysis

Before analyzing the DNS data, we present a brief theoretical analysis of various Eulerian components of acceleration. It is straightforward to prove that, in homogeneous turbulence, the correlation between an irrotational and a solenoidal vector is always zero (see the Appendix). From this property, it follows that

$$\langle \mathbf{a}_I \cdot \mathbf{a}_S \rangle = 0, \quad (5)$$

$$\langle \mathbf{a}_I \cdot \mathbf{a}_L \rangle = 0. \quad (6)$$

Additionally, using statistical stationarity, we can show (see the Appendix) that

$$\langle \mathbf{a}_S \cdot \mathbf{a}_L \rangle = 0. \quad (7)$$

Since $\mathbf{a} = \mathbf{a}_I + \mathbf{a}_S$, it also follows from Eqs. (6) and (7) that

$$\langle \mathbf{a} \cdot \mathbf{a}_L \rangle = 0, \quad (8)$$

i.e., the Lagrangian acceleration $D\mathbf{u}/Dt$ is uncorrelated to the Eulerian acceleration $\partial u/\partial t$. This property directly yields the following result:

$$\langle \mathbf{a}_L \cdot \mathbf{a}_C \rangle = -\langle |\mathbf{a}_L|^2 \rangle. \quad (9)$$

These relations allow us to write the acceleration variance as

$$\langle |\mathbf{a}|^2 \rangle = \langle |\mathbf{a}_I|^2 \rangle + \langle |\mathbf{a}_S|^2 \rangle, \quad (10)$$

$$\langle |\mathbf{a}|^2 \rangle = \langle |\mathbf{a}_C|^2 \rangle - \langle |\mathbf{a}_L|^2 \rangle. \quad (11)$$

Thus, while the acceleration variance is given by the sum of variances of the pressure gradient and viscous terms, it is also obtained via a direct cancellation of convective and local components. We will now explore how the scaling of all these Eulerian contributions affect the scaling of acceleration variance itself.

B. Properties of \mathbf{a}_I and \mathbf{a}_S

It is well known that acceleration variance is dominated by the irrotational pressure gradient contribution and the corresponding viscous contribution is negligible, i.e., $|\mathbf{a}_I| \gg |\mathbf{a}_S|$ [13,24]. We first reaffirm this result in Fig. 1(a), which shows the fractional contributions of \mathbf{a}_I and \mathbf{a}_S , and also the correlation $\langle \mathbf{a}_I \cdot \mathbf{a}_S \rangle$, which is zero as expected. It is evident that $\langle |\mathbf{a}|^2 \rangle \approx \langle |\mathbf{a}_I|^2 \rangle$. We can readily show that

$$\langle \mathbf{a} \cdot \mathbf{a}_I \rangle = \langle \mathbf{a}_C \cdot \mathbf{a}_I \rangle = \langle |\mathbf{a}_I|^2 \rangle, \quad \langle \mathbf{a} \cdot \mathbf{a}_S \rangle = \langle \mathbf{a}_C \cdot \mathbf{a}_S \rangle = \langle |\mathbf{a}_S|^2 \rangle, \quad (12)$$

which demonstrate that both the pressure gradient and viscous contributions predominantly arise from the convective component. This is not surprising since $\langle \mathbf{a}_I \cdot \mathbf{a}_L \rangle$ and $\langle \mathbf{a}_S \cdot \mathbf{a}_L \rangle$ are both zero [from Eqs. (6) and (7)]. We shall elaborate on this point in the next subsection.

It is worth noting that, while the contribution from the viscous term \mathbf{a}_S is negligible in comparison to \mathbf{a}_I , it is nevertheless finite and intimately connected to the fundamental dynamics of turbulence. In particular, its variance can be written as [11,13]

$$\langle |\mathbf{a}_S|^2 \rangle / a_K^2 = -\frac{35}{2} \frac{\mathcal{S}}{(15)^{3/2}}, \quad (13)$$

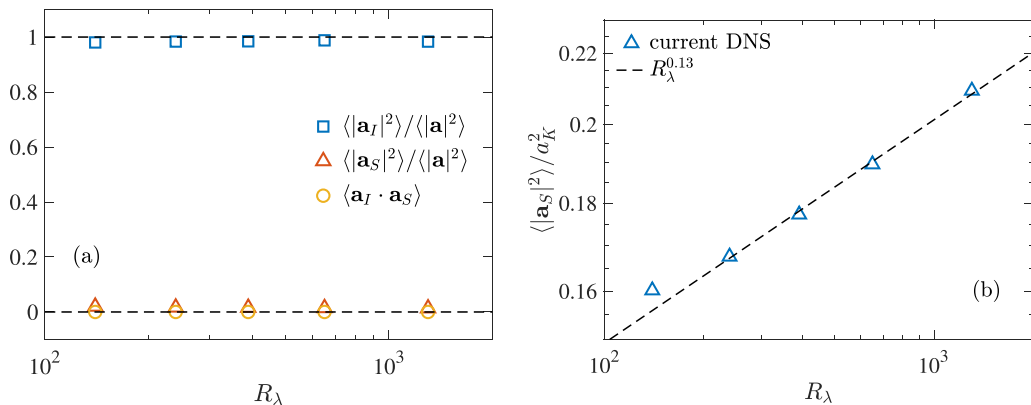


FIG. 1. (a) Fractional contributions of the irrotational pressure gradient and solenoidal viscous terms to the acceleration variance, as well as their mutual correlation, as functions of R_λ . (b) Variance of the solenoidal viscous acceleration normalized by the Kolmogorov scales, as a function of R_λ . For all data points, the statistical error bars are smaller than the marker size.

where $a_K = \langle \epsilon \rangle^{3/4} \nu^{-1/4}$ and \mathcal{S} is the skewness of longitudinal velocity gradients, which is always negative in turbulence, characterizing the energy cascade from large to small scales [38,39]. The skewness can also be related to vortex stretching [40] and is known to weakly increase in magnitude as $R_\lambda^{0.13}$ [41]. This scaling matches the prediction from extending Eulerian multifractals to Lagrangian variables [22,39,42]. Figure 1(b) shows a satisfactory agreement with this result (except at the lowest Reynolds number).

However, our recent work [21] computed the acceleration variance $\langle |\mathbf{a}_I|^2 \rangle$ and found it to vary as $R_\lambda^{0.25}$. We expressed the acceleration analytically in terms of the fourth order velocity structure functions [43,44] and showed that

$$\langle |\mathbf{a}|^2 \rangle / a_K^2 \approx \langle |\mathbf{a}_I|^2 \rangle / a_K^2 \sim R_\lambda^{0.25}. \quad (14)$$

The data from various sources, including our own DNS, show excellent agreement with this prediction (see [21]; we also reaffirm it below). It then follows that an extension of Eulerian multifractals to explain intermittency of Lagrangian quantities is fraught with uncertainties.

C. Properties of \mathbf{a}_L and \mathbf{a}_C

It follows from Eq. (11) that acceleration variance results from direct cancellation between the variances of \mathbf{a}_C and \mathbf{a}_L . This cancellation is consistent with the random sweeping hypothesis [45,46], which states that the small scales of turbulence are swept past an Eulerian observer on a much shorter time scale than the time scale governing their dynamical evolution. The nominal validity of this hypothesis is also implicitly reflected in the result that $\mathbf{a} = D\mathbf{u}/Dt$ and $\mathbf{a}_L = \partial\mathbf{u}/\partial t$ are uncorrelated [see Eq. (8)].

The convective acceleration $\mathbf{a}_C = \mathbf{u} \cdot \nabla \mathbf{u}$ essentially represents a correlation between the velocity and its gradients. Given the general understanding that the former characterizes the large scales and the latter the small scales, we can assume that the two are essentially uncorrelated (provided R_λ is sufficiently high). Thus simple scaling arguments suggest that $|\mathbf{a}_C| \sim u'/\tau_K$, where u' is the root-mean-square (rms) velocity and τ_K is the Kolmogorov time scale, characterizing the rms of velocity gradients. We then have

$$\langle |\mathbf{a}_C|^2 \rangle / a_K^2 = c R_\lambda, \quad (15)$$

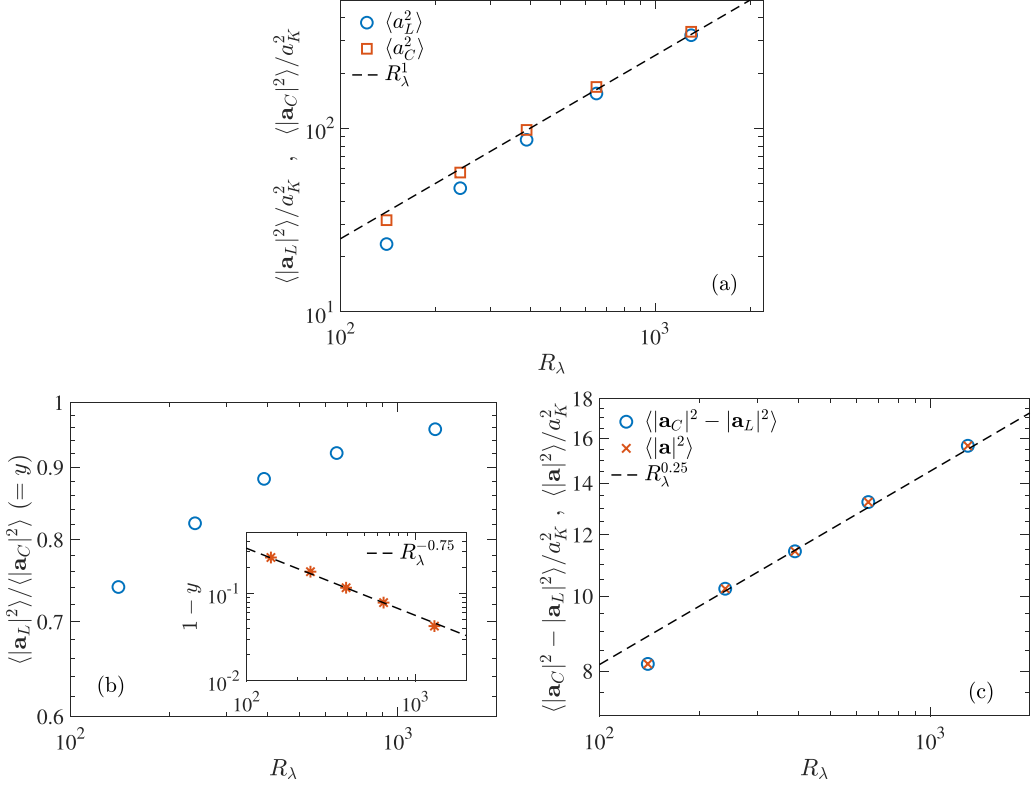


FIG. 2. (a) Variances of local and convective acceleration, normalized by Kolmogorov scales, as a function of R_λ . (b) Ratio of variance of local and convective acceleration; inset shows deficit of the ratio from unity. (c) Difference between the convective and local acceleration as a function of R_λ , compared directly with acceleration variance. For all data points, the statistical error bars are smaller than the marker size.

where c is some proportionality constant and we have utilized the classical estimate $u'/u_K \sim R_\lambda^{1/2}$ [39] (u_K being the Kolmogorov velocity scale). To the first order, it can be also expected that $\langle |\mathbf{a}_L|^2 \rangle$ also follows a similar scaling. This can be inferred indirectly by noting that

$$\langle |\mathbf{a}_L|^2 \rangle = \langle |\mathbf{a}_C|^2 \rangle - \langle |\mathbf{a}|^2 \rangle \quad (16)$$

$$= \langle |\mathbf{a}_C|^2 \rangle - \langle |\mathbf{a}_I|^2 \rangle - \langle |\mathbf{a}_S|^2 \rangle, \quad (17)$$

where observations show that the scaling of $\langle |\mathbf{a}_C|^2 \rangle$ dominates over the other two components.

Figure 2(a) shows the variances of local and convective acceleration. It can be immediately seen that $\langle |\mathbf{a}_C|^2 \rangle / a_K^2$ follows a simple linear scaling in R_λ , as anticipated in Eq. (15). On the other hand, $\langle |\mathbf{a}_L|^2 \rangle / a_K^2$ approaches this scaling as R_λ increases, but noticeably deviates at low R_λ . Using the results in Eqs. (13)–(17), the precise scaling of $\langle |\mathbf{a}_L|^2 \rangle$ can be quantified as

$$\langle |\mathbf{a}_L|^2 \rangle / a_K^2 = cR_\lambda - c_1R_\lambda^{0.25} - c_2R_\lambda^{0.13}, \quad (18)$$

where c_1 and c_2 are proportionality constants. Evidently, the deviations at lower R_λ can be understood in terms of these additional terms. It also follows from here that the ratio $\langle |\mathbf{a}_L|^2 \rangle / \langle |\mathbf{a}_C|^2 \rangle$ has the form $1 - (c_1/c)R_\lambda^{-0.75} - (c_2/c)R_\lambda^{-0.87}$. Asymptotically, when normalized by Kolmogorov variable, $\langle |\mathbf{a}|^2 \rangle = \langle |\mathbf{a}_C|^2 \rangle - \langle |\mathbf{a}_L|^2 \rangle \approx c_1R_\lambda^{-0.25}$.

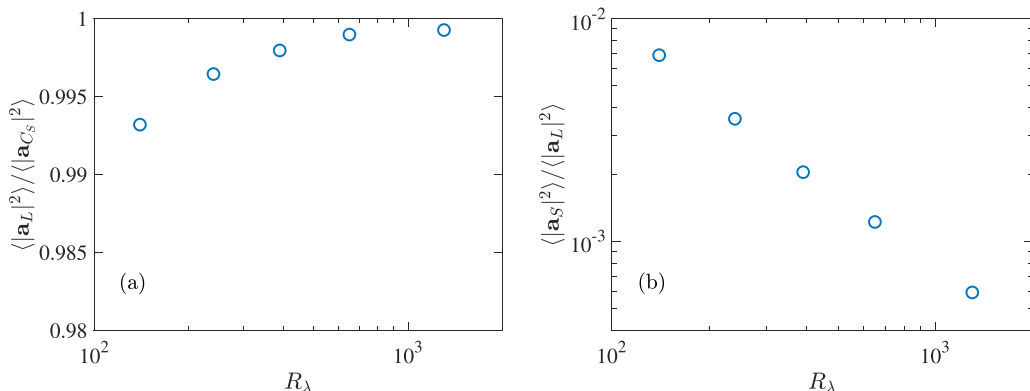


FIG. 3. (a) Ratio of variance of local acceleration to that of solenoidal component of convective acceleration, as a function of R_λ . (b) Ratio of variance of solenoidal viscous acceleration to that of local acceleration. For all data points, the statistical error bars are smaller than the marker size.

To verify the behaviors of \mathbf{a}_L and \mathbf{a}_C we plot in Fig. 2(b) the ratio $y = \langle |\mathbf{a}_L|^2 \rangle / \langle |\mathbf{a}_C|^2 \rangle$ as a function of R_λ . The inset shows $1 - y$, which is in excellent agreement with a power law $R_\lambda^{-0.75}$. The ratio steadily approaches unity, which demonstrates that, asymptotically, only the $R_\lambda^{0.25}$ term contributes to Lagrangian acceleration. This is also confirmed by Fig. 2(c), which shows $\langle |\mathbf{a}_C|^2 \rangle - \langle |\mathbf{a}_L|^2 \rangle$ normalized by Kolmogorov scales with the acceleration variance data from [21]—both sets of points are indistinguishable and in excellent agreement with $R_\lambda^{0.25}$ scaling.

D. Further analysis of the role of \mathbf{a}_C

The near cancellation between \mathbf{a}_L and \mathbf{a}_C can be further analyzed by noticing that \mathbf{a}_L is solenoidal, whereas \mathbf{a}_C is not; thus they can never completely cancel each other. Further, since \mathbf{a}_I is irrotational and \mathbf{a}_S is solenoidal, we can write [24]

$$\mathbf{a}_{C_I} = \mathbf{a}_I, \quad (19)$$

$$\mathbf{a}_L + \mathbf{a}_{C_S} = \mathbf{a}_S, \quad (20)$$

where we have decomposed \mathbf{a}_C into irrotational and solenoidal components, i.e., $\mathbf{a}_C = \mathbf{a}_{C_I} + \mathbf{a}_{C_S}$. Such a decomposition can readily be implemented in Fourier space using the Helmholtz decomposition, i.e., for a vector \mathbf{V} with Fourier coefficient $\hat{\mathbf{V}}$, the Fourier coefficients of irrotational and solenoidal parts are, respectively, given as

$$\hat{\mathbf{V}}_I(\mathbf{k}) = (\mathbf{k} \cdot \hat{\mathbf{V}})\mathbf{k}/k^2, \quad \hat{\mathbf{V}}_S(\mathbf{k}) = \hat{\mathbf{V}} - \hat{\mathbf{V}}_I, \quad (21)$$

where \mathbf{k} is the wave vector and $k = |\mathbf{k}|$. Note that irrotationality is imposed in Fourier space by the condition $\mathbf{k} \times \hat{\mathbf{V}}_I = \mathbf{0}$ and solenoidality by $\mathbf{k} \cdot \mathbf{V}_S = 0$ (both of which can be easily verified).

From the above decomposition, it trivially follows that $|\mathbf{a}_{C_I}|^2 = |\mathbf{a}_I|^2$ and $\langle |\mathbf{a}_C|^2 \rangle = \langle |\mathbf{a}_{C_I}|^2 \rangle + \langle |\mathbf{a}_{C_S}|^2 \rangle$ and we can also show that

$$\langle \mathbf{a}_L \cdot \mathbf{a}_{C_S} \rangle = -\langle |\mathbf{a}_L|^2 \rangle, \quad (22)$$

$$\langle |\mathbf{a}_S|^2 \rangle = \langle |\mathbf{a}_{C_S}|^2 \rangle - \langle |\mathbf{a}_L|^2 \rangle. \quad (23)$$

Thus the very small solenoidal component \mathbf{a}_S results from near perfect cancellation between \mathbf{a}_{C_S} and \mathbf{a}_L . We quantify this in Fig. 3. Panel (a) shows that the ratio $\langle |\mathbf{a}_L|^2 \rangle / \langle |\mathbf{a}_{C_S}|^2 \rangle$ steadily approaches unity as R_λ increases (though it cannot be strictly unity since $|\mathbf{a}_S|$ is always finite). In panel (b), the ratio $\langle |\mathbf{a}_S|^2 \rangle / \langle |\mathbf{a}_L|^2 \rangle$ is shown, which steadily decreases with R_λ , as expected.

In summary, based on the present analysis, we can essentially write $|\mathbf{a}_C| \gtrsim |\mathbf{a}_{C_S}| \gtrsim |\mathbf{a}_L| \gg |\mathbf{a}| \approx |\mathbf{a}_I| = |\mathbf{a}_C| \gg |\mathbf{a}_S|$. Perhaps surprisingly, $\langle |\mathbf{a}|^2 \rangle \approx \langle |\mathbf{a}_I|^2 \rangle \sim R_\lambda^{0.25}$, while variances of the components \mathbf{a}_C and \mathbf{a}_L have far stronger dependencies on R_λ ; for example, $\langle |\mathbf{a}_C|^2 \rangle$ essentially scales as R_λ .

IV. CONCLUSIONS

The most interesting result of the analysis is that the local and convection terms of acceleration are anticorrelated and both of them depart from both K41 and also from the multifractal predictions. In particular, the variance of both essentially scale linearly with R_λ . The two terms are, however, strongly anticorrelated. Thus the difference between the two, which specifies the Lagrangian acceleration, scales as $R_\lambda^{0.25}$ [21]. This result, which is an indication that the two terms are separately much more intermittent than their algebraic (vector) sum, is also at odds with Kolmogorov's and Lagrangian extensions of Eulerian multifractals, but not nearly as much as their sum. The scaling $\langle |\mathbf{a}_C|^2 \rangle \sim R_\lambda$ comes from the assumption that \mathbf{u} and $\nabla \mathbf{u}$ are uncorrelated, so essentially $\langle |\mathbf{a}_C|^2 \rangle$ follows the same scaling as $\langle |\mathbf{u}|^2 \rangle$. The interpretation is that the small scales are simply swept by the large scale velocity without getting affected, which is consistent with the random sweeping hypothesis. The only part that is consistent with multifractals is the solenoidal viscous part, but its overall contribution to acceleration is essentially negligible. Overall, our results demonstrate that the behavior of Lagrangian acceleration and its Eulerian components is unlikely to be captured by phenomenological approaches, such as multifractals, due to underlying nontrivial correlations—highlighting the need for alternative models that also take into account the Navier-Stokes dynamics.

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APPENDIX: VANISHING CORRELATIONS BETWEEN VARIOUS EULERIAN CONTRIBUTIONS TO ACCELERATION

Let us assume that vector \mathbf{A} is irrotational and vector \mathbf{B} is solenoidal; this implies $\nabla \times \mathbf{A} = \mathbf{0}$ and $\nabla \cdot \mathbf{B} = 0$ (or $\partial B_i / \partial x_i = 0$). For the former, we can write $A_i = \partial \phi / \partial x_i$, where ϕ is some scalar quantity. Thus the correlation can be simplified as

$$\langle \mathbf{A} \cdot \mathbf{B} \rangle = \left\langle \frac{\partial \phi}{\partial x_i} B_i \right\rangle \quad (\text{A1})$$

$$= \left\langle \frac{\partial(\phi B_i)}{\partial x_i} - \phi \frac{\partial B_i}{\partial x_i} \right\rangle \quad (\text{A2})$$

$$= \frac{\partial \langle \phi B_i \rangle}{\partial x_i} - \left\langle \phi \frac{\partial B_i}{\partial x_i} \right\rangle \quad (\text{A3})$$

$$= 0, \quad (\text{A4})$$

where the first term is zero from statistical homogeneity and the second term is zero since $\partial B_i / \partial x_i$.

Thus, for the components of acceleration, we can write

$$\langle \mathbf{a}_I \cdot \mathbf{a}_S \rangle = 0, \quad (\text{A5})$$

$$\langle \mathbf{a}_I \cdot \mathbf{a}_L \rangle = 0. \quad (\text{A6})$$

For the correlation between \mathbf{a}_S and \mathbf{a}_L , the following steps have to be considered:

$$\langle \mathbf{a}_S \cdot \mathbf{a}_L \rangle = \left\langle v \frac{\partial^2 u_i}{\partial x_k \partial x_k} \cdot \frac{\partial u_i}{\partial t} \right\rangle \quad (\text{A7})$$

$$= v \left\langle \frac{\partial}{\partial x_k} \left(\frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial t} \right) - \frac{\partial u_i}{\partial x_k} \cdot \frac{\partial}{\partial t} \left(\frac{\partial u_i}{\partial x_k} \right) \right\rangle \quad (\text{A8})$$

$$= v \left\langle \frac{\partial}{\partial x_k} \left(\frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial t} \right) \right\rangle - \frac{v}{2} \left\langle \frac{\partial}{\partial t} \left(\frac{\partial u_i}{\partial x_k} \right)^2 \right\rangle \quad (\text{A9})$$

$$= 0. \quad (\text{A10})$$

The last step follows from the fact that the first term is zero from statistical homogeneity, whereas the second term is zero from statistical stationarity. Since $\mathbf{a} = \mathbf{a}_I + \mathbf{a}_S$, it also follows that

$$\langle \mathbf{a} \cdot \mathbf{a}_L \rangle = 0. \quad (\text{A11})$$

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