Gallery of Fluid Motion

Mandelbrot granular raft

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In this paper we explain how one can generate a fractal granular raft from toroidal vortices using a spinning top, a tank of glycerol and a curing elastomer. As the spinning top rotates in the glycerol tank, a primarily rotational flow is established. The spinning top is 50 mm in diameter and is commercially available as the “Helene spinning top.” The transparent tank is a 10.2 cm $\times$ 10.2 cm $\times$ 10.2 cm cube made of acrylic. For the sake of flow visualization, we add a small volume of dyed fluid elements that follow the primary flow and spin in the enclosed glycerol tank. However, as shown in Fig. 1, the colored trace of the added liquid also reveals the presence of secondary flows in the form of toroidal vortices that roll around the spinning top. As shown by Taylor [1], the centripetal acceleration of fluid particles in the primary flow is the driving mechanism behind this instability that leads to the emergence of “ever-spinning” [2] toroidal vortices and a three dimensional velocity-field. Secondary flows emerge at a spinning rate of 40 rpm that corresponds to a Taylor number of $\mathrm{Ta} \equiv 4R^4\omega^2/\nu^2 \simeq 1500$. The size of the toroidal vortices is set by the geometrical constraints of the tank. Laser visualization of the flow shows that the centripetally driven instability leads to secondary flows with axial velocities that are comparable to the tangential velocity of the primary flow.

Instead of the miscible dyed glycerol, we can also add a small volume of an immiscible liquid such as a curing elastomer. To this end, we use a two-part vinylpolysiloxane (VPS) solution as the curing elastomer. After mixing, the Si-H groups in the cross linker react with the vinyl groups in the base. Gelation is achieved after 10 minutes and the elastomer is fully cured after 20 minutes. The elastomer has a density of 1.1 $\mathrm{g.cm^{-3}}$ and is therefore slightly lighter than the background liquid glycerol. In the liquid state, the elastomer has a nonzero interfacial tension with glycerol that is around 38 mN.m. As shown in Fig. 2(a), due to the nonzero interfacial tension between glycerol and elastomer, the added volume of liquid elastomer forms elongated filaments that finally break up

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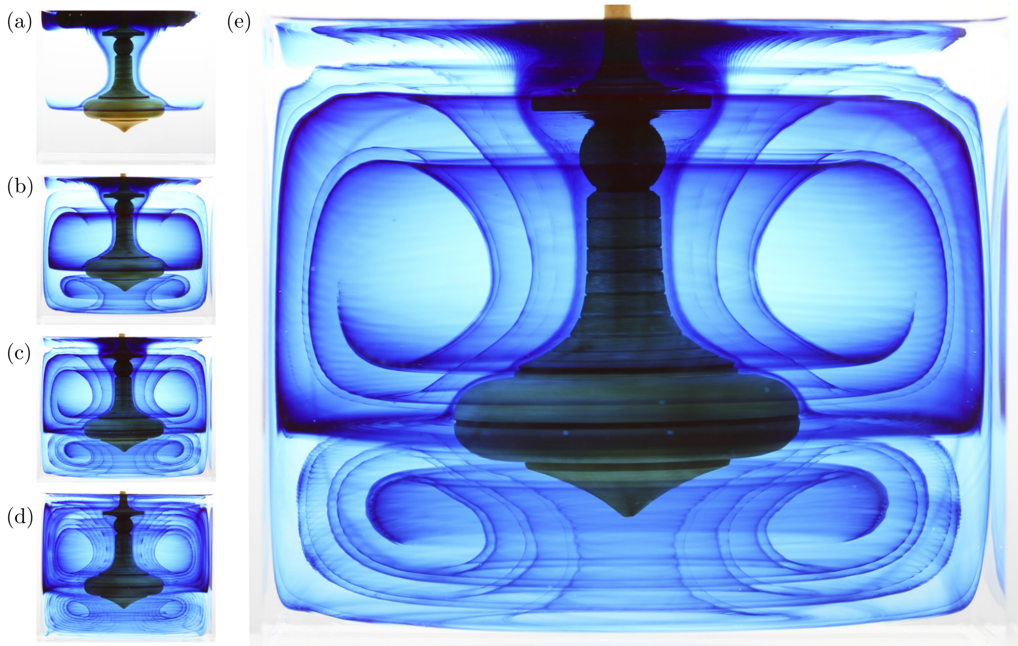

FIG. 1. Visualization of the secondary flows around a spinning top via addition of a small volume of dyed glycerol: panels (a)–(d) show the temporal evolution of the loci of the dyed fluids elements. We can observe the 3D structure of these spiralling toroidal vortices in the magnified view of panel (e). The 50 mm diameter spinning top spins with a rotation rate of 150 rpm.

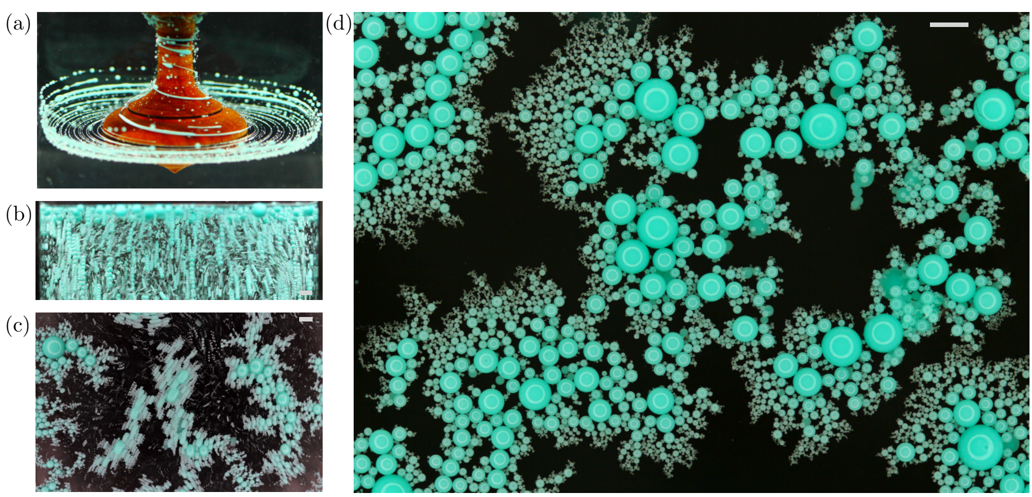

FIG. 2. Formation of a fractal granular raft: panel (a) shows the fragmentation of the immiscible elastomeric liquid within the glycerol tank. Fragmented droplets gradually cure into buoyant beads that ultimately rise to the liquid/air interface upon flow cessation. Panels (b) and (c) show time-lapse overlays for, respectively, side and top views of the tank. As the particles reach the liquid/air interface, they aggregate into clusters due to capillary forces. Panel (d) shows the top view of the formed fractal raft at the liquid/air interface after 24 hours of upside sedimentation. A time-lapse movie of this slow process is included in the original video. The gray scale bar is 2 mm in length.

into small droplets and fragments. Secondary flows carry the larger ligaments/droplets through the fragmentation process [3] multiple times, until the droplets are small enough that resisting capillary stresses can maintain their shape against viscous stresses in the background glycerol liquid. The entire fragmentation process lasts only few minutes, during which the elastomer remains in its uncured liquid state. As the chemical process of gelation proceeds, the secondary flows keep the curing droplets moving within the tank in their individual 3D Lissajous/Bowditch orbits [4]. Once curing is complete, we stop the flow and remove the spinning top from the tank. As shown in the overlaid time lapse side view, Fig. 2(b), upon flow cessation cured buoyant beads start rising to the surface in an upside sedimentation process throughout the viscous medium. Larger beads rise to the liquid/air interface first and gradually migrate toward each other due to the "cheerios effect" [5]. Top- view of the liquid/air interface in Fig. 2(c) shows that this capillary-mediated aggregation process leads to the formation of particle islands, which grow in size and eventually merge into each other. After 24 hours of slow aggregation and spatial rearrangement, the buoyant particles form a fractal granular raft with a shape that is similar to a Mandelbrot set [6], as highlighted in Fig. 2(d). Similar to other granular rafts [7], these fractal rafts also exhibit elastic behavior and can buckle or fracture under applied stresses [8]. The studied phenomenon has a close connection to emulsion-based fabrication of microparticles in biomedical applications and a comprehensive understanding of the size distributions of the fabricated particles can be beneficial to such applications.

In summary, we have shown how the fragmentation of a curing elastomer in secondary flows around a spinning geometry in a glycerol tank generates a collection of solid buoyant beads that gently rise to the interface and aggregate into a floating fractal raft.

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[1] G. I. Taylor, VIII. Stability of a viscous liquid contained between two rotating cylinders, Philosophical Transactions Royal Society London. Series A, Containing Papers Mathematical Physical Character **223**, 289 (1923).

[2] A. Bergman, M. Bergman, and M. Legrand, *The Windmills of Your Mind* (RCA Victor, 1969).

[3] E. Villermaux, Fragmentation, Annu. Rev. Fluid Mech. **39**, 419 (2007).

[4] J. Lovering, Anticipation of the Lissajous Curves, Proceedings of the American Academy of Arts and Sciences **16**, 292 (1880).

[5] D. Vella and L. Mahadevan, The "cheerios effect", Am. J. Phys. **73**, 817 (2005).

[6] B. B. Mandelbrot, C. J. G. Evertsz, and M. C. Gutzwiller, *Fractals and Chaos: The Mandelbrot Set and Beyond* (Springer, New York, NY, 2004), Vol. 3.

[7] S. Protière, Particle rafts and armored droplets, Annu. Rev. Fluid Mech. **55**, 459 (2023).

[8] C. Peco, Y. Liu, C. Rhea, and J. E. Dolbow, Models and simulations of surfactant-driven fracture in particle rafts, Int. J. Solids Struct. **156-157**, 194 (2019).