Invited Articles

Gust encounters of rigid wings: Taming the parameter space

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One of the primary challenges in the study of gust encounters lies in isolating, defining, and parametrizing a specific problem in a highly unsteady, three-dimensional, and multiscale flow. Recent efforts have decomposed the complex gusty environment typical of atmospheric turbulence and wakes into three canonical gust types: transverse gusts, vortex gusts, and streamwise gusts. By applying analytical, experimental, and numerical methods, a comprehensive picture of the flow fields and force histories typical of gust encounters has been achieved, shedding light on how the defining parameters in the problem affect unsteady forcing. The focus of the current work is on vortex formation and lift production on rigid two-dimensional wings. Despite these simplifications, analytical and low-order modeling of gust encounters resulting in massively separated flows remains a challenge, largely because the force response of the wing is highly sensitive to the growth and motion of vorticity in the flow. It also remains to be seen how well canonical gusts represent real-world gust encounters, and whether there is some region of the parameter space where linear superposition of these inherently nonlinear flows results in a reasonable approximation of the true problem. This article provides an overview of a limited selection of recent and classic work on large-amplitude gust encounters to motivate a discussion of current challenges in parametrizing and modeling these flows, as well as thoughts on working toward a long-term goal of mitigating gust responses.

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I. INTRODUCTION

Many challenges in modern flight center around the operation of air vehicles in unsteady and unpredictable environments. These include delivery drones operating in close proximity to people and property in an urban environment characterized by building airwakes and complex wind patterns, as well as manned vehicles performing reconnaissance or search and rescue missions in complex terrain and weather. New challenges in the control of aerodynamic forcing arise when flight operations take place in an environment where flow disturbances are of the same order of magnitude as the airspeed of the vehicle. A gust of wind in this type of environment is not a small disturbance, and may result in large-scale flow separation over the lifting surfaces of the vehicle. Similar scenarios are found in many other applications. Unmanned vehicles underwater may encounter wakes, tidal and river turbines may experience large fluctuations in inflow, and urban wind turbines must contend with winds of rapidly varying magnitudes and directions. The significance of a large-amplitude gust encounter is thus not limited to aircraft operation, but extends to many different applications in which air or water vehicles operate in boundary layers, turbulence, weather, or wakes.

Over the past several decades, most work on gust encounters has focused on small disturbances in which the flow can be reasonably assumed to remain attached to the lifting surface. Küssner [1],

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von Kármán and Sears [2], Sears [3], and others derived theories for wing-gust encounters based on potential flow. These results have recently been found to be even more broadly applicable than might be expected, and Küssner's model (and convolutions thereof) has been found to provide a reasonable prediction of the lift produced in some large-amplitude gust encounters, where the flow clearly separates and becomes dominated by coherent leading- and trailing-edge vortices [4–8]. Many recent studies of gust encounters have focused on exploring the parameter space over which linear theories like these can be applied; comparing, contrasting, and defining different types of gusts and gust encounters; explaining why or to what extent these theories hold despite massive flow separation; and expanding or developing new models to understand and predict aerodynamic force production in separated flows resulting from gust encounters on rigid wings. The long-term goal of this work is to identify the flow phenomena responsible for undesirable transients in wing loading and the physics that might be exploited to mitigate them, first in two-dimensional canonical setups and then in more complex models of real-world unsteady environments.

II. WHAT IS A GUST ENCOUNTER?

The current state of the art in aircraft design [9] and operation [10] includes the effects of continuous turbulence via the statistical von Kármán wind turbulence model [11,12] as well as those of discrete gusts that might be encountered in a typical atmosphere [13]. Recent interest in gust encounters has built on the idea of a canonical discrete gust, now with a focus on the physics of much larger flow disturbances, i.e., gusts of the same order of magnitude as the airspeed of the vehicle and therefore strong enough to incite flow separation on rigid wings. Discrete gusts are further taken to be nominally transient (i.e., isolated excursions from the steady state), though periodic gusts are also considered. It should also be noted that in the context of the current work, gust encounters are defined as cases in which the disturbance is introduced via the external flow, not in the motion of the wing. While wing motions are sometimes taken as an approximation of a gust encounter, they are not, in general, equivalent problems.

To define the modern large-amplitude gust encounter problem, it is first necessary to develop a common nomenclature regarding the gust flow and the process of the wing interacting with this flow. Building on previous work [13], it is intuitive that some of the key parameters in a wing-gust encounter will be the strength of the gust (i.e., the magnitude of the flow disturbance) and the size of the gust (i.e., the length scale or time duration of the gust encounter). Two key nondimensional parameters are thus the *gust ratio* and the *encounter width*. The gust ratio, GR, is the magnitude of the flow disturbance, V, normalized by the freestream, U_∞,

$$GR = \frac{V}{U_\infty}.$$

The gust ratio characterizes the likelihood of flow separation. If $GR \lesssim 0.1$, the gust may be considered a small perturbation to the flow and thus be treated with linear equations, whereas larger values of GR indicate a flow that is likely to experience large-scale separation and thus violate the assumptions of linear theory. The encounter width is the width of the gust flow, w, normalized by the wing chord, c,

$$W_e = \frac{w}{c}.$$

Very small encounter widths represent flow disturbances that are unlikely to have a large impact on aerodynamic forcing as they are mechanically filtered out by the wing, while very large encounter widths represent flows that may be considered quasisteady. In between these two extremes lies the range of flows of current interest, where the width of the flow disturbance is of the same order of magnitude as that of the wing chord, and thus unsteady force production is expected.

Less obvious but also important parameters include effects such as the velocity profile of the gust flow (and thus the resulting change in effective angle of attack experienced by the wing) and the rate at which the encounter occurs (i.e., the rate of change of effective angle of attack due to the convection speed of the gust or the rate at which the wing passes through the gust). Intuitively, it would be possible to achieve different responses to a fixed gust by passing through it faster or slower, thus changing the rate at which the wing interacts with the gust and the rate at which the effective angle of attack of the wing varies as it passes through the gust flow. While the gust ratio parameter does capture the effect of the velocity ratio [i.e., the maximum effective angle of attack experienced by the wing, max (α_{eff})], it does not capture that of the history of the flow, $\alpha_{\mathrm{eff}}(t)$, or the effective pitch rate, $\dot{\alpha}_{\mathrm{eff}}$. Thus the question remains as to how best to incorporate these effects into a concise characterization of a gust encounter.

For a number of reasons, a fundamental challenge in studying gust encounters is identifying and parametrizing the gust flow itself. Wind gusts arise from atmospheric turbulence, air wakes, and similar phenomena that are not typically characterized at the time or length scales of interest for wing-gust encounters. Furthermore, the magnitude, time, and length scales of interest vary widely with the scale of the vehicle and its flight speed. A gust of a magnitude that would be a small perturbation to a manned vehicle in forward flight would be a large disturbance to the same vehicle in hover or to a smaller, slower vehicle. Similarly, the relevant time and length scales of a large-amplitude gust vary with vehicle and flight speed. A gust that is large enough to engulf an entire delivery drone and thereby incite large-scale flow separation may affect only a small portion of a larger aircraft and thus be effectively filtered out by the wing. Finally, what is the shape of a gust flow? The velocity profile of a gust flow varies widely and depends on the origin of the gust. A shear layer emanating from the edge of a building has a different velocity profile from a vortex shed into the wake, and both of these are likely to be complicated by surrounding flow structures and boundary conditions.

There remains much to be done in terms of identifying types of gusts and the relevant defining parameters. To facilitate recent collaborative efforts, a set of canonical gusts has been defined, along with a parametrization of the typical force response [14]. One aim of this work was to relate characteristics of the gust encounter to the severity of the response, a challenging task due to the sheer number of variables in the problem. It is possible to avoid this task by using data-driven methods to uncover relationships between wing-gust encounters and their aerodynamic responses, but this would require an extensive data collection effort carefully designed to capture all of the relevant physics. While these types of methods are likely to be instrumental in the development of control algorithms for gust mitigation, analytical studies of canonical problems remain critical to gaining insight into the underlying physics that might be exploited for rapid control.

III. TYPES OF CANONICAL GUST ENCOUNTERS

There are a myriad of ways a gust flow might develop, and even more ways that a wing might pass through a gust. In an attempt to make the problem more tractable, an effort has been made to identify and define three types of canonical two-dimensional gusts: transverse gusts, vortex gusts, and streamwise gusts. In recent work, these canonical gust types are defined relative to the length scale of the wing, and it should be noted that the gust type may change if evaluated over a different length scale. For example, a large vortex shed from a bluff body may act as a vortex gust on a wing of chord similar to the vortex diameter, but more like a pair of transverse gusts on a much smaller wing.

To study gust encounters in the laboratory, many research groups have produced unsteady flow environments using a variety of methods including pitching or otherwise active wings, plates, and cylinders [15–17], shuttered wind tunnels [7,18,19], and active grids [20,21]. These efforts have been successful in generating vortex or streamwise gusts in the form of airwakes or oscillations in the freestream. Producing a well-defined transverse gust in a wind tunnel is a more challenging endeavor, but it has nonetheless been achieved [19,21,22]. Most recent studies of transverse gusts, however, have been performed either in towing tanks [4–6,23] or numerically [24–27].

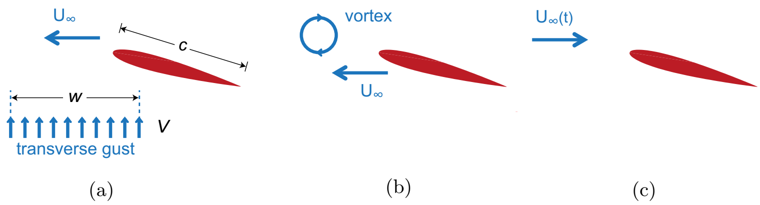

FIG. 1. Schematic of three types of gust encounters. From Ref. [28]. (a) Transverse gust, (b) Vortex gust, (c) Streamwise gust.

Transverse gusts, illustrated in Fig. 1(a), represent a wing passing through a well-defined updraft or a strong shear layer. Examples of these types of flows include atmospheric phenomena and large-scale wakes. These commonly occur in urban environments as well as near naval vessels, mountains, and other complex terrain. In a canonical model of the transverse gust encounter, a rigid wing is held at a fixed geometric angle of attack in steady forward motion before passing through a well-characterized updraft. Previous and ongoing work on transverse gusts has focused on sine-squared velocity profiles (representing a real-world 1-cosine gust) [4,23] or top-hat velocity profiles (following the canonical gusts of classical linear theories) [5,6,27].

The length scale of a transverse gust flow is fairly straightforward to identify, but there remains some uncertainty as to whether the maximum amplitude of the gust, the average gust flow disturbance, or some other metric is the best way to parametrize the magnitude of the gust flow in the context of a wing-gust encounter. To date, most work has taken the maximum amplitude of the gust disturbance as the defining metric, this being directly related to the maximum effective angle of attack experienced by the wing [13]. However, it should be noted that transverse gusts of the same gust ratio (as defined by the maximum amplitude of the gust flow) but different velocity profiles result in a different buildup of lift and ultimately a different maximum lift force on the wing. Figure 2 shows a comparison of the measured lift on a flat plate wing at zero incidence passing through top-hat and sine-squared gusts. While the basic structure of the flow (i.e., separation and the formation of a strong leading-edge vortex) is the same, the rate at which the leading-edge vortex strengthens and lift increases differs. The magnitude of the lift curves becomes more similar if a top-hat gust of $GR = 0.5$ is compared to the sine-squared gust of $GR = 1$, i.e., for two gusts with the same gust ratios based on average rather than maximum gust velocity, but the shape of the curves remains the same [6]. As shown in Fig. 2, convolution of Küssner's model matches well with experimental measurements of the sine-squared gust at a gust ratio of 1, but less well for the top-hat gust. At lower gust ratios, $GR \leqslant 0.5$, good agreement with Küssner was found for both gusts [4,6].

The canonical vortex gust, illustrated in Fig. 1(b), represents wing-vortex interactions where the diameter of the vortex is the same order of magnitude as the chord of the wing. These interactions are commonly found in the wakes of other wings and blades (e.g., rotorcraft, wind and tidal turbines, formation flight), the wakes of structures and bluff bodies (e.g., ship-airwake interactions, urban environments), and in other unsteady environments (e.g., atmospheric or seabed turbulence). It should be noted that while an isolated vortex as shown in Fig. 1(b) is perhaps a convenient abstraction of a vortex gust encounter, most external flows are irrotational and thus most vortices exist in opposite-signed pairs. If the flow shown here is irrotational, the clockwise vortex shown in the figure must have a corresponding counterclockwise vortex (or distribution of vorticity) elsewhere in the flow. One focus of recent vortex gust encounter work has been fundamental flows with nominally two-dimensional (i.e., high aspect ratio) vortices oriented such that the vortex core is parallel to the leading edge of the wing and its pair is sufficiently far from the wing so as to not be a primary consideration [15–17,29–33]. There also exists substantial work on vortices oriented in the streamwise direction (i.e., normal to the leading edge of the wing), largely motivated by interaction with a wingtip vortex in formation flight) [34–36].

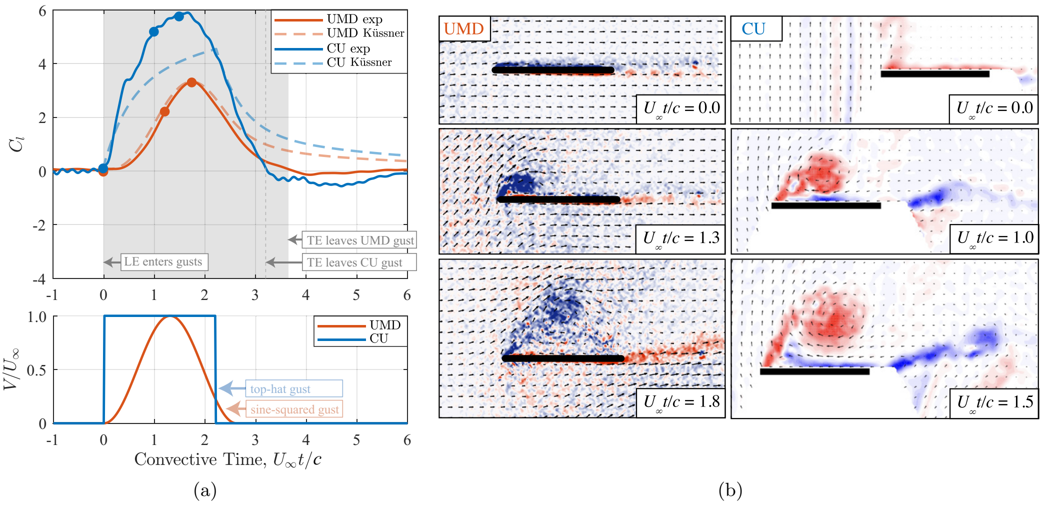

FIG. 2. Results from a top-hat transverse gust encounter in the University of Cambridge (CU) towing tank and a sine-squared gust encounter in the University of Maryland (UMD) towing tank. Both test models are a flat plate wing at zero incidence passing through a gust of $GR = 1$. From Ref. [28]. (a) Lift coefficient histories from experimental measurements and analytical results from Küssner's model applied to sine-squared and top-hat gusts. (b) Color contours of vorticity (note inverted colors) overlaid on velocity fields as measured using time-resolved particle image velocimetry measurements of a sine-squared (*left*) and top-hat (*right*) gust encounter.

While the transverse gust described above has one clear degree of freedom beyond the gust ratio and encounter width, namely the shape of the velocity profile, the vortex gust has more. The width of the vortex gust may be taken either as the radius of the vortex or, especially for series of vortices, the half-wavelength of the disturbance. Furthermore, while deformation of the gust flow as a wing passes through it is clearly visible in the case of the transverse gust, it does not always have a large influence on force production [5]. This is not the case for a vortex gust encounter, where the trajectory of the vortex is drastically altered by the presence of the wing, and thus so is the force history. One example of this is given by Chen and Jaworski [32] and Peng and Gregory [37] in their comparison of the effect of vortex inflow position and the resulting variations in vortex trajectory and strength on a single point (numerical) or physical (experimental) vortex as it interacts with the boundary layer of the wing (Fig. 3). Because the trajectory of the vortex as it passes over the wing directly affects the flow induced at the wing, it also affects the unsteady forcing on the wing. If the vortex collides with the wing, it may break up, but how it does is likely to depend on exactly how and where this interaction occurs.

Of course real-world vortex gusts may not be transient. They may be periodic, random, or some combination thereof. Fortunately, it appears that (at least for a series of discrete coherent vortices) a transient vortex gust encounter produces a response very similar to a single period of a periodic vortex encounter [38]. For a series of vortices significantly smaller than the wing chord, the resulting force response of the wing appears to be the superposition of the effect of the wake and the lift history of the airfoil [39]. This is likely to remain true so long as the flow separation is not too extreme or the frequency of vortex encounters is low enough to allow for reattachment between encounters.

The third and final canonical gust encounter defined here is that of a streamwise gust as illustrated in Fig. 1(c). A streamwise gust encounter occurs when a wing at fixed incidence experiences time variations in the streamwise flow. Transient streamwise gusts are difficult to create in a laboratory and so are often modeled by moving the test model rather than the freestream [40–42], though in this

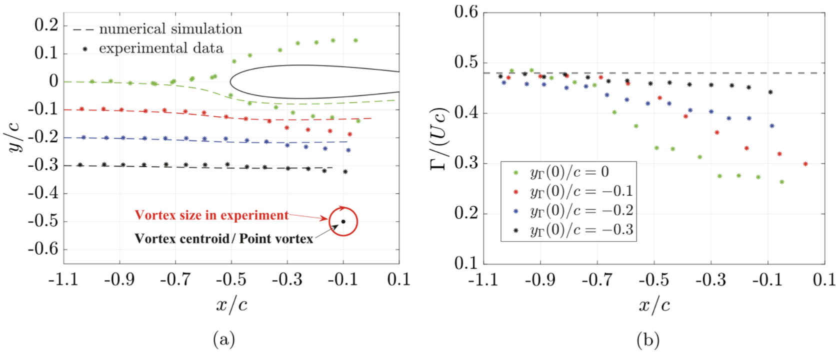

FIG. 3. Numerical [32] and experimental [37] results for a vortex interacting with a 12% thick airfoil. From Ref. [32]. (a) Effect of inflow position on vortex trajectory over the wing. (b) Variations in vortex strength throughout the encounters due to boundary layer interaction.

case care must be taken to account for the lack of buoyancy effects (due to the lack of an accelerating flow) [7,43] and the addition of added mass effects (if the test model is accelerated) [44]. Periodic [18,21] or random [45–47] streamwise gusts are typically generated in louver-equipped unsteady wind tunnel facilities, but they have also been performed in water [48]. This type of gust models the streamwise fluctuations experienced by lifting surfaces in tides, flapping and surging wings, rotor blades, and atmospheric turbulence. Here, the effective angle of attack of the wing remains constant as the relative flow angle does not change, but both a time and a length scale of the flow disturbance can be defined, and thus both the gust ratio and encounter width can be found.

Because it is possible to have a large-amplitude streamwise gust without a dramatic increase in effective angle of attack, flow may remain attached and the effect of the gust encounter on the health of the boundary layer and process of flow separation is more complicated. Recent work in this area has investigated how separation metrics like the leading edge suction parameter (LESP) [49] can be used to model or predict separation in gust encounters [25,50]. Williams *et al.* [46], for example, measured the LESP on a wing in random streamwise disturbances modeled after the von Kármán gust spectrum, and they found it to be a reliable indicator of whether the flow is attached or separated (Fig. 4). The addition of flaps to a wing may alter the value of the LESP, but it does not appear to hinder its effectiveness in indicating separation [51], thus the LESP and similar metrics may prove to be instrumental in the development of control for gust response mitigation.

IV. FORCE IN A GUST ENCOUNTER

If the long-term goal of studying gust encounters is to find effective ways to mitigate or exploit the transient forcing that a wing experiences, the most important fundamental questions are those of when, where, and why transient forces arise. One approach to answering these questions is to apply analytical and low-order models to the flow and use these as a tool to evaluate the underlying flow physics. By understanding the limitations and assumptions of a model while comparing its results to those of experiments or high-fidelity computations, it is possible to isolate physical contributions to forcing and evaluate the potential of gust mitigation techniques.

The simplest models of force production in a gust encounter use potential flow to compute lift in an attached flow, generally by modeling the effective angle of attack of the wing due to a combination of a rigid gust flow and a planar shed wake. In a large-amplitude gust encounter, however, the assumptions of potential flow theories no longer hold. In these types of flows, impulse methods [53] are particularly useful as they allow for consideration of the effects of strong shed vortices [54–56] as well as vorticity contained in the gust flow itself, though care must be taken to account for all of the vorticity in the flow only once.

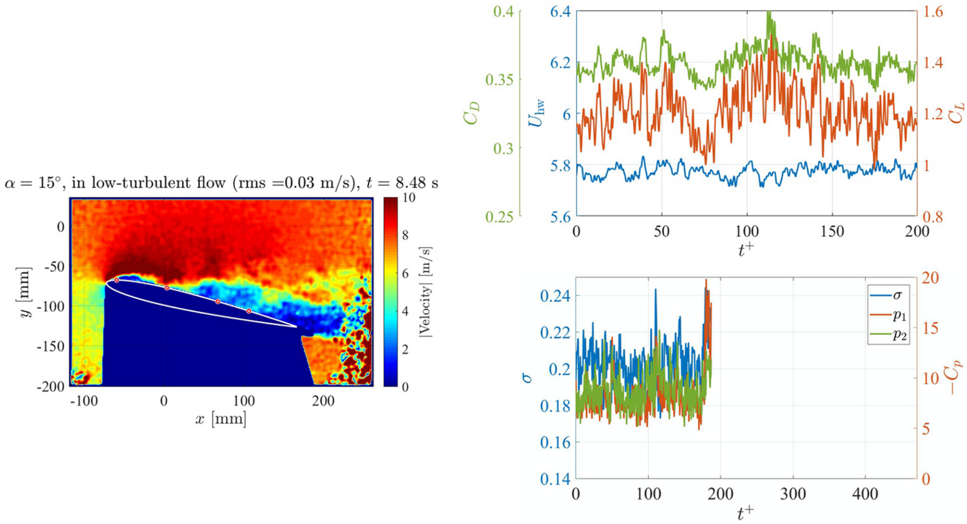

FIG. 4. Contours of streamwise velocity, and a time series of drag, lift, and velocity for a wing in a randomly varying freestream. Lift and drag vary in time with fluctuations in the streamwise velocity, but the suction parameter clearly rises from $\sigma = 0.2$ to 0.23 when the flow reattaches. From Ref. [52].

Impulse models also allow for the categorization of different sources of force in a separated flow, thereby informing low-order models and revealing physics that might be exploited for control and mitigation of force transients. The force exerted by the fluid on the body (i.e., wing) is given by the rate of change of impulse in a flow, $\vec{F} = -\rho d\vec{P}/dt$, where the impulse P is directly related to the vorticity in the flow, and ρ is the density of the fluid. By Kelvin's circulation theorem, the total circulation in the flow does not change, i.e., $\frac{d}{dt}(\Gamma_\omega + \Gamma_\gamma) = 0$, where the terms represent the sums of the free vorticity in the flow and that bound to the wing in a vortex sheet. For an inviscid flow (or a fully separated flow in which the effects of viscosity are confined to the sharp edges of a flat plate wing), the circulation of the bound vortex sheet representing the body in the flow, $\Gamma_\gamma = \int \gamma_\gamma(x)dx$, is equal and opposite to the sum of the contributions from vorticity scattered throughout the flow field, $\Gamma_\omega = \int \gamma_\omega(x)dx$ [53–55,57]. That is, γ_ω can be thought of as containing the "mirror image" of the free vorticity in the flow necessary to satisfy Kelvin's circulation theorem and enforce the no through-flow boundary condition at the wing due to Γ_ω. Another contribution to the bound vortex sheet is that of γ_b, which contains the potential flow result of the no through-flow boundary condition at the wing due to body motion, but this does not explicitly appear in the equation above because although the distribution of vorticity may change in time, the net circulation of this sheet is zero, i.e., $\Gamma_b = \int \gamma_b(x)dx = 0$.

The force produced on a wing passing through a strong gust can thus be thought of as a combination of noncirculatory effects due to inviscid flow (i.e., potential flow due to body motion) as represented by γ_b, and vorticity distributed throughout the flow field represented by γ_ω. The latter can be further broken up into two contributions: one that is shed from the wing and one that is due to the external flow [58]. Each of these contributions is reflected in a bound vortex sheet of its own, $\gamma_{\rm shed}$ and $\gamma_{\rm ext}$, and, together with γ_b, they make up the total vortex sheet bound to the wing, γ_γ. This leads to an expression for the transverse force on the wing,

$$F_y = -\rho \frac{dP_y}{dt}, \tag{1}$$

where

$$P_y = -\int x\gamma_\gamma(x)dx \tag{2}$$

and

$$\gamma_\gamma = \underbrace{\gamma_b + \gamma_{\text{ext}}}_{\gamma^{\text{nc}}} + \underbrace{\gamma_{\text{shed}}}_{\gamma^{\text{c}}}. \tag{3}$$

The circulatory force contribution, γ^{c}, results from the portion of the bound vortex sheet with nonzero circulation. For an airfoil in steady flow, the circulatory force is given by thin airfoil theory as $L = -\rho U\Gamma$, where $\Gamma = \int \gamma^{\text{c}}(x)\,dx \neq 0$, and γ^{c} represents a bound vortex sheet along the surface of the wing made up of the mirror images of contributions from vorticity shed from the wing into the wake, $\gamma^{\text{c}}_{\text{shed}}$. Note that in this formulation, $\gamma^{\text{c}}_{\text{shed}}$ accounts for all vorticity shed for all time, and so it includes any attached-flow bound vorticity whose integrated value is equal and opposite to that of the starting vortex as well as any other leading- and/or trailing-edge vortices.

At early times during a large-amplitude gust encounter, and especially for sharp-edged gusts, the primary force is noncirculatory, i.e., force that is not related to a net circulation about the body and so $\int \gamma^{\text{nc}}(x)\,dx = 0$. This force arises from changes in the portion of the bound vortex sheet with zero net circulation, including that due to inviscid added mass, γ^{nc}_b, as well as any free vorticity in the external flow related to the gust itself, $\gamma^{\text{nc}}_{\text{ext}}$. Because force is a function of the time rate of change of vorticity distribution, changes in either the strength or the shape of the bound vortex sheet can result in a noncirculatory force, hence the contribution of force from a vortex sheet of net zero magnitude.

Added mass is one common noncirculatory force that arises due to the no through-flow condition on the surface of a wing undergoing time-varying motion with a component normal to its surface (i.e., translation, rotation, or deformation) [44]. This narrow definition of added mass requires that force be proportional to acceleration to fit the form $F = ma$ and thus be classified as an added mass force. The acceleration must be of the body (rather than the surrounding fluid) because if the surrounding fluid were to be accelerated independent of the body, free vorticity would be generated, and it would thus be accounted for by $\gamma^{\text{nc}}_{\text{ext}}$. For a wing in steady motion passing through a gust, there is therefore no added mass force as the bound vortex sheet due to potential flow does not change in time, i.e., $d\gamma^{\text{nc}}_b/dt = 0$. However, there may be a force due to the relative strength and motion of vorticity in the gust flow, i.e., $d\gamma^{\text{nc}}_{\text{ext}}/dt \neq 0$. Wings that are maneuvering or actuated as they pass through the gust (e.g., pitching or plunging wings, or wings equipped with leading- or trailing-edge flaps), perhaps in an attempt to mitigate the aerodynamic gust response, do experience an added mass force while these maneuvers take place as this results in a nonzero change in γ^{nc}_b for the duration of that actuation [59].

The vortex sheet and thus force that arises due to external vorticity in the gust flow, $\gamma^{\text{nc}}_{\text{ext}}$, is directly related to the gust velocity profile, the turbulence, and other unsteadiness in that flow, and the deformation of the gust flow in response to the wing. Note that because the external flow is generally assumed to be irrotational, the sum of this vorticity is zero, and $\Gamma_{\text{ext}} = -\int \gamma^{\text{nc}}_{\text{ext}}(x)dx = 0$, so γ_{ext} is considered a noncirculatory contribution. However, changes in the distribution of vorticity in this sheet nevertheless give rise to a force contribution [58]. In a sharp-edged transverse gust, for example, the gust vorticity is concentrated in shear layers at the edges of the gust. Küssner [1] and von Kármán and Sears [2] account for the effects of these shear layers, but since the result is noncirculatory and, when the wing is fully immersed in the gust, equivalent to a potential flow solution for a flat plate normal to a freestream, they attribute the resulting force to added mass rather than external vorticity [58]. Furthermore, while Küssner's model takes these shear layers to be rigid, experiments [5] have demonstrated significant gust deformation. Somewhat surprisingly, however, deformation of the gust shear layers does not appear to significantly affect the force contribution of the gust vorticity for $GR \lesssim 1$ so long as the wing is thin [5]. For bodies of significant volume, i.e., not infinitely thin flat plates, and especially at gust ratios greater than 0.5, shear layer thickness

TABLE I. Summary of unsteady potential flow models for rigid two-dimensional wings.

Disturbance	Periodic	Transient
Wing motion	Theodorsen [60]	Wagner [63], von Kármán and Sears [2]
Transverse	Sears [3]	Küssner [1], Miles [64]
Streamwise	Isaacs [65,66], Greenberg [67]	
Transverse and streamwise	Atassi [62]	

and distortion become significant and may need to be considered in the development of accurate low-order models [58].

V. MODELS OF GUST ENCOUNTERS

Models of gust encounters are of great interest, both to aid in better understanding the highly separated flow that develops in a large-amplitude gust encounter, and ultimately to predict and control for gust responses. One challenge in the current work, however, is that the simplest and fastest models are based on potential flow and are thus formally limited in their applicability to the linear case in which a gust encounter can be considered a small disturbance, and flow remains attached. While this assumption generally holds for a large air vehicle operating in a weakly unsteady environment, it does not hold for the large-amplitude gusts of current interest.

If flow can be assumed to remain attached, however, thin airfoil theory can be used to build an analytical model of the flow, resulting in an expression for the net unsteady force, typically decomposed into a superposition of the effect of vorticity on the wing and in its wake. The effect of the vorticity on the wing can be integrated directly, accounting for attached flow and added mass, whereas a correction must be applied to that in the wake to account for the viscous physics not present in the otherwise inviscid model. These corrections (e.g., those of Theodorsen [60] and Sears [3]) are applied to both the phase and magnitude of the force response, and are a function of the reduced frequency of the wing motion or incoming gust. In later extensions of this work by Goldstein and Atassi [61] and Atassi [62], the effect of the presence of the wing on the gust flow was considered, resulting in a formulation in which flow disturbances in both the transverse and streamwise directions can be accounted for. Several such models, some of which are given in Table I, have been derived for both transient and periodic disturbances, both for airfoils moving (i.e., pitching, plunging, and/or heaving) in a steady or uniform flow and for static airfoils experiencing unsteady inflows (i.e., gust encounters) in the streamwise (horizontal) and/or transverse (vertical) directions. The foundational assumption in each of these, however, is that the flow disturbance is small, and flow remains attached to the wing. Furthermore, many of these models do not account for the effect of a flow disturbance that does not exist uniformly across the airfoil chord. The notable exceptions to this are Küssner [1], who gives a solution for a sharp-edged transverse gust, and von Kármán and Sears [2], who draw an analogy to a broken-line airfoil that experiences a rise in effective angle of attack over an increasing portion of the wing as it enters the gust.

The time-varying lift on a thin airfoil undergoing a small impulsive change in angle of attack was first described by Wagner [63]. Von Kármán and Sears [2] further explained the delay in lift for an analogous problem of a wing at fixed incidence impulsively started from rest. They treated the bound vortex on the wing and the starting vortex in the wake as a vortex pair, the momentum of which is proportional to the product of the circulation and distance between the two vortices. As the wing moves away from the starting vortex, the momentum of the system increases and the rate of increase of momentum is equal to the force. Using this vortex-pair model, von Kármán and Sears [2] extended Wagner's theory to unsteady motions by allowing the airfoil to shed a series of vortices into the wake. For n vortices of strength Γ strung out in a planar wake at locations x in a fluid of density ρ, the lift and moment per unit span are

$$L = -\rho \frac{d}{dt} \sum_{i=1}^{n} \Gamma_i x_i\,, \quad M = -\frac{\rho}{2} \frac{d}{dt} \sum_{i=1}^{n} \Gamma_i x_i^2, \tag{4}$$

where the moment is taken about the origin of the coordinate system, and both results require two-dimensional flow, motion only along the x-axis, and that the Kutta condition holds [2]. In this formulation, it is clear that the wake plays a key role in force production and that shed vorticity cannot be neglected. Any low-order flow model of unsteady aerodynamic forcing must therefore account for detached vortices, their strength, and their convection. This seemingly simple requirement presents a significant challenge as circulation production on the wing is closely related to the evolution of the wake, and thus many models break down after short times [68].

Further thin airfoil theories have been derived specifically for gust encounters. The idealized sharp-edged gust problem, in which a wing passes through a rigid transverse gust with a step-function velocity profile, was first presented by Küssner [1]. The fundamental difference between this problem and Wagner's impulsively pitching airfoil is that, in Wagner's problem, the angle of attack changes instantaneously along the entire airfoil chord. In the sharp-edged gust problem, however, the angle of attack varies progressively along the airfoil chord as the wing enters the gust. Experiments on flat-plate wings pitching to high incidence have demonstrated that both the pivot point and pitch rate have a substantial effect on the unsteady forcing [69–77]. This result suggests that the rate of gust penetration is a critical parameter in a wing-gust encounter, and so motivates further consideration of von Kármán and Sears's [2] broken-line airfoil analogy. If a wing enters a sharp-edged gust disturbance gradually, a different buildup of lift and pitching moment occurs than if the entire wing is instantaneously engulfed [78]. These results are in line with the many previous suggestions that the timescale of the flow unsteadiness, and thus the rate at which a wing enters a gust, can affect the timing and magnitude of a forcing transient.

Despite violation of the underlying assumptions, linear analytical models have been widely applied to the modern large-amplitude gust problem with remarkable success. For example, Küssner's [1] model and convolutions thereof provide a reasonable result for $GR \leqslant 0.5$ for sharp-edged gusts [5,6]; to much higher gust ratios for smoother gusts [4], though it should be noted that the model is more accurate on gust entry than on gust exit, likely due to the additional free vorticity in the flow and the more highly distorted gust shear layer on exit. For vortex gusts, inviscid models perform well if the vortex is offset from the centerline of the airfoil by 20% chord or more or if the angle of attack is less than 20° [32,37,79]. Linear inviscid theories also work well for streamwise gusts at low angles of attack if the boundary layer remains fully attached. In this case, the persistence of attached flow means that boundary layer dynamics become important and phase lags may arise, though force amplitudes are reasonably well predicted to fairly large amplitudes [7,8].

Since a large-amplitude gust encounter is clearly a nonlinear flow, and because there is a huge parameter space to be investigated, many recent flow modeling efforts have turned toward more complex approaches including discrete vortex models, data-driven methods, and combinations of the two. Discrete vortex models can be a useful tool for unsteady and separated flows. They become computationally expensive at long times, but aggregating vortices and modeling shear layers rather than individual vortices can reduce their dimensionality [80]. Further improvements and physical insight can be gained by incorporating real-time measurements and thereby compensating for physics that are otherwise neglected in the inviscid model. For example, Darakananda *et al.* [25] have demonstrated that a vortex model augmented with pressure measurements and an ensemble Kalman filter flow estimator can provide excellent predictions of vorticity, surface pressure, and force response to a flow disturbance. Fully data-driven approaches can also provide good predictions without requiring a parametrized gust input, an admitted downside to classical methods, but this comes at a cost as these methods provide less physical insight into the flow.

Recent high-fidelity computations of large-amplitude gust encounters have applied the unsteady Reynolds-averaged Navier-Stokes equations to evaluate a broader parameter space than is possible in experiments, including varying Mach, Froude, and Reynolds numbers, as well as the length scale, amplitude, and turbulence content of the gust. Modern computer hardware, along with the maturity of solvers with advanced turbulence closures for separated flows [81], has made it possible to capture the highly unsteady separated flows that result from large-amplitude transverse gust encounters [27]. However, without the addition of cross-derivative terms needed for gust ratios that result in large-scale flow separation ($GR \gtrsim 0.4$–0.5 for transverse gusts), numerical dissipation can result in extremely large meshes and high computational cost. One alternative is to use jet boundary conditions both above and below the wing; this approach has been shown to replicate experimental gust responses at lower cost [27]. Even at low Reynolds numbers (e.g., 40 000), physical gust encounters typically contain significant turbulence, and thus turbulence closures are recommended to capture similar effects in computational models.

VI. MITIGATING THE EFFECTS OF GUST ENCOUNTERS

Modeling and predicting the evolution of separated flows in gust encounters is only the first step toward the long-term goal of mitigating the aerodynamic response of a wing to such an encounter, and many challenges remain. First, an incoming gust in a laboratory setting is often both expected and fully known *a priori*. How can a gusty inflow be sensed (and, if necessary, the resulting flow estimated) with enough advance notice to act? Secondly, given the large flow disturbances and large-scale flow separation that occur in the problems of current interest, what types of actuators have enough control authority and a rapid enough response time to effect the desired force regulation? On the surface, this might seem to be an issue left to the flow control community, but in reality, given the fast time response and large actuator authority required to maintain controlled aerodynamic forcing in a large gust, it is possible that control must be effected via fundamentally different physics than many actuators in common use today.

Finally, although preliminary closed-loop control based on effective angle of attack has been promising [59,82], control mechanisms to attenuate unsteady force responses in large-amplitude gusts largely remain unexplored. Modeling the effect of actuators on the flow field is one necessary step toward model-based control and a better understanding of the closed-loop system. In some cases, lift disturbances may be short-lived and mild in magnitude, and disturbance rejection may be unnecessary or traditional techniques applied without knowledge of the incoming gust flow. However, for increasing gust duration and magnitude, the existence of coherent vortices in the flow plays a greater role in aerodynamic forcing. Indeed, mitigating the wing's gust response could be effectively achieved by exploiting these vortices, for example, by pitching to delay leading-edge vortex separation. To achieve this level of flow interaction, it is necessary to sense and estimate the state of the flow field in real time. Rapid flow-field estimation and low-order modeling can be performed [83–87], but one particular challenge is that of how to deal with uncertainty. High levels of uncertainty plague all stages of the problem, from sensing an inherently unsteady gust flow (perhaps with poorly defined boundaries or incoherent flow structures) to the results of any action taken to affect unsteady forcing (perhaps modeled by less than perfect aerodynamic models). Future efforts in this area would likely benefit greatly from an integrated approach simultaneously addressing the modeling of the nonlinear unsteady aerodynamics of a large-amplitude gust response, uncertainty in the gust inflow, and control design.

VII. CHALLENGES AND FUTURE WORK IN GUST ENCOUNTERS

Interest in large-amplitude gust encounters has surged in recent years and, building on extensive previous work on more general unsteady aerodynamics, dynamic stall, flapping wings, and other aggressive wing maneuvers, some of the more fundamental questions regarding typical responses to a large-amplitude gust encounter have now been answered. Much like in the case of wings undergoing large unsteady motions, large-amplitude gust encounters where the gust ratio is $O(1)$ typically result in large-scale flow separation, the roll-up of separated shear layers into coherent leading- and trailing-edge vortices, and a vortex-dominated flow. As a wing enters a strong gust, the formation of these vortices combined with interactions with the vorticity in the gust flow lead to a rapid buildup of aerodynamic force on the wing. A good understanding of how, when, and to what extent these flows can be modeled using inviscid theory is beginning to emerge. It is now clear that while existing analytical models can perform surprisingly well for some cases, more accurate and more general models require consideration of viscous effects including accurate predictions of flow separation and the growth and motion of both the leading-edge vortex and wake, presenting a significant challenge for analytical methods and, at least at this point in time, benefiting from some element of empirical or data-driven approaches.

It should be noted that the three types of gusts identified and defined here as canonical models are very much idealized two-dimensional representations of real-world gust encounters. How often do these canonical gusts accurately represent the flow structures that a vehicle might encounter? Or, put another way, what flow structures make up the unsteady aerodynamic environment in an urban setting, a ship airwake, unsettled weather, or complex terrain? To date, work on the flow physics of gust encounters has necessarily focused on canonical models to allow for tractable problems and comparisons with theory. However, it remains to be seen to what extent canonical gusts dominate a gusty environment or, more importantly, the aerodynamic response of a wing to such an environment. While there have been multiple efforts to measure or compute unsteady winds, there does not appear to be an in-depth categorization of the relatively small-scale flow structures that are of primary interest for wing-gust encounters. This is a challenging task for many reasons, not the least of which is the enormous amount of data that would have to be acquired and processed, but the results of such an endeavor would inform and focus future gust-encounter research.

Acknowledging that real-world gust encounters are not likely to be entirely represented by canonical problems, one open question of great interest is to what extent more complex real-world flows can be decomposed for analysis and the resulting flow fields or force histories reassembled to approximate the original flow. That is, for large-amplitude gust encounters where flow can be expected to separate and the governing equations are nonlinear, is there some region of the parameter space where the nonlinearities can be neglected and the superposition of solutions to canonical gust encounter problems is a reasonable approximation of the true problem? Experiments on wing motions in canonical vortex gust encounters suggest that this may be possible [39], but results are less promising for more complex problems. For example, in a study of the airwakes of ships at sea, Dooley *et al.* [88] found that ship motion could be neglected at low sea states, but superposition of the airwake and ship motion failed at high sea states, and the only acceptable way to model this flow involved a fully coupled solution.

Reynolds number is also likely to affect how superposable a set of flows are. Higher Reynolds number flows are made up of a broader range of turbulent structures, introducing the question of which structures most affect forcing. Gust encounters are of interest for a wide range of vehicle scales, but the fundamental flow physics of gust encounters are most easily studied in controlled laboratory conditions where practicalities require that most experiments are performed at Reynolds numbers $O(10^4–10^5)$. Experiments are therefore representative of small unmanned air vehicles, but how do the gust flows and resulting interactions observed here scale to the larger length scales and higher Reynolds numbers typical of manned aircraft? Dooley *et al.* [89] found that ship airwake laboratory experiments at Reynolds numbers $O(10^6)$ capture the most critical flow features, but differences, even in the mean flow, still arise between $O(10^6)$ and $O(10^8)$. What features of the wake must be captured in a scaled experiment or numerical simulation to produce meaningful results?

To date, Reynolds number concerns have largely been avoided in the study of canonical transverse and vortex gusts by focusing on coherent large disturbances and flat plate wings so that separation undoubtedly occurs at the leading edge of the wing. However, streamwise gusts (where flow separation may not occur despite strong flow disturbances), gust recovery (i.e., the process of flow reattachment as the wing exits the gust), and flows over thick and blunt leading-edged airfoils necessitate a more thorough consideration of the effects of viscosity. One approach to this problem has been the use of data-driven models such as that of Goman and Khrabrov [90]. This approach may be suitable for use in a control loop, and it has the potential for gust response mitigation [91,92], but it is limited to the conditions over which the model is trained. Another approach is that of predicting flow separation using theory aided by empirical results. As mentioned previously, LESP has shown promise as an indication of flow separation, but recent results suggest that the critical value of LESP at the point of separation can vary with flow conditions [25,51].

Fortunately, LESP need not be entirely informed by experiments; it can also be predicted by thin airfoil theory [51]. However, while the instantaneous value of LESP is a reliable indicator of flow separation, it is also necessary to predict the time elapsed between the point at which a wing exceeds its static stall angle and the onset of large-scale flow separation because the critical LESP value varies with the history of the flow [51]. For cases in which boundary layer unsteadiness does not have a large impact on the timing of vortex formation (e.g., low reduced frequency disturbances or strong trailing wakes), the timing of leading-edge flow separation and vortex formation can be predicted by a combination of an unsteady inviscid flow solver (e.g., panel methods) and quasisteady treatment of the boundary layer equations coupled with a separation criterion based on a critical value of the Falkner-Skan parameter [93]. Early results suggest that this method is of similar accuracy to others reported in the literature, but it has the benefit of requiring only a single empirical parameter that theoretically does not depend on the airfoil geometry. Despite numerous and ongoing efforts to accurately and robustly predict the growth and separation of a leading-edge vortex, there is no one perfect solution, and the search will likely continue for a long time to come. Realistically, the best-suited method may depend on the parameter space and application of interest.

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