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Dissipation scaling in constant-pressure turbulent boundary layers

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Using results from previous direct numerical simulations and experiments in the outer region of spatially evolving turbulent boundary layers, we compute the streamwise evolution and the wall-normal variation of the dissipation parameter C_ε, namely, the turbulent kinetic energy dissipation rate, normalized by appropriate powers of the local turbulent kinetic energy and integral length scale. For $\mathrm{Re}_\theta \gtrsim 10\,000$ (Re_θ is a Reynolds number on the freestream velocity and the local momentum thickness), C_ε is essentially constant in the streamwise direction, but varies by up to 50% in the wall-normal direction. For $\mathrm{Re}_\theta < 10000$, C_ε is additionally found to vary in the streamwise direction and is inversely proportional to the local turbulence Reynolds number Re_λ.

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I. INTRODUCTION

The solution of the otherwise unsolvable statistical equations of turbulence may be achieved with the use of turbulence models [1–5]. The formulation of turbulence models requires the use of pertinent scales for the velocity and size of the most energetic turbulent motions, whose values are normally determined as parts of the solution. The velocity scale is universally derived from the turbulent kinetic energy (TKE) per unit mass k, or a related property, and an integral length scale L serves as a suitable length scale. Whereas k and individual turbulent stresses appear directly in the Reynolds equations, there is no fundamental principle by which one may connect L to the rest of the properties of the turbulence. A third parameter of essential importance is the TKE viscous dissipation rate ε. Unlike L, ε is connected to k and the other turbulence properties through the TKE equation and so may be included in the solution unknowns. Dimensional analysis may readily provide a surrogate length scale as $\mathcal{L} \propto k^{3/2}/\varepsilon$, but this would introduce an *ad hoc* assumption for the equivalence of $\mathcal{L}$ and L. This assumption is in fact one of the most pivotal conjectures in turbulence analysis. Its justification is based on an assumed equilibrium between the rate at which TKE is fed externally to large-scale motions and the rate at which it is dissipated by motions of the smallest dynamically significant size, indeed a size that is much smaller than L [6–8]; the process of interscale energy transfer is commonly referred to as the energy cascade.

Since its early introduction [9], the equivalence $L \propto \mathcal{L}$ has been used widely in turbulence literature, however, it has received little direct experimental support and its basic premise has met with little scrutiny. Whereas the equivalence $L \propto \mathcal{L}$ is plausible for canonical flows far away from their origin, where the turbulence structure has sufficient time to evolve to a natural (presumably asymptotic) state, its applicability to flow regions where the turbulence structure undergoes transformation seems to be questionable. It should also be noted that the equivalence $L \propto \mathcal{L}$ does not necessarily imply that the equilibrium postulate is true (see [10]). A few recent studies, focused on the scale equivalence assumption in spatially developing canonical turbulent flows [11–14], found substantial upstream regions where this assumption failed, yet had well defined scaling, which suggested that the flow structure was organized in some way.

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The turbulent boundary layer (TBL) and the turbulent pipe flow are by far the most intensely studied turbulent flows, because of their relevance to a plethora of industrial, environmental, and other engineering applications. It is therefore surprising that the available literature shows no apparent concern for the confirmation of the scale equivalence assumption; an exception is the observation that $L/\mathcal{L}$ at a fixed point on the centerline of a turbulent pipe flow depends on the inlet velocity [15]. We have therefore endeavored to examine the validity of this assumption and to explore the formulation of alternative relationships between k, ε, and L in a spatially developing TBL. In view of the enormous volume of published data, we had presumed that this would be a simple task. An exhaustive literature search, however, identified no publicly available sets of k, ε, and L values during the early stages of TBL development. In the present article, we consider a TBL under conditions that have been documented previously, but use data that requires additional processing by the corresponding original authors. The two cases we consider are a set of TBL results obtained by direct numerical simulation (DNS) at relatively low Reynolds numbers [16,17] and experimental data at higher Reynolds numbers [18].

The turbulence Reynolds number Re_λ is a dimensionless property that has been found to be a most appropriate measure of the local relative strength of a turbulent flow and suitable for comparing different parts of an evolving flow and flows with different geometries. While acknowledging that different definitions may be found in the literature, we will define this property as $\mathrm{Re}_\lambda = (2k/3)^{1/2}\lambda/\nu$, where λ is the Taylor microscale and ν is the kinematic viscosity of the fluid. Unlike L, which has no direct relationship with k and ε, λ is defined in terms of these parameters as $\lambda \propto \sqrt{\nu k/\varepsilon}$.

The length scale ratio $L/\mathcal{L}$ may be conveniently expressed as the nondimensional dissipation parameter

$$C_\varepsilon = \varepsilon L/(2k/3)^{3/2} \propto L/\mathcal{L}, \tag{1}$$

which clearly demonstrates that length scale equivalence [1,3–5,9,19,20] is tantamount to $C_\varepsilon =$ const. Now consider a canonical turbulent flow, which evolves in the streamwise direction and may or may not be homogeneous in a transverse plane. Far away from the origin, it is plausible to expect that all turbulence properties would evolve at commensurate rates so that $C_\varepsilon \approx$ const, at least along a mean streamline and possibly transversely as well. Before such a state is achieved, however, it seems likely that there would be some region where the flow structure has evolved sufficiently for partial self-similarity to hold, but where the evolutions of k, ε, and L may be lagging one another; then $C_\varepsilon \not\approx$ const, but depends on the local conditions, which are best expressed by Re_λ. A simple relationship, of a type that is common in similarity analyses, would be the power law

$$C_\varepsilon \propto \mathrm{Re}_\lambda^\alpha. \tag{2}$$

One may note that for $\alpha \approx 0$, Eq. (2) reverts to the common assumption of $C_\varepsilon =$ const. The dissipation parameter may be also expressed as $C_\varepsilon \propto (\lambda/L)^{-1}\mathrm{Re}_\lambda^{-1}$, from which one may derive an expression equivalent to Eq. (2) as

$$\lambda/L \propto \mathrm{Re}_\lambda^{-(1+\alpha)}. \tag{3}$$

Equation (2) with $\alpha \neq 0$ has been fitted successfully to significant regions of the few canonical flows that have been examined in this respect, with these flows eventually achieving states in which $\alpha \approx 0$. Decaying grid turbulence was found to have a region with $\alpha \approx -1.0$ [13]. Turbulent axisymmetric wakes were also found to have a region with $\alpha \approx -1.0$ when Re_λ was sufficiently large [11] and two distinct subregions with $\alpha \approx -0.8$ and -0.5, respectively, for lower Re_λ [12]. Most relevant to this work are the findings in uniformly sheared flows (USFs), which bear strong structural similarities with the outer regions of TBLs and contain hairpinlike structures as dominant eddies, much like outer TBLs [21]. In USF [14] the power law exponent took first the value $\alpha = -0.6$ and then changed sign, before finally settling to zero as the turbulence continued to evolve. It was suggested by the authors [14] that these scalings may be a consequence of the fact that large-scale structures tend to adjust to a change in flow conditions more slowly than motions with smaller scales.

II. CONSTANT-PRESSURE TURBULENT BOUNDARY LAYER DATA SETS

We now proceed to examine the evolution of C_ε in constant-pressure TBLs and the possible existence of regions within which a power law of the type in Eq. (2) may be fitted to the data. As it is customary in TBL studies, we identified the state of development of each flow by the value of the local Reynolds number $\mathrm{Re}_\theta = U_\infty \theta/\nu$, which is based on the freestream velocity U_∞ and the local momentum thickness θ. This parameter is equivalent to a dimensionless streamwise coordinate. Whereas the DNS study resolved all turbulence properties, including the TKE dissipation rate, at all scales and in the entire flow domain, the experimental study only provided measurements of the streamwise components of the TKE and its dissipation rate obtained by single hot-wire anemometers at specific locations. For consistency, we restricted our comparisons to properties that were adequately resolved in both studies. Wall-normal variation of properties are expressed in terms of the dimensionless distance from the wall y/δ, where δ is the corresponding 99% boundary layer thickness. Streamwise variation of properties are investigated along lines of constant y/δ, which are plausible loci of self-similar conditions; three such lines were considered, having $y/\delta = 0.30$, 0.50, and 0.70, which all fall within the outer boundary layer and are sufficiently distant from the average location of the turbulent-nonturbulent interface, so the results are not expected to be contaminated significantly by intermittency and freestream characteristics. The turbulent kinetic energy k was surrogated by its streamwise component $\overline{u^2} \equiv u'^2$; the Taylor microscale was estimated as $\lambda = u'/\overline{(\partial u/\partial x)^2}^{0.5}$, with the streamwise velocity derivative in the experimental study determined via Taylor's frozen flow approximation from measurements of the temporal velocity derivative; the turbulence Reynolds number and the TKE dissipation rate were consequently determined as $\mathrm{Re}_\lambda = \lambda u'/\nu$ and $\varepsilon = 15\nu u'^2/\lambda^2$; the integral length scale L was calculated by integrating the streamwise velocity autocorrelation function to its first zero crossing; finally, the dissipation parameter was evaluated as $C_\varepsilon = \varepsilon L/u'^3$.

The main objective of the DNS study we considered [16] was to map in detail the process of bypass transition in a boundary layer that was adjacent to a mildly turbulent freestream. Simulated grid-generated turbulence was injected in the freestream near the start of the boundary layer with an intensity of 3%, which decayed to 0.8% at the downstream end of the computational domain. The spatial resolutions in the streamwise and wall-normal directions were less than 2η for $y/\delta \geqslant 0.25$, where $\eta = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov microscale. Results were available in the range $80 \leqslant \mathrm{Re}_\theta \leqslant 3000$, but we only considered values at seven downstream locations, corresponding to $\mathrm{Re}_\theta = 670$, 1000, 1410, 1750, 1997, 2533, and 2892. The boundary layer thickness values at these locations were $\delta/\theta_0 = 84.65$, 117.39, 157.61, 190.00, 214.08, 265.36, and 298.71, where θ_0 was the inlet momentum thickness. Based on the skin friction coefficient values and on visualizations of the flow structure along the boundary layer, we may assert that the transition process was essentially completed upstream of the location with $\mathrm{Re}_\theta = 670$, so the results at all seven locations considered here were within the turbulent regime. Turbulent boundary layer characteristics in the DNS study were found to be in excellent agreement with experimental results at comparable Re_θ [22].

The experimental data considered in this work were collected in the High-Reynolds-Number Boundary Layer Wind Tunnel at the University of Melbourne, which has a 27-m-long working test section [18]. The freestream had a velocity of $U_\infty = 20$ m/s and a turbulence intensity of 0.05%. Measurements were taken with a 2.5-μm-diam, boundary-layer-type hot-wire probe, having a single sensor with a length of $l \approx 0.54$ mm, which achieved a spatial resolution of $l/\eta \leqslant 6$ for $y = 0.3\delta$, $l/\eta \leqslant 5$ for $y = 0.5\delta$, and $l/\eta \leqslant 4$ for $y = 0.7\delta$. It is noted that temporal resolution was also satisfied (see [18] for further details). Data were acquired at ten downstream locations, having $\mathrm{Re}_\theta = 7200$, 9300, 11 200, 12 700, 15 600, 17 200, 21 700, 26 100, 33 300, and 34 900; the corresponding TBL thickness were $\delta = 45.0$, 60.1, 72.8, 81.9, 102, 115, 145, 176, 229, and 242 mm.[1]

[1]The SP40 case in Ref. [18].

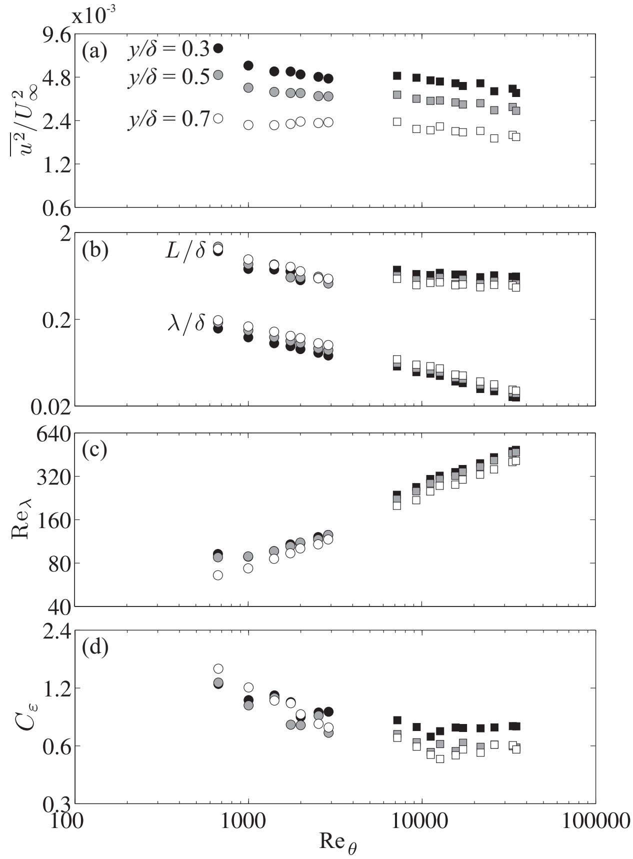

FIG. 1. Streamwise evolutions of turbulence parameters for the DNS (circles) and experimental data sets (squares). (a) Normalized variance of the streamwise turbulent velocity fluctuations, (b) normalized Taylor microscale and streamwise integral length scale, (c) turbulence Reynolds number, and (d) dissipation parameter. Symbols with black, gray, and white fill denote values at $y/\delta = 0.3$, 0.5, and 0.7, respectively.

III. RESULTS

Figure 1 shows the spatial evolutions of relevant turbulence properties in the outer TBL for both the DNS and the experimental data sets. All properties have been shown in logarithmic rather than linear axes, as the former are more suitable for displaying clearly the entire ranges of plotted data. One may readily notice that, despite the difference in freestream conditions and the gap in the corresponding streamwise ranges, all properties followed consistent trends from one data set to the other. The variance of the streamwise turbulent velocity fluctuations is seen to decrease with increasing streamwise distance as well as with increasing wall-normal distance. The Taylor microscale, normalized by the local boundary layer thickness, decreased at a relatively fast rate in the streamwise direction and increased at a much slower rate in the wall-normal direction. The normalized integral length scale also decreased in the streamwise direction, but had an inconsistent wall-normal trend for the DNS data, while the experimental values slowly decreased with distance from the wall. These results demonstrate that the boundary layer thickness grew faster than either of these two length scales. Another interesting observation is that the streamwise rate of decrease of λ/δ was constant in the entire range of data shown, while L/δ initially decreased at the same rate as λ/δ before markedly changing trend for $\mathrm{Re}_\theta \gtrsim 10\,000$. The resulting change in the evolution rate of the ratio λ/L possibly reflects a change in the energy cascade process [13].

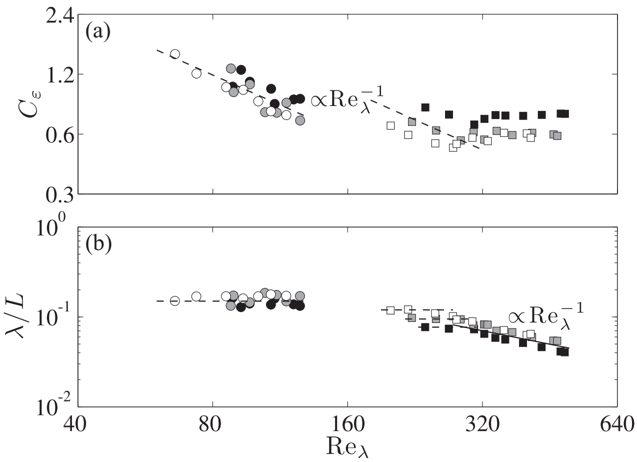

FIG. 2. Scaling of (a) the dissipation parameter and (b) the ratio λ/L with Re_λ for the DNS (circles) and experimental data sets (squares). Black, gray, and white symbols are for $y/\delta = 0.3$, 0.5, and 0.7, respectively.

We now turn our attention to the evolutions of Re_λ and C_ε, also shown in Fig. 1, so that we may explore the validity of Eq. (2). With the exception of two values at the most upstream DNS station (which may possibly carry a residual effect of transition), Re_λ increased monotonically and at the constant rate of $\mathrm{Re}_\lambda \propto \mathrm{Re}_\theta^{0.75}$. In contrast, C_ε decreased at a nearly constant rate for $\mathrm{Re}_\theta \lesssim 10\,000$ and then seemed to settle at a constant value, which depended on the normalized distance from the wall. These results permit no possibility for C_ε to be constant in the entire TBL, not even along a similarity line with a constant y/δ. Local scaling laws for this parameter are obviated by Fig. 2(a), which is a plot of C_ε vs Re_λ in logarithmic scales. Equation (2) describes well the data in two regions of the TBL: For $\mathrm{Re}_\theta \lesssim 10\,000$ ($\mathrm{Re}_\lambda \lesssim 300$), $C_\varepsilon \propto \mathrm{Re}_\lambda^{-1}$ (i.e., $\alpha = -1$), whereas for $\mathrm{Re}_\theta \gtrsim 10\,000$ ($\mathrm{Re}_\lambda \gtrsim 300$), $C_\varepsilon \approx$ const (i.e., $\alpha = 0$). The validity of these laws is confirmed by Fig. 2(b), which shows that λ/L was nearly constant for $\mathrm{Re}_\theta \lesssim 10\,000$ and inversely proportional to Re_λ for $\mathrm{Re}_\theta \gtrsim 10\,000$, in accordance with Eq. (3).

Figure 2(a) shows that the proportionality coefficients of the dissipation scaling laws varied along the wall-normal direction. These variations are hard to describe for the $\mathrm{Re}_\theta \lesssim 10\,000$ region (i.e., where $\alpha = -1$), but Fig. 3 clearly shows that for $\mathrm{Re}_\theta \gtrsim 10\,000$ (i.e., where $\alpha = 0$), C_ε was a function of y/δ only and independent of Re_θ. The data in the outer TBL ($0.1 \leqslant y/\delta \leqslant 1.0$) are described well by the quadratic function $C_\varepsilon = 1.85(y/\delta)^2 - 2.27(y/\delta) + 1.26$. This result is very significant

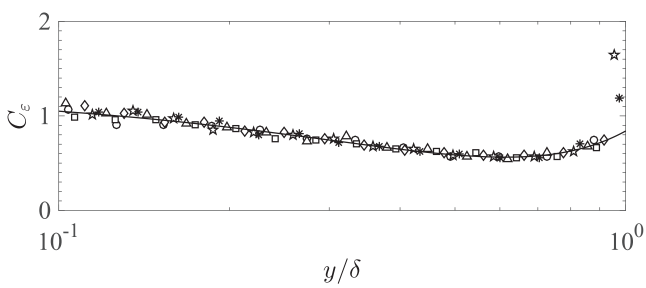

FIG. 3. Wall-normal variation of the dissipation parameter C_ε for $\mathrm{Re}_\theta = 12\,700$ (circles), 15 600 (squares), 17 200 (upward triangles), 21 700 (downward triangles), 26 100 (diamonds), 33 300 (asterisks), and 34 900 (stars); this figure contains values at many more y/δ locations than previous ones.

because it shows that there is no significant three-dimensional region within a TBL, within which C_ε may be considered to have a uniform value. Although, as demonstrated in the preceding paragraph, for $\mathrm{Re}_\theta \gtrsim 10\,000$ the length scale $\mathcal{L}$, which is obtained by dimensional analysis from k and ε, is indeed proportional to L, which is a measure of the physical size of the energy containing eddies, the proportionality coefficient varies by as much as nearly 50% in the wall-normal direction.

IV. CONCLUSION

We have shown explicitly that, far away from its origin ($\mathrm{Re}_\theta > 10\,000$), a constant-pressure TBL had an outer part in which the scales L and $\mathcal{L}$ were equivalent, although their ratio depended on the normalized distance from the wall. More importantly, we have also shown that a substantial upstream region of a TBL ($\mathrm{Re}_\theta < 10\,000$) had an outer part in which the dissipation parameter was inversely proportional to the turbulence Reynolds number. It is noteworthy that all previous DNSs of TBLs, as well as a large number of experimental studies, have been confined within the latter range of Re_θ. To dismiss the possibility that the presently found scaling is the result of viscous effects, we note that, even at the most upstream location we considered, the energy spectra from the DNS exhibited an identifiable inertial subrange with a slope close to $-5/3$.

The nonconstancy of the dissipation parameter in substantial regions of the TBL is consistent with similar findings in several other turbulent flows. It is again noted that the value of the exponent α in the TBL was different from that in USF, despite the strong structural similarities in these two flows. This observation leads to a conclusion that dissipation parameter scaling may not be universal, but rather depends on the conditions under which the dominant structures of a flow evolve towards a state of C_ε constancy.

The importance of the present work is significant, because it evaluates a cornerstone conjecture of turbulence analysis against direct analytical and experimental evidence in the TBL, arguably the most intensely studied turbulent flow. Many flows in nature and in engineering systems have TBLs, which can be expected to have a varying C_ε in their entirety or in extensive regions. Examples include (i) atmospheric boundary layers in urban and other complex domains; (ii) flows with heat convection in electrical and electronic components, nuclear reactor fuel channels, and industrial heat exchangers; (iii) flows past aircraft, ships, trains, and other vehicles; and (iv) cardiovascular and pulmonary flows. Modeling of such flows can benefit from the present findings, but much work would be necessary for the development and validation of future turbulence models that account for the nonconstancy of C_ε. In closing, we will highlight a simple example where this conjecture appears in turbulence modeling and how one may begin to improve the model. In doing so, we acknowledge that we have not tested this model and we do not claim it to be applicable to any specific system or suitable as an alternative general-purpose model.

Consider turbulence models that employ the eddy (turbulent) viscosity concept [5]. Such models define the eddy viscosity as

$$\nu_T \propto k^{1/2} L \tag{4}$$

and its modeling as $\nu_T = C_\mu k^2/\varepsilon$, where C_μ is an empirical constant. One may, however, rearrange the eddy viscosity definition in Eq. (4) by substituting L in terms of C_ε from Eq. (1) and then use Eq. (2) to eliminate C_ε so that the eddy viscosity appears in terms of k, ε, and the exponent α alone. The introduction of an empirical coefficient C_μ^*, which would need to be evaluated at conditions that would be relevant to a particular region of a flow, would then lead to a general eddy viscosity model as

$$\nu_T = C_\mu^* k^{(2+\alpha)} \varepsilon^{-(1+\alpha/2)} \nu^{-\alpha/2}. \tag{5}$$

For the case where $\alpha = 0$, this model would revert to the standard k-ε model, with the possible provision that wall-normal dependence of the turbulence structure would somehow be incorporated into the proportionality coefficient; for the region of a TBL where $\alpha = -1$, however, it would become $\nu_T = C_\mu^* k \varepsilon^{-0.5} \nu^{0.5}$.

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