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Dissipation in unsteady turbulence

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Recent experiments and simulations have shown that unsteady turbulent flows display a universal behavior at short and intermediate times, different from classical scaling relations. The origin of these observations is explained using a nonequilibrium correction to Kolmogorov's energy spectrum, and the exact form of the observed universal scaling is derived.

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I. INTRODUCTION

Taylor's dissipation rate estimate [1] and Kolmogorov's inertial-range scaling [2] are cornerstones of the description of turbulent flows. In recent experiments [3] it was observed that in a number of situations where Taylor's estimate is not valid, another universal expression fits the data, depending both on the local flow features and the initial conditions. In this Rapid Communication we will show that these observations can be explained using an unsteady correction to Kolmogorov's inertial-range spectrum.

Kolmogorov's concepts, introduced in the 1940's, state that the energy distribution among scales at sufficiently high Reynolds number is completely determined by the scale size and the energy flux through scales, for scale sizes sufficiently small compared to the most energetic eddies, and sufficiently large compared to the smallest, dissipative scales. In this range the energy spectrum is approximately given by the relation

$$E(\kappa,t) = C_K \epsilon(t)^{2/3} \kappa^{-5/3}, \tag{1}$$

where $\epsilon(t)$ is the average dissipation rate, κ the wave number, and $C_K \approx 1.5$ a constant. This relation is observed, to a good approximation, in a wide range of turbulent flows. Taylor's dissipation rate estimate,

$$\epsilon(t) = C_\epsilon \frac{U(t)^3}{L(t)}, \tag{2}$$

relates the dissipation rate, which is in principle a small-scale quantity, to the dynamics of the large-scale quantities U, the rms velocity, and L the integral length scale [4,5]. The insight that the dissipation can be modeled using large-scale quantities allows for the formulation of simple engineering models that need not take into account the multiscale character of turbulence. Both relations are intimately related [6,7] and an estimate of the constant C_ϵ can be obtained using relation (1) (details are given below). The quantity C_ϵ can be expressed as a function of two distinct Reynolds numbers, through the relation

$$C_\epsilon \sim \frac{R_L(t)}{R_\lambda(t)^2}, \tag{3}$$

where

$$R_L(t) = \frac{U(t)L(t)}{\nu}, \quad R_\lambda(t) = \sqrt{15\frac{U(t)^4}{\nu\epsilon(t)}}, \tag{4}$$

where it can be noted that C_ϵ is independent of the viscosity ν.

Recent experimental studies at Imperial College London, considering decaying wind-tunnel turbulence behind different types of turbulence-generating grids [8–11], have focused on both the regions of the flow near the grid, and farther away from it. Their results seem to show that far away from the grid, if the initial Reynolds number is large enough, the classic result (2) is obtained. However, in an adjacent region, closer to the grid, another seemingly universal law is observed,

$$C_\epsilon \sim \frac{\sqrt{R_L(0)}}{R_\lambda(t)}, \tag{5}$$

where $R_L(0)$ is determined by the initial conditions. Other research groups confirmed the results in independent grid-turbulence experiments [12–14] and direct numerical simulations (DNS) [15]. The scaling observed in these experiments seems more general than the case of freely decaying grid turbulence only, since experiments and simulations of the wakes generated by plates with both regular and irregular edges show the same tendency [16–18]. Recently, it was shown that in yet another different type of turbulent flow, where the kinetic energy is maintained at a certain level through an external forcing, the fluctuations of kinetic energy and dissipation around the long-time-averaged state can be described by the same law [19].

It is noted that expression (5) is radically different from (3), since expression (5) depends on the initial conditions and the local flow properties, whereas (3) only involves local quantities. Since, as stated before, C_ϵ can be related to Kolmogorov's energy spectrum (1), (5) might suggest a departure from (1) during the transient, but this is not observed. Our analysis explains these puzzling results. In particular, it is shown that the observation of (5) is related to a subdominant correction to Kolmogorov's energy spectrum first proposed in Ref. [20]. In the next section we reproduce a simple derivation of this nonequilibrium correction. In Sec. III, the correction to the dissipation rate estimate is determined. In Sec. IV, the derived relations are compared to existing experimental and numerical results. Section V concludes this paper.

II. DERIVATION OF THE NONEQUILIBRIUM ENERGY SPECTRUM

We reproduce here the simplest possible derivation of the nonequilibrium correction to the energy spectrum. The same results were obtained by Refs. [21,22] using similarity arguments, using Kovaznay's closure in Ref. [23], and using more sophisticated closures in Refs. [20,24].

We start from the evolution equation for the kinetic energy spectrum at high Reynolds numbers at scales where both production and dissipation mechanisms can be neglected,

$$\partial_t E(\kappa,t) = -\partial_\kappa \Pi(\kappa,t), \tag{6}$$

where $\Pi(\kappa,t)$ is a flux of energy which should vanish at $\kappa = 0$ and $\kappa = \infty$. This relation states that in a steady state, where the left-hand side vanishes, in the inertial range, the flux is conserved (and thus independent of κ), so that the right-hand side also vanishes.

We make the assumption that we can decompose the energy spectrum into its *equilibrium* and *nonequilibrium* parts,

$$E(\kappa,t) = \overline{E}(\kappa,t) + \widetilde{E}(\kappa,t). \tag{7}$$

It is extremely important for the following to note that both parts are a function of time and that this is not a separation of the energy distribution in a steady and an unsteady part. The determination of the equilibrium part of the flow will be discussed in Sec. IV A. For the moment we will content ourselves by defining the equilibrium part of the turbulence as the part for which the flux is not a function of scale $\overline{\Pi}(\kappa,t) = \epsilon(t)$ and therefore $\partial_\kappa(\overline{\Pi} + \widetilde{\Pi}) = \partial_\kappa \widetilde{\Pi}$, yielding for the evolution of the spectrum,

$$\partial_t \overline{E}(\kappa,t) = -\partial_\kappa \widetilde{\Pi}(\kappa,t), \tag{8}$$

where we focus on the analytically tractable case of small nonequilibrium, $|\partial_t \widetilde{E}(\kappa,t)| \ll |\partial_t \overline{E}(\kappa,t)|$. It is at this point that we need the introduction of an assumption on the functional form of the flux.

In the present derivation, we consider Kovaznay's model [25] for the flux,

$$\Pi(\kappa,t) = C_K^{-3/2}\kappa^{5/2}E(\kappa,t)^{3/2}. \tag{9}$$

The choice of this model will limit our considerations to the inertial-range interval of the energy distribution. More complicated closures would be needed to take into account a realistic infrared range for small κ, or more complex situations where anisotropy or inhomogeneity are present. Expression (9) immediately yields, when $\Pi(\kappa,t) = \epsilon(t)$, that $\overline{E}(\kappa,t)$ is given by

$$\overline{E}(\kappa,t) = C_K\epsilon(t)^{2/3}\kappa^{-5/3}. \tag{10}$$

Introducing (7) into (9) yields for small perturbations

$$\Pi(\kappa,t) = C_K^{-3/2}\kappa^{5/2}\overline{E}(\kappa,t)^{3/2}\left(1 + \frac{3}{2}\frac{\widetilde{E}(\kappa,t)}{\overline{E}(\kappa,t)}\right) = \epsilon(t)\left(1 + \frac{3}{2}\frac{\widetilde{E}(\kappa,t)}{\overline{E}(\kappa,t)}\right). \tag{11}$$

Substituting this into expression (8) gives upon integration

$$\widetilde{E}(\kappa,t) = C_K\Omega_\epsilon(t)\epsilon(t)^{1/3}\kappa^{-7/3}, \tag{12}$$

with

$$\Omega_\epsilon(t) = \frac{2C_K}{3}\frac{\dot{\epsilon}(t)}{\epsilon(t)}. \tag{13}$$

It is this new frequency Ω_ϵ in the dynamics which allows one to find the $k^{-7/3}$ scaling in Eq. (12) as a first linear correction to classical scaling, as for the shear-stress spectrum in homogeneous shear flow, where the mean-velocity gradient is introduced as the typical frequency [26]. The small parameter in our derivation is $\widetilde{E}(\kappa,t)/\overline{E}(\kappa,t)$. Combining the Kolmogorov scaling with (12), one finds that

$$\frac{\widetilde{E}(\kappa,t)}{\overline{E}(\kappa,t)} = \Omega_\epsilon(t)\epsilon(t)^{-1/3}\kappa^{-2/3}, \tag{14}$$

showing that the validity of the approximation should improve as the wave number increases.

Both spectra (10) and (12) are a function of time. The equilibrium part describes thus not necessarily a steady state, and temporal fluctuations are therefore not purely described by (12), since if they are slow enough, they will have time to adapt to the equilibrium distribution (10). The observation of the nonequilibrium scaling (12) is not straightforward, since it is subdominant with respect to the Kolmogorov spectrum (10). Conditional averaging allows one, however, to extract the unsteady energy spectrum, as was illustrated in Ref. [23], where a clear $\kappa^{-7/3}$ wave-number spectrum was observed in a statistically steady turbulent flow simulation. The difference in wave-number scaling between $\overline{E}(\kappa,t)$ and $\widetilde{E}(\kappa,t)$ is the origin of the observations of a nonclassical, but universal, transient scaling of the dissipation rate. We will elaborate on that in the following. We note that the possible relevance of the spectrum suggested by Yoshizawa (12) to estimate the dissipation rate in unsteady turbulence was already mentioned in Ref. [27].

III. DERIVATION OF THE NEW DISSIPATION SCALING

To simplify the considerations we assume the two scalings (10) and (12) to hold in the wave-number interval between κ_0, the forcing scale, and κ_η, the Kolmogorov scale, given by

$$\kappa_\eta \sim \frac{\epsilon(t)^{1/4}}{\nu^{3/4}}. \tag{15}$$

Outside this interval the kinetic energy is assumed to be zero, for analytical convenience. We have also considered more complicated spectra adding a more realistic infrared energy range (as in Ref. [7]) and the results hereafter were shown to be robust. All quantities will be decomposed into their equilibrium part, indicated by an overbar, and their nonequilibrium part, indicated by an overtilde. For instance, $C_\epsilon = \overline{C}_\epsilon + \widetilde{C}_\epsilon$, $\epsilon = \overline{\epsilon} + \widetilde{\epsilon}$, etc. The equilibrium kinetic energy is computed by integrating expression (10),

$$\overline{k}(t) = \int \overline{E}(\kappa,t)d\kappa = (3/2)\overline{U}(t)^2, \tag{16}$$

and, similarly, the nonequilibrium energy is obtained from Eq. (12). The integral length scale is defined by

$$L(t) = \frac{3\pi}{4}\frac{\int \kappa^{-1}E(\kappa,t)d\kappa}{\int E(\kappa,t)d\kappa} \equiv \frac{3\pi}{4}\frac{\mathcal{I}(t)}{k(t)}, \tag{17}$$

where we introduced $\mathcal{I}(t) \equiv \int \kappa^{-1}E(\kappa,t)d\kappa$ for later convenience. The dissipation can be computed from the energy spectrum by

$$\epsilon(t) = 2\nu \int \kappa^2 E(\kappa,t)d\kappa, \tag{18}$$

where all integrals are evaluated on the interval $[\kappa_0,\kappa_\eta]$. Carrying out these integrals using (10) to evaluate $\overline{U}$ and $\overline{L}$ [expressions (16) and (17)], and substituting these expressions in Kolmogorov's and Taylor's expressions (1) and (2), it is immediately found that the equilibrium value of the normalized dissipation rate is

$$\overline{C}_\epsilon = \frac{3\pi}{10}C_K^{-3/2} \approx 0.51. \tag{19}$$

This value is thus the inertial-range estimate of C_ϵ, assuming a spectrum given by (1) on the interval $[\kappa_0,\kappa_\eta]$. Despite such gross assumptions on the shape of the energy spectrum, its value is actually close to the value observed in direct numerical simulations of forced high Reynolds numbers turbulence where values around 0.5 are observed [5]. In the following, we will omit the time dependence of the different quantities to lighten the notation. It should, however, be kept in mind, as we stressed before, that both the equilibrium and the nonequilibrium quantities can depend on time.

Since $C_\epsilon \sim \epsilon\mathcal{I}/k^{5/2}$, we can write without any approximations

$$\frac{C_\epsilon}{\overline{C}_\epsilon} = \frac{\left(1+\frac{\widetilde{\epsilon}}{\overline{\epsilon}}\right)\left(1+\frac{\widetilde{\mathcal{I}}}{\overline{\mathcal{I}}}\right)}{\left(1+\frac{\widetilde{k}}{\overline{k}}\right)^{5/2}}. \tag{20}$$

The different quantities in this expression are obtained by integrating expressions (16)–(18) over the interval $[\kappa_0,\kappa_\eta]$, using the spectra (10) and (12) for the equilibrium and nonequilibrium contributions, respectively. For instance, it is found that

$$\frac{\widetilde{\epsilon}}{\overline{\epsilon}} = \frac{2\Omega_\epsilon}{\epsilon^{1/3}\kappa_\eta^{2/3}} \quad \text{and} \quad \frac{\widetilde{k}}{\overline{k}} = \frac{\Omega_\epsilon}{2\epsilon^{1/3}\kappa_0^{2/3}}, \tag{21}$$

where we have assumed $\kappa_0 \ll \kappa_\eta$. Since in the equilibrium state $\kappa_0/\kappa_\eta \sim R_\lambda^{-3/2}$, we find that

$$\frac{\widetilde{\epsilon}}{\overline{\epsilon}} \sim R_\lambda^{-1}\frac{\widetilde{k}}{\overline{k}}, \tag{22}$$

which is a direct consequence of the $k^{-7/3}$ scaling of $\widetilde{E}(k)$. This shows that the $\widetilde{\epsilon}/\overline{\epsilon}$ term in Eq. (20) is negligible. This indicates also that the temporal dissipation rate fluctuations observed in Refs. [19,27] are mainly related to the equilibrium distribution $\overline{E}(k,t)$ of the flow and negligibly contribute to the nonequilibrium part of the dissipation rate. We further find that $\widetilde{\mathcal{I}}/\overline{\mathcal{I}} = (10/7)\widetilde{k}/\overline{k}$. The expression for large Reynolds numbers is therefore

$$\frac{C_\epsilon}{\overline{C}_\epsilon} \approx \frac{\left(1 + \frac{10}{7}\frac{\widetilde{k}}{\overline{k}}\right)}{\left(1 + \frac{\widetilde{k}}{\overline{k}}\right)^{5/2}}. \tag{23}$$

Evaluating the Reynolds number, one finds analogously

$$\frac{R_\lambda}{\overline{R}_\lambda} \approx \left(1 + \frac{\widetilde{k}}{\overline{k}}\right), \tag{24}$$

where $\overline{R}_\lambda$ is given by (4) using the equilibrium values $\overline{U}$ and $\overline{\epsilon}$. To obtain these two expressions we have thus only assumed that the Reynolds number is high and that the energy spectrum can be represented by (10) and (12) between κ_0 and κ_η. We consider the case where $\widetilde{k}/\overline{k}$ is small, for which the nonequilibrium scaling (12) was derived, so that we can use a Taylor expansion to rewrite (23) as

$$\frac{C_\epsilon}{\overline{C}_\epsilon} \approx \left(1 + \frac{\widetilde{k}}{\overline{k}}\right)^{-15/14} = \left(\frac{R_\lambda}{\overline{R}_\lambda}\right)^{-15/14}. \tag{25}$$

and this is our prediction for the Reynolds number dependence of the normalized dissipation rate. To appreciate the similarity with the experimentally observed power law (5), one needs to realize that $\sqrt{R_L(0)} \sim \overline{R}_\lambda$ [combining expressions (2) and (4)] and that $\overline{C}_\epsilon$ is a constant, so that this expression can be rewritten as

$$C_\epsilon \sim \left(\frac{\sqrt{R_L(0)}}{R_\lambda(t)}\right)^{15/14}, \tag{26}$$

and we find to a good approximation expression (5). Indeed, the difference between (5) and (26) will in most cases be small enough to fall into experimental error bars or the convergence of statistical averages in simulations. We further mention here also that in the experimental and numerical investigations reported in Ref. [3], the possibility was left open that the exponents are not exactly, but only close to, the ones in expression (5).

IV. COMPARISON WITH EXISTING RESULTS

A. Determining the equilibrium state

At this point we will compare to existing results from the literature. A subtle point is how one can identify the equilibrium part of a flow. The quantities that we need to determine first are the equilibrium values $\overline{U}$, $\overline{L}$, and $\overline{\epsilon}$. We have considered here isotropic turbulence. For such flows the equilibrium state is the Kolmogorov constant flux state, where

$$\bar{\Pi}(\kappa,t) = \epsilon(t). \tag{27}$$

Such a state needs an energy input at large scales which is in equilibrium with the dissipation $\epsilon(t)$ at small scales. Probably the best approximation of a constant flux state can be obtained in DNS with an external forcing term. In practice, due to the finite size of the simulated domain, fluctuations of the energy input and the dissipation rate will always lead to a certain amount of imbalance. The time average of the energy injection will, however, balance the time-averaged energy dissipation, so that for such flows the equilibrium values of $\overline{U}$, $\overline{L}$, and $\overline{\epsilon}$ are obtained by time averaging.

The comparison of our prediction with existing experimental results on grid-generated turbulence in a wind tunnel is not straightforward since in the vicinity of the grid the turbulence is not statistically homogeneous, nor isotropic. It is in this production zone where the kinetic energy is injected into the flow by the shear layers generated by the wakes of the grid bars. The constant flux state, where the dissipation is in equilibrium with the production, corresponds to the point in the flow where the kinetic energy attains its maximum. At short distances beyond this point the equilibrium spectrum

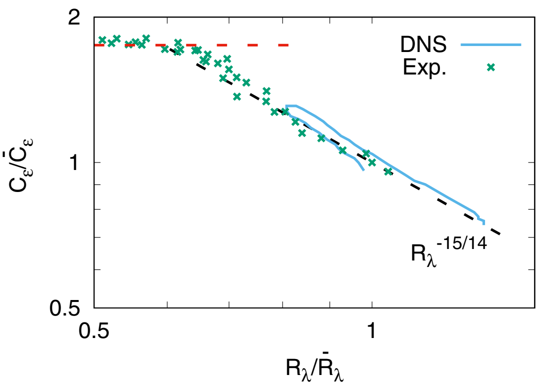

FIG. 1. The prediction of the normalized dissipation rate as a function of the Reynolds number ratio $R_\lambda/\overline{R}_\lambda$, expressions (25) and (29), compared to numerical [19] and experimental [10] results.

can be considered constant in time, compared to the nonequilibrium part. For larger times the equilibrium spectrum will evolve. We do not have access to experimental results for the equilibrium energy distribution at later times and we will therefore use the flow at the energy peak to estimate the equilibrium values of the different quantities in our comparison with experiment.

B. The dissipation scaling

As mentioned above, for a forced DNS in a statistically steady state the equilibrium is straightforwardly identified by time averaging. Furthermore, in a periodic domain, an instantaneous space average will tend to the same ensemble average, if the volume over which it is averaged contains a sufficient number of flow structures. In practice, this is never the case and temporal fluctuations will be observed around a long-time-averaged flow [19]. These box-averaged fluctuations are not necessarily in equilibrium and will thereby give rise to an evolution of C_ϵ. We have plotted in Fig. 1 the results of Fig. 3 of Ref. [19] for the fluctuations of C_ϵ around its average value for their highest Reynolds number ($700 < R_\lambda < 1000$) as a function of the ratio of the Reynolds number to its time-averaged value which we call $\overline{R}_\lambda$. It is observed that those results are in perfect agreement with our prediction.

We have also attempted a comparison with the experimental results reported in Ref. [10]. We have replotted in Fig. 1 the data from their Fig. 6, where the Reynolds number varies in the range $290 > R_\lambda > 111$. As argued above, we have considered their first data point, corresponding to the peak value of the kinetic energy, as the equilibrium state determining $\overline{R}_\lambda$. At this point a value of $C_\epsilon \approx 0.5$ is found for the equilibrium value of the normalized dissipation rate. It is observed that the experimental results, as the simulation, reproduce the theoretical prediction (25) exactly.

At this point, an open question is whether our analysis is relevant for decaying turbulence at long times, where C_ϵ settles to a constant value, different from its equilibrium value. We will consider the case where the kinetic energy decays following a power law $k = k_0(t/t_0)^{-n}$. The precise values of the reference quantities k_0 and t_0 are not important in the following. Deriving this expression for k twice to obtain expressions for ϵ and $\dot{\epsilon}$ gives $\dot{\epsilon}/\epsilon = -(n+1)/t$ and $\epsilon/k = n/t$. Using these relations, integrating Eqs. (10) and (12), and eliminating κ_0 from the expressions, it is immediately found that

$$\frac{\widetilde{k}}{\overline{k}} = -\frac{2}{9}\frac{n+1}{n}, \tag{28}$$

and therefore we find for the dissipation rate constant using (25),

$$\frac{C_\epsilon}{\overline{C}_\epsilon} \approx \left(\frac{9n}{7n-2}\right)^{15/14}. \tag{29}$$

The value of n is in general contained in the range $1 \leqslant n \leqslant 2$, leading to the ratio $1.8 \geqslant C_\epsilon/\overline{C}_\epsilon \geqslant 1.54$, which is a rather realistic range of values when compared to experiments and simulations [7]. In Fig. 1 we have added the asymptotic value $C_\epsilon/\overline{C}_\epsilon = 1.75$ corresponding to a decay exponent $n = 1.2$, typical for decaying grid-generated turbulence, which fits the long-time data accurately. Self-similar decay is therefore, in our framework, not an equilibrium, but a state where the disequilibrium is a constant fraction of the total kinetic energy, $\widetilde{k}(t)/\overline{k}(t) \neq f(t)$.

These ideas explain why in the experimental and numerical results in Ref. [3] the Reynolds number decays before Taylor's expression is observed. Indeed, the imbalance is not a low-Reynolds number effect and in the experiments and simulations the Reynolds number is in principle high enough to observe both Taylor's and Kolmogorov's scaling. However, the evolution of both R_λ and C_ϵ is a function of $\widetilde{k}/\overline{k}$. In turbulent flows in which the kinetic energy at long times decays following a power law, this latter quantity evolves from zero to a constant value, given by expression (28). The Reynolds number decays thus during the nonequilibrium transient from its initial value to a value

$$\frac{R_\lambda}{\overline{R}_\lambda} \approx \frac{7n-2}{9n}. \tag{30}$$

When this phase is attained and both R_λ and $\overline{R}_\lambda$ decay following power laws, this ratio remains constant.

C. Time evolution of turbulent length scales

Following the above arguments, we can also predict how the ratio of the integral to Taylor length scale evolves during the nonequilibrium phase. The Taylor scale is given by

$$\lambda = \sqrt{\frac{10\nu k}{\epsilon}}. \tag{31}$$

Combining this relation with the definition (17) for L, we find using the same arguments as for C_ϵ, that

$$\frac{\lambda/L}{\overline{\lambda/L}} = \left(\frac{R_\lambda}{\overline{R}_\lambda}\right)^{1/14}. \tag{32}$$

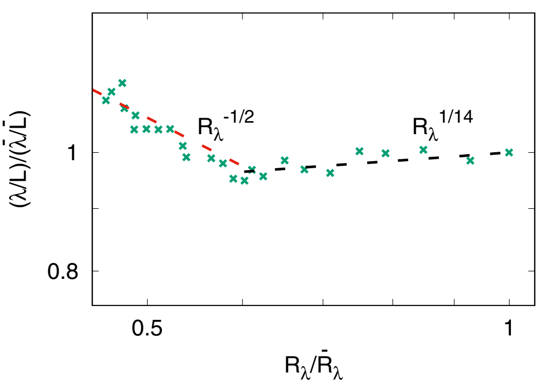

FIG. 2. The prediction of the ratio of the Taylor scale to the integral scale on the Reynolds number ratio $R_\lambda/\overline{R}_\lambda$, compared to experimental [10] results.

This small value of the exponent explains why it was observed that the length-scale ratio in the experiments remained approximately constant in the region of the flows where the nonequilibrium scaling was observed. In Fig. 2 it is observed that this power-law accurately describes the data. When the Reynolds number drops to the value corresponding to expression (30), the classical dependence is retrieved, where

$$\frac{\lambda/L}{\overline{\lambda/L}} = \left(\frac{R_\lambda}{\overline{R_\lambda}}\right)^{-1/2}. \tag{33}$$

V. CONCLUSION

We have presented a simple framework which allows one to interpret the nonequilibrium scaling observed in practically all the experiments and simulations mentioned in Ref. [3]. The present analysis is important for the modeling and understanding of turbulent flows since the nonequilibrium transient can be long and in many situations a self-similar decay might not even be reached before the flow is perturbed by the influence of boundaries, or because the Reynolds number has decayed too much for (1) and (2) to be valid. Given the agreement with experiments and simulations, the analytical results from the present investigation suggest that the normalized dissipation in a wide class of unsteady turbulent flows can be described by the same, fairly simple, relations.

Expression (25) constitutes the main result of the present work. However, it is not the exact value of the exponent, which is close to the experimental observations, that is of interest. Indeed, its precise value can change slightly as a function of the detailed shape of the energy spectrum. We have checked this by assuming more realistic shapes for the energy containing range, and the results are robust, but the power-law exponent can somewhat change. What is of greater importance is that the foregoing analysis gives a firm theoretical basis for the transient behavior of turbulent flows. The only nontrivial ingredient in the derivation is the shape of the unsteady energy spectrum $E(\kappa,t)$ [expression (12)]. The present analysis complements thereby recent investigations, suggesting that spectral imbalance [27,28] and large-scale coherence [29] are behind the universal scaling of C_ϵ in nonequilibrium turbulence.

Since this, rather simple, framework for unsteady turbulence allows one to explain practically all the experimental observations in the transient, unsteady phase of developing turbulent flows [3], it is plausible that engineering models can be improved by taking these ideas into account. We further think that the understanding of more complicated flows can greatly benefit from the insights obtained in this Rapid Communication. For this to be successful, the ideas developed here for isotropic turbulence should be extended to other configurations, such as shear flows and turbulent boundary layers. Defining an equilibrium flow for anisotropic and inhomogeneous flows is more delicate, but since the nonequilibrium scaling for the dissipation also describes the turbulent wakes of plates [16–18], we think that at least part of the present ideas can be transposed to more complicated flows.

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