Quantum thermal management devices based on strong coupling qubits

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We study the performance of a thermal management device with small scales by considering a strong coupling between quantum qubits. A small change of the thermal current at the base will cause a great change to the thermal current at the emitter and collector, reaching its promise for large thermal amplification. The competition between the quantum coherence and the incoherence induces a significant variation in the amplification factor and consequently relates the thermal controls with quantum effects. The results obtained here will provide a feasible scheme for the realization of quantum thermal management devices.

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I. INTRODUCTION

Quantum thermodynamics [1,2] is an interdisciplinary study of quantum mechanics and thermodynamics, which can be traced back to early quantum mechanics [3–5]. In recent years, significant advances in this field have been done, especially in the area of nonequilibrium thermodynamics [6–8] and quantum information [9,10]. Among them, quantum thermodynamics will not only help us study quantum effects in small systems, but also contribute to a deeper understanding of the relationship between classical and quantum physics [11,12].

As an important platform for studying thermal control [13,14], quantum devices have attracted wide attention, such as quantum heat engines and refrigerators [15–22]. These quantum mechanical models play a role similar to their classical analogs by generating and controlling heat currents. Among these, a model of the quantum thermal transistor consisting of three interacting subsystems was proposed in Ref. [23]. Quantum thermal transistors have the ability to control the thermal current of the collector and the emitter through the adjustments of the thermal current at the base [24–27].

The quantum thermal transistor is analogous to an electronic bipolar one with the properties of negative differential thermal resistance (NDTR) [26]. NDTR plays a very important role in the development of thermal transistors. Recent works show the advantages of NDTR properties. In Refs. [28,29], the authors built a model of thermal transistors based on the near-field radiation of the thermal photon. In Ref. [30], a thermal transistor was formed by quantum dots, where NDTR properties arose because of the Coulomb blockade. Reference [24] aimed to study the role of quantumness among the quantum thermal transistors. In the devices consisting of quantum systems, exploring the role of quantum properties is a fundamental topic. Although thermal transistors have been extensively studied, the study of quantum properties is not comprehensive enough, especially for the general thermal transistors with strong coupling qubits [31–33].

Interference between multiple transitions in nonequilibrium environments may give rise to nonvanishing steady quantum coherence. However, the role of quantum coherence in quantum thermal devices has not been addressed. In this paper, compared with Refs. [23,31], we design a different quantum thermal management device based on strong coupling qubits. We will show that quantum coherence [20,34–39] leads to NDTR and helps improve the thermal amplification effects. The thermal amplification effects are improved by adjusting the couplings among qubits and the dissipations between qubits and baths.

To clearly present the motivation behind our study and the main research results, the rest of paper is organized as follows. In Sec. II, we establish the model of quantum thermal management devices and drive its evolution equation. In Sec. III, we focus our attention on the thermal amplification effect induced by the quantum coherence. In Sec. IV, the appropriate parameter settings are required to obtain the thermal amplifiers with satisfactory performance. Finally, we draw our conclusions and set up the path to further investigations.

II. THE MODEL AND MASTER EQUATION

Figure 1 shows the model of a quantum thermal management device. It is made up of three qubits E, B, and C, which interact with three baths at temperatures T_E, T_B, and T_C ($T_E > T_B > T_C$), respectively. The base connected with qubit B is capable of regulating and controlling the heat currents of the emitter connected with qubit E and the collector connected with qubit C. The free Hamiltonian of the three qubits is

$$H_0 = H_E + H_B + H_C, \tag{1}$$

where $H_i = E_i\sigma_i^z/2$ $(i = E, B, C)$ with $\sigma_i^z = |0\rangle\langle 0| - |1\rangle\langle 1|$. When the interactions among qubits vanish, each qubit is in a thermal state $\tau_i = n_i|0\rangle\langle 0| + \bar{n}_i|1\rangle\langle 1|$, where $n_i = 1/[1 + \exp(\beta_i E_i)]$, $\bar{n}_i = 1 - n_i$, and the reciprocal of temperature $\beta_i = 1/T_i$. T_i is the temperature of bath i and the Boltzmann constant k_B is set to unity throughout the paper.

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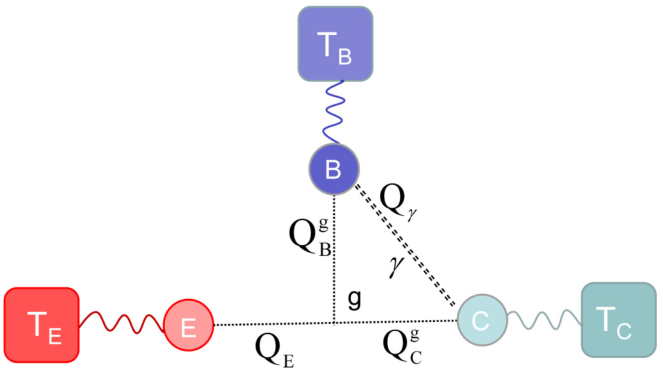

FIG. 1. The schematic diagram of a quantum thermal management device. The coherence and the heat current Q_{BC} are generated by a two-qubit interaction with strength γ between qubits B and C. Heat currents Q_E^g, Q_B^g, and Q_C^g are caused by the tripartite interaction with the weak interaction strength g.

Quantum coherence is built by introducing a two-qubit interaction,

$$H_\gamma = \gamma(\sigma_B^+\sigma_C^- + \sigma_B^-\sigma_C^+), \tag{2}$$

where $\sigma_i^+ = |0\rangle\langle 1|$ and $\sigma_i^- = |1\rangle\langle 0|$. The interaction coupling strength γ is assumed to be comparable with the free Hamiltonian ($\gamma \sim E_i$), and is much larger than the coupling between the qubit and the bath. Therefore, we assume that the dissipations do not destroy the eigenstate governed by the Hamiltonian $H_{BC} = H_B + H_C + H_\gamma$. Consequently, the dressed states of qubits B and C are used to describe the nonequilibrium steady-state statistics of the system.

The two eigenstates of H_{BC} with two-qubit coherence are $|\psi_{01}\rangle = \cos\frac{\theta}{2}|01\rangle + \sin\frac{\theta}{2}|10\rangle$ and $|\psi_{10}\rangle = \cos\frac{\theta}{2}|10\rangle - \sin\frac{\theta}{2}|01\rangle$ with $\theta = \arctan(2\gamma/\Delta E)$ and $\Delta E = E_C - E_B$. We choose these two eigenstates to build a virtual qubit v [22] and adjust the values of E_B, E_C, and γ to ensure that the virtual qubit v possesses the energy-level spacing $E_v = E_E$, which meets the requirements of the resonance between the qubit $\boldsymbol{E}$ and the virtual qubit v. The task of thermal management can be performed by an arbitrarily weak interaction which allows three qubits to resonantly exchange energy. The corresponding interaction Hamiltonian is given by

$$H_g = g(\sigma_E^+\sigma_v^- + \sigma_E^-\sigma_v^+), \tag{3}$$

where $\sigma_v^\pm$ are the raising and lowering operators of the virtual qubit v. Therefore, the total Hamiltonian of the three-qubit system is given by

$$H = H_E + H_B + H_C + H_\gamma + H_g. \tag{4}$$

The three-qubit system becomes irreversible due to the coupling between the qubit and the bath. Based on the Born-Markov approximation, which involves the assumptions that the environment is time independent and the environment correlations decay rapidly in comparison to the typical timescale of the system evolution, one can get the quantum dynamics of the system:

$$\dot{\rho} = -i[H,\rho] + D_E[\rho] + D_B[\rho] + D_C[\rho]. \tag{5}$$

The operators $D_i[\rho](i = E, B, C)$ denote the dissipative parts associated with the emitter, base, and collector, which have the form of

$$D_i[\rho] = \gamma_i(\varepsilon_i)\big[A_i(\varepsilon_i)\rho A_i^+(\varepsilon_i) - \tfrac{1}{2}\{\rho, A_i(\varepsilon_i)A_i^+(\varepsilon_i)\}\big], \tag{6}$$

where the jump operator $A_i(\varepsilon_i)$ is closely interrelated with the coupling between the system and the bath i and the transition energy level is ε_i. The decay rate $\gamma_i(\varepsilon_i) = P_i(\varepsilon_i)\bar{n}_i(\varepsilon_i)$ when $\varepsilon_i < 0$ and $\gamma_i(\varepsilon_i) = P_i(\varepsilon_i)n_i(\varepsilon_i)$ when $\varepsilon_i > 0$, where $P_i(\varepsilon_i)$ denotes the dissipation rate and is related to the spectral density of the bath. In the wide-band approximation, $P_i(\varepsilon_i) = P_i$, which is assumed to be independent of ε_i.

The steady-state populations of the open quantum system can be obtained by setting the left-hand side of Eq. (5) equal to zero, i.e., $\dot{\rho}_s = 0$. The steady-state energy currents are determined by the average energy going through the system,

$$\dot{E}(\infty) = \sum_{i=E,B,C} \mathrm{Tr}\{HD_i[\rho(\infty)]\} = Q_E + Q_B + Q_C = 0, \tag{7}$$

which is satisfied with the first law of thermodynamics. The heat fluxes Q_E, Q_B, and Q_C are defined with respect to their own dissipative operators, $Q_i = \mathrm{Tr}\{HD_i[\rho(\infty)]\}$. The heat current of the emitter is directly dependent on the three-body interaction and is given by

$$Q_E = -\tfrac{1}{4}gdE_E = Q_E^g, \tag{8}$$

where $d = \frac{48(\bar{n}_E\tilde{n}_C\bar{\tilde{n}}_B - n_E\bar{\tilde{n}}_C\tilde{n}_B)}{9P^2+(14+4\sum_{i\neq j}\Omega_{ij})g^2}Pg$, $\tilde{n}_v = \cos^2\frac{\theta}{2}n_{vv} + \sin^2\frac{\theta}{2}n_{v\mu}$ $(v,\mu = B, C)$, $n_{v\mu} = 1/[1+\exp(\varepsilon_v/T_\mu)]$, $\varepsilon_B = (E_C+E_B)/2 - \sqrt{(\Delta E/2)^2+\gamma^2}$, $\varepsilon_C = (E_C+E_B)/2 + \sqrt{(\Delta E/2)^2+\gamma^2}$, $\bar{\tilde{n}}_v = 1 - \tilde{n}_v$, $\Omega_{EC} = n_E\bar{\tilde{n}}_C + \bar{n}_E\tilde{n}_C$, $\Omega_{CB} = \tilde{n}_C\bar{\tilde{n}}_B + \tilde{n}_B\bar{\tilde{n}}_C$, $\Omega_{BE} = n_E\tilde{n}_B + \bar{n}_E\bar{\tilde{n}}_B$, and $P = P_E + P_B + P_C$. The heat current of the collector is given by

$$Q_C = -Q_\gamma + Q_C^g \tag{9}$$

and the heat current from the base is written as

$$Q_B = Q_\gamma - Q_B^g. \tag{10}$$

In Eqs. (9) and (10), each heat current has been divided into two categories. Q_γ is connected to the coherence generated by the internal coupling γ between the qubits B and C, i.e.,

$$Q_\gamma = P_BP_C\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2} * \left\{\frac{1}{P_B'}(n_{BB} - n_{BC})\varepsilon_B - \frac{1}{P_C'}(n_{CC} - n_{CB})\varepsilon_C\right\}, \tag{11}$$

where $P_B' = P_B\cos^2\frac{\theta}{2} + P_C\sin^2\frac{\theta}{2}$, $P_C' = P_C\cos^2\frac{\theta}{2} + P_B\sin^2\frac{\theta}{2}$. The heat current produced by the three-body interaction with the interaction strength g is given by

$$Q_{B/C}^g = \frac{1}{4}gdP_{B/C}\left(\frac{\varepsilon_{B/C}}{P_{B/C}'}\cos^2\frac{\theta}{2} - \frac{\varepsilon_{C/B}}{P_{C/B}'}\sin^2\frac{\theta}{2}\right). \tag{12}$$

At high temperature limits, the heat currents can be simplified into

$$Q_E \propto E_E[\Delta(\beta_E)E_E + \Delta(\beta_{BC})E\cos\theta], \quad (13)$$

$$Q_B \propto E_B[\Delta(\beta_E)E_E + \Delta(\beta_{BC})E\cos\theta] - \Delta(\beta_{BC})\frac{P}{8}\gamma^2\left(\frac{E^2}{\gamma^2+\frac{\Delta E^2}{4}}+1\right), \quad (14)$$

and

$$Q_C \propto E_C[\Delta(\beta_E)E_E + \Delta(\beta_{BC})E\cos\theta] + \Delta(\beta_{BC})\frac{P}{8}\gamma^2\left(\frac{E^2}{\gamma^2+\frac{\Delta E^2}{4}}+1\right), \quad (15)$$

where $E=(E_C+E_B)/2$, $\beta=(\beta_C+\beta_B)/2$, $\Delta(\beta_E)=\beta-\beta_E$, and $\Delta(\beta_{BC})=\beta_C-\beta_B$. Equations (13)–(15) show that at high temperatures, the heat transfer $[Q \propto \Delta(1/T)]$ obeys another linear law in irreversible thermodynamics [22,40–43].

The coherence of the qubit system can be measured by the nondiagonal elements of the virtual qubit v [22,34], i.e.,

$$C(\rho_v) = \left|\frac{\widetilde{n}_C - \widetilde{n}_B}{\widetilde{n}_C + \widetilde{n}_B - 2\widetilde{n}_C\widetilde{n}_B}\right| \sin\theta. \quad (16)$$

III. THE THERMAL AMLIFICATION EFFECT

A thermal management device behaves as a thermal transistor when it is capable of switching or amplifying the heat currents at the collector and the emitter via a small change in the base heat current. A thermal amplifier requires a high amplification factor $\alpha_{E/C}$, which is defined as the ratio of the change rate of the heat current at the emitter (collector) to that of the heat current at the base. The heat currents Q_E and Q_C are connected to Q_B, which can be adjusted by the base temperature T_B. The amplification factor $\alpha_{E/C}$ explicitly reads

$$\alpha_{E/C} = \frac{\partial Q_{E/C}}{\partial Q_B}. \quad (17)$$

The amplification factor $\alpha_{E/C}$ is a key parameter to determine the performance of the thermal management device. When $|\alpha_{E/C}| > 1$, a small change in Q_B stimulates a large variation in Q_E or Q_C. According to the conservation law of energy Eq. (7) and the definition in Eq. (17), we can prove that $\alpha_E + \alpha_C + 1 = 0$.

We consider the heat currents from the baths into the qubit system as positive. If T_E and T_C are fixed values and T_B is adjustable, the thermal conductances of the three terminals are defined as

$$\sigma_i = -\frac{\partial Q_i}{\partial T_B} = \sigma_{i\gamma} + \sigma_{ig}, \quad (18)$$

where $\sigma_{ij} = -\frac{\partial Q_{ij}}{\partial T_B}$ $(i = E, C, B; j = \gamma, g)$. $\sigma_{i\gamma}$ are the thermal conductances relying on the coherence effect due to the internal coupling γ, and σ_{ig} are the thermal conductances associated with the three-body interaction g. By using Eq. (18), the amplification factor can be rewritten as

$$\alpha_{E/C} = -\frac{\sigma_{E/C}}{\sigma_C + \sigma_E} = \frac{\sigma_{E/C}}{\sigma_B}. \quad (19)$$

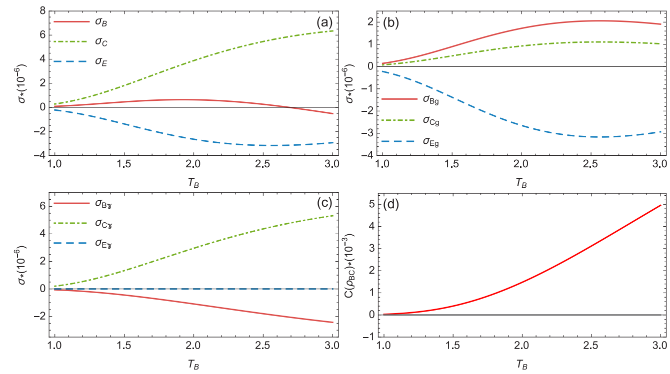

FIG. 2. The thermal conductances (a) σ_i, (b) σ_{ig}, (c) $\sigma_{i\gamma}$, and (d) the coherence between the qubits B and C as functions of the base temperature T_B. We choose the parameters $T_E = 10$, $T_C = 1$, $P_E = 0.01$, $P_B = 0.001$, $P_C = 0.0001$, $E_E = 6$, $E_B = 10$, $g = 0.01$, and $\gamma = 2.8$.

It indicates that $|\alpha_{E/C}| > 1$ when one of the thermal conductances is negative, i.e., $\sigma_C < 0$ or $\sigma_E < 0$. The system cannot play as a thermal amplifier unless negative differential thermal conductance exists.

We keep T_E and T_C invariant and plot the thermal conductances (a) σ_i, (b) σ_{ig}, (c) $\sigma_{i\gamma}$, and (d) the coherence between the qubits B and C as functions of the base temperature T_B in Fig. 2. As shown in Fig. 2(a), $|\sigma_B|$ is much smaller than σ_C and $|\sigma_E|$. The amplification effect exists and a small change of the base heat current $|Q_B|$ or the base temperature T_B is able to dramatically change the heat currents Q_E and $|Q_C|$ of the emitter and the collector.

As mentioned above, the thermal conductance can be divided into two separate parts. Figures 2(b) and 2(c) display the thermal conductances σ_{ig} and $\sigma_{i\gamma}$ varying with the base temperature T_B. The magnitude of σ_{Bg} is comparable with σ_{Cg} and σ_{Eg} and is a monotonically increasing function of T_B, meaning that it is unlikely to create an autonomous thermal amplifier with large amplification factors. For the two thermal conductances σ_{Bg} and $\sigma_{B\gamma}$ of the base, $\sigma_{Bg} > 0$ [Fig. 2(b)], whereas $\sigma_{B\gamma}$ originating from the coherence is negative [Fig. 2(c)], ensuring that we achieve a small σ_B [Fig. 2(a)]. Quantum coherence exists [Fig. 2(d)], allowing us to modify the thermodynamic behavior through the quantum control. Such a phenomenon makes large thermal amplifications possible.

The curves of the heat currents Q_i $(i = B, C, E)$, two different branches Q_B^g and Q_γ of the base heat current, base thermal conductance σ_B, and amplification factors α_E and α_C as functions of the base temperature T_B are illustrated in Fig. 3. It is observed from Fig. 3(a) that the base heat current $|Q_B|$ is significantly smaller than the emitter heat current Q_E and the collector heat current $|Q_C|$, and a small change of the heat current $|Q_B|$ corresponds to a large change of the heat currents Q_E and $|Q_C|$. This leads to a noticeable amplification effect for the heat currents Q_E and Q_C. Two different branches $-Q_B^g$ and Q_γ of the base heat current are shown in Fig. 3(b), which can explain the change of Q_B. Figures 3(c) and 3(d) show that when the base thermal conductance σ_B is zero, the amplification factors α_E and α_C diverge. It should be noted that the amplification effect in Fig. 3(d) is the result

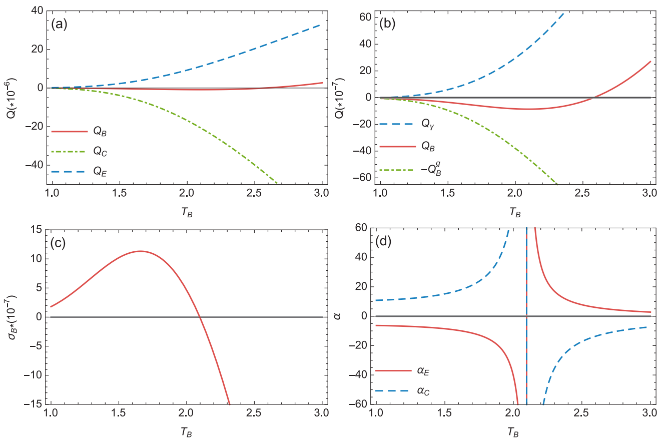

FIG. 3. (a) The heat currents Q_B, Q_C, and Q_E, (b) the two different branches $-Q_B^g$ and Q_γ of the base heat current and Q_B, (c) the base thermal conductances σ_B, and (d) the amplification factors α_E and α_C as functions of the base temperature T_B. We choose the parameters $T_E = 10$, $T_C = 1$, $P_E = 0.01$, $P_B = 0.001$, $P_C = 0.0001$, $E_E = 6$, $E_B = 10$, $g = 0.01$, and $\gamma = 2.5$.

of the negative differential thermal conductance, which can be explained qualitatively as follows: The base heat current Q_B comes from two different branches as seen from Eq. (10). One is the negative heat current $-Q_B^g$, which is through the internal interaction g with a given temperature difference $T_E - T_B$. The other part is the positive heat current Q_γ, which is through the quantum coherence operation (corresponding to the internal interaction γ) for a given temperature difference $T_B - T_C$. At first, there is no heat current between the base and the collector, i.e., $Q_\gamma = 0$, when the base temperature is the same as the collector temperature. However, the emitter has heat current flowing into the base by the internal interaction g, leading to a negative base heat current $-Q_B^g$. Thus, there exists the relation $Q_B = -Q_B^g < 0$. Then, as the base temperature increases, $|-Q_B^g|$ induced by the internal interaction g continues to increase due to negative differential thermal conductance, although the temperature difference $T_E - T_B$ is reduced. Consequently, the base heat current Q_B continues to decrease until Q_γ increases to a greater degree. At this point, Q_B reaches a minimum value. After that, Q_B increases with the increase of T_B. Specific analysis shows that the amplification factors α_E and α_C diverge and lead to an infinite amplification factor at $T_B \approx 2.1$, which is due to the fact that Q_B has a minimum value at this point. In other words, these phenomena can be understood with the fact that $\sigma_B = 0$ [as seen from Eq. (19)]. Under this condition, an infinitesimal change in $|Q_B|$ makes considerable differences in Q_E and $|Q_C|$.

IV. OPTIMIZATION OF THE AMPLIFICATION EFFECT

This type of quantum system allows for designing thermal amplifiers with appealing performance, but the appropriate parameter settings are required. Adjusting the base energy level E_B can directly control the thermal conductances. Figures 4(a) and 4(b) reveal the influence of the energy level of the base energy level E_B on the performance of the thermal amplification device. The curves of the amplification factors α_E and α_C as functions of E_B are illustrated in Fig. 4(b). It is

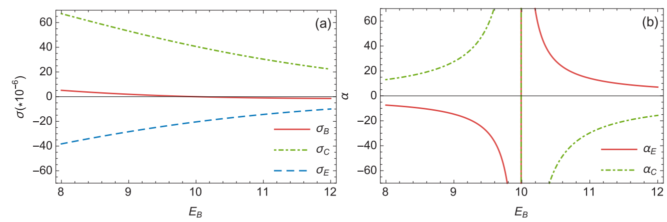

FIG. 4. (a) The thermal conductances σ_B, σ_C, and σ_E, and (b) the amplification factors α_E and α_C as functions of the base temperature T_B. The base temperature $T_B = 2.1$, and the values of other parameters are the same as those used in Fig. 3.

shown that $|\alpha_E|$ and α_C are larger than 1 and become infinite at the point of $E_B \approx 10$. Therefore, the thermal management devices exhibit a good amplification effect, which results from $\sigma_E < 0$ according to Eq. (19). As shown in Fig. 4(a), the thermal conductances $|\sigma_E|$ and σ_C of the emitter and the collector are significantly larger than the base thermal conductances $|\sigma_B|$, and σ_E is negative at the same time. This leads to the thermal amplification effect of the thermal amplification device. Noting that $\sigma_B \to 0$ at the point of $E_B \approx 10$, which induces the infinite amplification factors α_E and α_C.

Figure 5 reveals the influence of the decoherence rate P_B of the base and the strong coupling γ on the performance of the thermal amplification device. Figure 5(a) shows the amplification factors α_E and α_C as functions of P_B. It is found that the decrease of P_B can significantly increase the amplification factors α_E and α_C. Reducing P_B can effectively suppress the heat current Q_γ as indicated in Eq. (11). This results in a decrease of the base heat current Q_B, which is beneficial for us to obtain large amplification factors. Furthermore, reducing the decoherence rate P_B is convenient for us to induce the effect of the amplification.

On the contrary, increasing the strong coupling γ can strengthen the amplification effect of the thermal device. The curves of the amplification factors α_E and α_C as functions of γ are shown in Fig. 5(b). The quantum coherence induced by the strong coupling between qubits allows us to modify the thermodynamic behavior, which makes large thermal amplifications possible. Namely, increasing the strong coupling γ is advantageous for the thermal management device to play the role of amplifier. It is noted that the phenomena in Fig. 5 satisfy the requirement that the base heat current $|Q_B|$

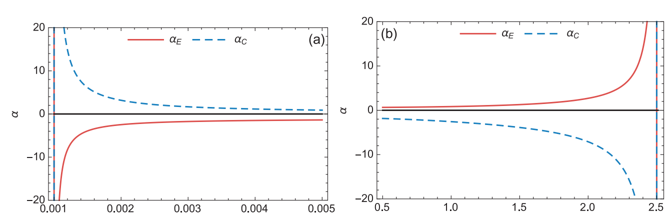

FIG. 5. The amplification factors α_E and α_C as functions of (a) the decoherence rate of the base P_B and (b) the strong coupling γ. When $P_B = 0.001$, an infinite amplification factor is obtained. At the point of $\gamma \approx 2.5$, an infinite amplification factor is observed. The base temperature $T_B = 2.1$, and the values of other parameters are the same as those used in Fig. 3.

is significantly smaller than the emitter heat current Q_E and the collector heat current $|Q_C|$, which is the expectation of a thermal management device.

It is noted that the parameters including T_i and E_i are set to be dimensionless throughout this paper. When the units of the parameters are given, the amplification factor becomes infinite for $T_B = 2$ mK and $E_B/\hbar = 10$ GHz. Such parametric values have been obtained in some reported experiments [44–46].

V. CONCLUSIONS

In summary, we propose a quantum thermal management device with a three-qubit model. The effects of the internal interaction and quantum coherence on the energy transfer processes are analyzed. Results show that quantum coherence gives rise to NTDR of the base and enables the thermal flow through the collector and emitter to be controlled by a small change in the heat current through the base. The effect of the thermal amplification can be significantly improved by adjusting the base energy level, reducing the base dissipation rate, and increasing the internal interaction. These results will stimulate interest for designing thermal management devices at nanoscale.

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