$\mathcal{PT}$-symmetric periodic structures with the modulation of the Kerr nonlinearity

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We study the $\mathcal{PT}$-symmetric periodic layered structure with the modulated Kerr nonlinearity. We demonstrate that such systems can be transformed from the full to the broken $\mathcal{PT}$ symmetry or in the opposite direction. These transitions in the periodic structure with a finite length may be observed indirectly with the help of the dependence of the transmitted wave intensity on the incident wave intensity. Furthermore, the bistability properties depend on a gain and loss value and the choice of an input surface. The reduction of a feedback between counterpropagating waves is the main impact of the complex $\mathcal{PT}$-symmetric refractive index variation on a bistability curve.

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I. INTRODUCTION

The concept of $\mathcal{PT}$ symmetry was considered for the first time during the investigation of quantum operator properties. It was shown that a non-Hermitian Hamiltonian may possess a real spectra, provided the Hamiltonian is symmetric with the successive action of the parity operator $\mathcal{P}$ and the time reversal operator $\mathcal{T}$. In quantum mechanics these operators are defined as $\mathcal{P}\psi(\boldsymbol{r},t)=\psi(-\boldsymbol{r},t)$ and $\mathcal{T}\psi(\boldsymbol{r},t)=\psi^*(\boldsymbol{r},-t)$, where $\psi(\boldsymbol{r},t)$ is a wave function [1]. The symmetry imposes the necessary condition on a complex potential $V(\boldsymbol{r})$ in the Schrödinger equation: $V(\boldsymbol{r})=V^*(-\boldsymbol{r})$.

The mathematical equivalence of the Schrödinger equation and the wave equation in paraxial approximation is the basis to propose optical $\mathcal{PT}$-symmetric systems [2,3]. In optics the dielectric permittivity is analogous to the complex potential and has the following $\mathcal{PT}$-symmetry condition: $\epsilon(\boldsymbol{r})=\epsilon^*(-\boldsymbol{r})$. The study of such optical systems is a rather attractive problem of modern photonics, since new features to be revealed may be used to control optical signals [4–7].

As an example of an optical $\mathcal{PT}$-symmetric system one should note a layered periodic structure, which possesses both the refractive index change and the balanced periodic variation of a gain and a loss along the propagation distance. Usually, the spatial distribution of the real part of the dielectric permittivity is approximated by an even harmonic function, while the imaginary part, by an odd one.

In the vicinity of the $\mathcal{PT}$-symmetric periodic structure band gap the interaction of forward and backward propagating waves has several peculiarities. Plane-wave transmittance through such active medium is nonreciprocal: the reflection from the opposite input surfaces is accompanied by either the backward wave absorption or its amplification [8]. When the gain and the loss exceeds some threshold value the $\mathcal{PT}$-symmetric system manifests one more useful feature—the possibility of lasing. For a semi-infinite periodic structure this threshold is proportional to the ratio of the real and the imaginary parts of the dielectric permittivity [9]. The $\mathcal{PT}$ symmetry is broken in this mode.

Nonlinearity presented in a periodic $\mathcal{PT}$-symmetric system complicates the interaction of an optical radiation with the periodic medium. One can mention some papers where the Kerr nonlinearity effect is considered [9–11]. In general, the case of the unbroken $\mathcal{PT}$ symmetry is discussed. For an example, the possibility of optical soliton formation and propagation is shown theoretically in [9]. Besides that, optical bistability can occur in the finite length active periodic structures [10,11]. The growth of switching intensity threshold with the increase of the gain and loss is reported here. The impact of the gain and loss saturation on bistability characteristics is studied as well.

In discussing the effects observed in nonlinear $\mathcal{PT}$-symmetric systems, one should necessarily mention the transitions between the states of the $\mathcal{PT}$ symmetry under the action of an intense light radiation. Such transitions have been already examined for the system of optical lattices [12]. The transitions between $\mathcal{PT}$-symmetry states in layered periodic structures have not been considered yet. This problem is discussed in the present paper provided the Kerr nonlinearity is varied periodically. The principal role of optical bistability which emerges in the system, allowing one to indirectly observe the transitions, is considered in detail.

To investigate the declared problems, we organize our paper as follows: In Sec. II we describe the system of the basic equations. The dispersion characteristics for different $\mathcal{PT}$-symmetric states and the lasing threshold for the finite length active structure are shown in Sec. III. Then, in Sec. IV bistability without and with a nonlinearity modulation is explored, and in Sec. V we present the conclusions.

II. BASIC EQUATIONS

Let us consider the propagation of an optical radiation in the finite length layered periodic dielectric structure with the following dependence of the refractive index n on the

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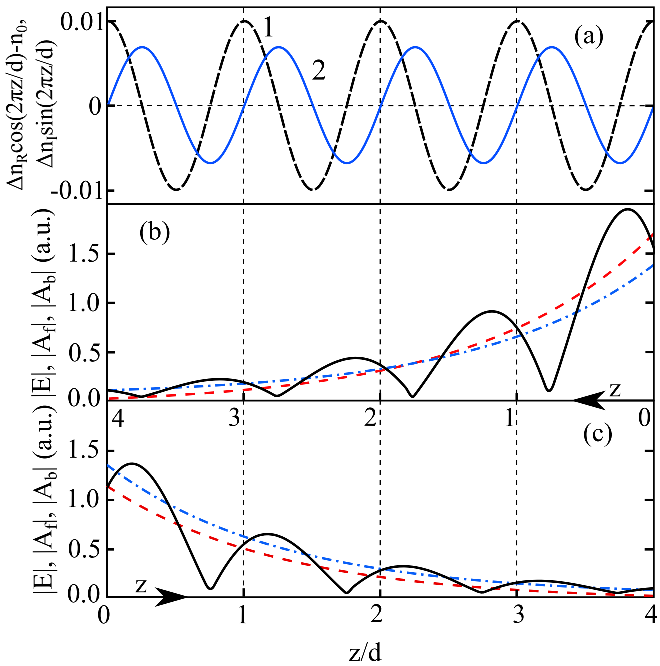

FIG. 1. (a) The spatial distribution of the real (1) and imaginary (2) parts of the refractive index in the $\mathcal{PT}$-symmetric periodic structures with $\Delta n_R > \Delta n_I$, $n_2 = \Delta n_2 = 0$. The distributions of the forward (dash-dotted), backward (dashed), and total optical field (solid) along coordinate z when light wave incidents on the right (b) and on the left (c) input surfaces. The carrier frequency of the light waves is in the vicinity of the first band gap. The arrows indicates the direction of the light propagation and the coordinate z.

coordinate z:

$$n(z) = n_0 + \Delta n_R \cos\left(\frac{2\pi}{d}z\right) \pm i\Delta n_I \sin\left(\frac{2\pi}{d}z\right) + n_2|E|^2 + \Delta n_2 \cos\left(\frac{2\pi}{d}z\right)|E|^2. \quad (1)$$

The first term in this expression corresponds to the average value of the refractive index of the medium n_0 while the following terms are small perturbations. The second one describes the refractive index variation along the medium with the period d [see Fig. 1(a), line 1]. The third term defines the periodic change of a gain and a loss [see Fig. 1(a), line 2]. The sign before Δn_I depends on the choice of an input surface. The incidence of a light radiation on the nominal left surface corresponds to the positive sign (z axis is directed to the right), and the negative sign is responsible for the nominal right incidence (z axis is directed to the left). The fourth and the fifth terms take into account the dependence of the refractive index on the intensity of the incident light radiation: n_2 denotes the average value of a nonlinear index, and Δn_2 refers to the value of a nonlinear index modulation along the periodic structure.

The variation of the refractive index (1) allows one to seek the solution of the appropriate wave equation as the superposition of the forward and backward propagating waves:

$$E(z,t) = A_f(z,t)\exp\left[i(k_0 z - \omega_0 t)\right] + A_b(z,t)\exp\left[-i(k_0 z + \omega_0 t)\right]. \quad (2)$$

Here ω_0 is the carrier frequency, and $k_0 = \pi/d$ is the propagation constant. The values $A_f(z,t)$, $A_b(z,t)$ are the slowly varying complex amplitudes of the forward and backward waves that satisfy the following system of equations:

$$+i\left(\frac{\partial A_f}{\partial z} + \frac{1}{v}\frac{\partial A_f}{\partial t}\right) + (\kappa \pm g)A_b + \gamma(|A_f|^2 + 2|A_b|^2)A_f + \xi(|A_b|^2 + 2|A_f|^2)A_b + \xi A_f^2 A_b^* = 0, \quad (3a)$$

$$-i\left(\frac{\partial A_b}{\partial z} - \frac{1}{v}\frac{\partial A_b}{\partial t}\right) + (\kappa \mp g)A_f + \gamma(|A_b|^2 + 2|A_f|^2)A_b + \xi(|A_f|^2 + 2|A_b|^2)A_f + \xi A_b^2 A_f^* = 0, \quad (3b)$$

where $v = c/n_0$ is the phase velocity of the incident optical wave, $\kappa = \Delta n_R\omega_0/c$ denotes the Bragg coupling parameter, $g = \Delta n_I\omega_0/c$ is the coupling coefficient arising from the imaginary part of the refractive index and is responsible for the $\mathcal{PT}$-symmetry state, and $\gamma = n_2\omega_0/c$ and $\xi = \Delta n_2\omega_0/c$ are the parameters that are responsible for self-phase modulation. The upper sign before the parameter g, that is responsible for the gain and loss, corresponds to the left incidence while the lower one coincides with the right incidence. This system of equations is derived as the expansion of the equations obtained in the paper [13] for a layered periodic structure with the modulated Kerr nonlinearity but without the $\mathcal{PT}$-symmetric term.

The first nonlinear term in the equations (3) determines the nonlinear shift of a band gap depending on the intensities of the forward and backward propagating waves. The meaning of the second nonlinear term becomes clear if one introduces an effective coupling parameter in the form $\overline{\kappa} = \kappa + \xi(|A_{f,b}|^2 + 2|A_{b,f}|^2)$. In fact, it means the change of the dielectric contrast and, consequently, the width of the band gap. It is caused by the nonuniform distribution by the optical field inside the nonlinear periodic medium. The growth of the radiation intensity leads to an increase or decrease of the band gap width in accordance with the nonlinearity type. The last term in (3) is responsible for the small change of the Bragg coupling, which depends on the phase difference between the counterpropagating waves.

III. DISPERSION RELATIONS AND LASING THRESHOLD

To consider the effect of the modulated nonlinear index on the dispersion characteristics, let us remember the key findings of the linear theory of periodic structures with the $\mathcal{PT}$-symmetric variation of the refractive index.

The nature of nonreciprocity lies in the mutual arrangement of the optical field maxima and the regions with gain and loss in the structure. The spatial distribution of the optical field formed by the superposition of the counterpropagating waves depends on which of the input surfaces light wave incidents. The Fig. 1(b) shows the spatial structure of the optical radiation that occurs when the light wave falls on the nominal right input surface of the periodic structure with $g < \kappa$, while Fig. 1(c) presents the incidence on the left input surface. The carrier frequency of the light waves is in the vicinity of the first band gap. For the right input surface the maxima of the total light field are located in regions with gain ($\Delta n_I < 0$) and, hence, both transmitted and reflected waves are amplified. On the contrary, for the nominal left incidence the optical field maxima are in layers with loss ($\Delta n_I > 0$). Therefore, the amplitude of the reflected radiation declines, resulting in a suppression of the interaction between the forward and backward waves. This, in turn, causes the growth of the transmitted wave amplitude. Such particular nonreciprocal propagation of the optical radiation in the medium strongly affects the dispersion of the periodic structure.

In a linear mode ($\gamma = 0,\ \xi = 0$) the solution in the form $A_f,\ A_b \sim A_{f0},\ A_{b0}\exp\left(i(qz - \Omega t)\right)$ is substituted in the system (3). Thus the following dispersion relation is obtained [9]:

$$q = \sqrt{\delta^2 - (\kappa^2 - g^2)}, \tag{4}$$

where $\delta = \Omega/\nu$ denotes the detuning from the Bragg resonance, and $\Omega = \omega_0 - \omega_B$, $\omega_B = \pi c/(n_0 d)$ is the Bragg frequency. When $g < \kappa$ the band gap width reduces but the band structure has essentially the shape of a passive periodic structure. The point when $g = \kappa$ is an exception ($\mathcal{PT}$-symmetry-breaking threshold). In this case the band gap vanishes and the dispersion curves for the forward and backward waves are identical to the dispersion curves expected for a homogeneous medium. Above the $\mathcal{PT}$-symmetric breaking threshold ($g > \kappa$) the area of real values of q appear; the latter corresponds to the complex values of δ. In this mode an absolute instability can develop and the unbounded growth of an initial disturbance takes place.

The finite length of a real periodic structure length should be taken into account to determine the threshold of the $\mathcal{PT}$ symmetry breaking. The self-excitation threshold in distributed feedback lasers can be found when the reflection and transmission coefficients go to infinity. The amplification of a light wave in such structures is uniform, and an excitation occurs at a frequency that differs from ω_B [14]. For the system in consideration, on the contrary, a lasing takes place when $\delta = 0$. In this case, the reflection coefficient of the structure with length L can be written as follows:

$$r = i\frac{\kappa + g}{\kappa - g}\tanh^2\sqrt{\kappa^2 - g^2}L. \tag{5}$$

The argument of the hyperbolic function becomes imaginary when $g > \kappa$, and the lasing condition comes down to equality $\cos\sqrt{\kappa^2 - g^2}L = 0$. Hence, the threshold value of the parameter g can be defined as

$$g_{th} = \sqrt{\kappa^2 + \left(\frac{\pi}{2L}\right)^2}. \tag{6}$$

Here the first term characterizes losses connected to reflection, and the second one determines losses arising from transmission. Thereby, there is a range of values $\kappa \leqslant g < g_{th}$ with the convective unstable system. When $g > g_{th}$, the system is absolutely unstable and the lasing occurs at the Bragg frequency.

Let us illustrate now the effect of the superimposed modulated nonlinearity on the dispersion characteristics of the $\mathcal{PT}$-symmetric periodic structure. We search the solution of the system of equations (3) in the stationary mode ($\partial/\partial t = 0$) in the form $A_f(z,t) = u_f \exp\left[i(qz - \Omega t)\right]$, $A_b(z,t) = u_b \exp\left[i(qz - \Omega t)\right]$. The parameter $f = u_b/u_f$ is introduced at the next step. It indicates how the total power $P_0 = u_f^2 + u_b^2$ is divided between the forward and backward waves. The constants u_f and u_b can be written in the form $u_f = [P_0/(1 + f^2)]^{1/2}$ and $u_b = f[P_0/(1 + f^2)]^{1/2}$ [15]. As a result, we obtain the following nonlinear dispersion relations:

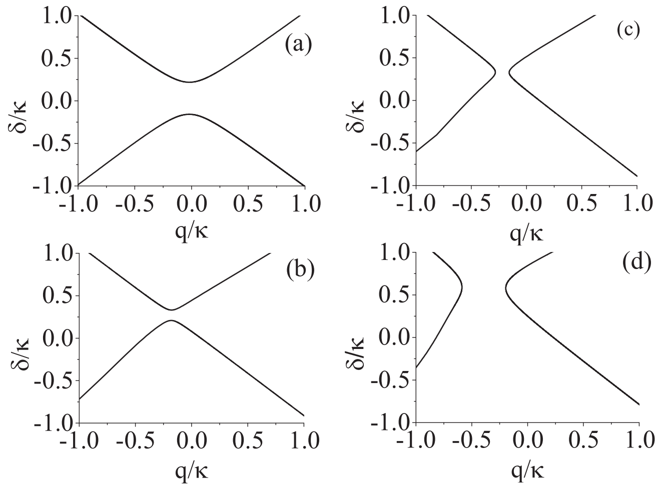

FIG. 2. The dispersion characteristics of the nonlinear $\mathcal{PT}$-symmetric periodic structure ($n_0 = 3.6$, $\kappa = 1000\ \text{m}^{-1}$, $g = 980\ \text{m}^{-1}$, $n_2 = 2 \times 10^{-17}\ \text{m}^2/\text{W}$, $\Delta n_2 = 2 \times 10^{-18}\ \text{m}^2/\text{W}$) depending on the incident radiation power: (a) $P_0 = 25\ \text{MW/cm}^2$, (b) $9P_0$, (c) $11P_0$, and (d) $20P_0$.

$$\delta = -\frac{\kappa(1+f^2)}{2f} - \frac{g(1-f^2)}{2f} - \frac{3\gamma P_0}{2} - \frac{\xi P_0(6f^2+f^4+1)}{2f(1+f^2)}, \tag{7a}$$

$$q = -\frac{(\kappa + \xi P_0)(1 - f^2)}{2f} - \frac{g(1 + f^2)}{2f} - \frac{\gamma P_0(1 - f^2)}{2(1 + f^2)}. \tag{7b}$$

In the case of the nonlinearity without modulation ($\gamma \neq 0,\ \xi = 0$), the relations (7) degraded to the expressions derived in [10]. It is noticed that such nonlinearity results in a shift of the band gap and the formation of a loop on the branch of the dispersion characteristics. The shift direction and the branch where the loop forms are determined by the Kerr nonlinearity type.

Figure 2 shows the dispersion curves for the layered structures with the modulated defocusing nonlinearity ($\gamma \neq 0,\ \xi \neq 0$) for the various input intensities. We took the parameters from the paper [11] as the basis for our calculation. The specific values of the parameters are as follows: $n_0 = 3.6$, $\kappa = 1000\ \text{m}^{-1}$, $g = 980\ \text{m}^{-1}$, $n_2 = 2 \times 10^{-17}\ \text{m}^2/\text{W}$, $\Delta n_2 = 2 \times 10^{-18}\ \text{m}^2/\text{W}$.

Figure 2(a) illustrates the absence of any distortions and shifts for the input intensity $P_0 = 25\ \text{MW/cm}^2$. The increase of the incident power up to $9P_0$ results in a shift of the band gap and reduction of its width [see Fig. 2(b)]. Then when the input power exceeds some certain critical value, the $\mathcal{PT}$ symmetry breaking takes place [see Fig. 2(c)]. It was found from the numerical calculations that the critical power can be approximated by linear dependence: $\xi P_0 = \kappa - g$. Further growth of the input power up to $20P_0$ is accompanied by an increase of the distance between the dispersion curves [see Fig. 2(d)].

In the structure with the modulated focusing nonlinearity ($\gamma > 0,\ \xi > 0$) a high intensive optical wave, on the contrary, allows one to put the system out of the initially broken $\mathcal{PT}$ symmetry and shift the band gap to a low-frequency area.

The transitions between the $\mathcal{PT}$-symmetric states are related to the Kerr nonlinearity modulation in a periodic structure. It changes the refractive index under intense optical radiation and, consequently, the effective coupling between the counterpropagating waves.

IV. BISTABILITY

To examine optical bistability, let us return to the finite length $\mathcal{PT}$-symmetric periodic structure. To proceed, we integrate the system (3) in the stationary mode with the help of the Runge-Kutta method. The following boundary conditions are set: $A_f(L) = A_0$, $A_b(L) = 0$, where A_0 is the amplitude of the transmitted wave. In these numerical calculations the parameters chosen in the previous section are used as well. The length of the structure is $L \approx 7$ mm.

The basic peculiarities of bistability depend on the input surfaces and the different ratios between parameters g and κ. These peculiarities can be explained rather properly with the transfer characteristics of the structure without the nonlinearity modulation. In this mode the coupling of the counterpropagating waves is fully determined by the ratio between g and κ [see Eqs. (3)]. At the first stage of the calculation the frequency of the incident light wave coincides with the Bragg frequency ($\delta = 0$). It is obvious that bistability patterns in this case are similar for the focusing and defocusing nonlinearity types. Therefore one may limit the consideration with one type only. Figures 3 and 4 illustrate the case of defocusing nonlinearity ($\gamma < 0,\ \xi = 0$).

Figure 3 shows the dependence of the intensity of the transmitted radiation (I_{tr}) on the intensity of the incident radiation (I_{inc}). When $g < \kappa$, the transfer characteristics possess the multistability inherent for passive periodic structures. The $\mathcal{PT}$-symmetric variation of the refractive index influences the optical bistability by the suppression of the feedback between the counterpropagating waves. This effect is illustrated by curves 1 and 2, which clearly show the reduction of the hysteresis loops.

The feedback in a passive periodic structure depends on the coupling parameter κ only. Taking into account the amplification and absorption properties of the $\mathcal{PT}$-symmetric structure, with the help of the parameter g it leads to the decrease of the feedback between the counterpropagating waves. As it has been mentioned already, the total field in the $\mathcal{PT}$-symmetric structure has different distributions for the left and right incidences. When light falls on the right input surface, the amplification partially diminishes the coupling of the forward wave with the backward one and, consequently, the feedback between them. An opposite situation takes place when light wave incidents on the left input surface. Here, the absorption increases the coupling of the forward wave with the backward wave. This fact induces the differences between the transfer characteristics for these input surfaces. However, due to the backward wave absorption for the left incidence, the hysteresis loop reduces as well.

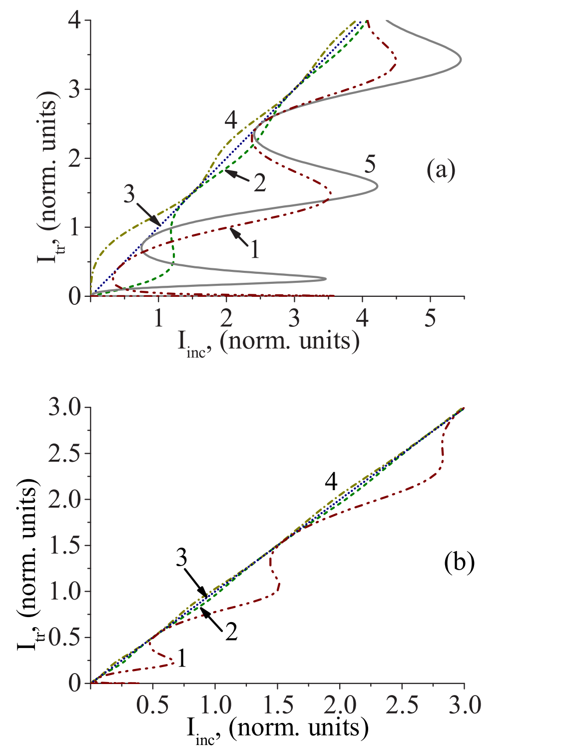

FIG. 3. The transfer characteristics when light wave incidents on the left (a) and right (b) input surfaces of the $\mathcal{PT}$-symmetric periodic structure without nonlinearity modulation ($\gamma < 0,\ \xi = 0,\ \delta/\kappa = 0$): (1) $g/\kappa = 0.8$, (2) 0.98, (3) 1, (4) 1.02, and (5) 0.

The transfer characteristics for the periodic structure with $g = \kappa$ (the exceptional point) are linear functions ($I_{tr} = I_{inc}$) regardless of the choice of input surfaces. When a light wave incidents on the right input surface, the amplification of the forward wave fully compensates the process of energy loss. Thus, the feedback between the counterpropagating waves is absent. In the case of the left incidence, the backward wave is absorbed completely and feedback also does not form. Consequently, in both cases the transmission of the forward

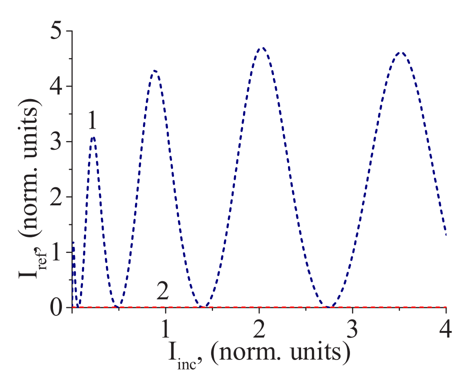

FIG. 4. The dependence of the reflected wave intensity on the incident wave intensity when light waves fall on the right (1) and left (2) input surfaces of the $\mathcal{PT}$-symmetric periodic structure without nonlinearity modulation ($\gamma < 0,\ \xi = 0,\ \delta/\kappa = 0,\ g/\kappa = 1$).

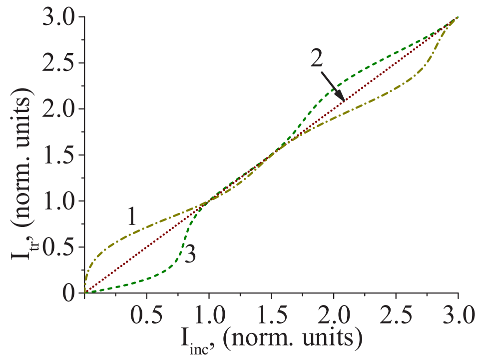

FIG. 5. The transfer characteristics when light wave incidents on the left input surfaces of the $\mathcal{PT}$-symmetric periodic structure with nonlinearity modulation (1) $\gamma > 0,\ \xi > 0,\ \delta/\kappa = 0,\ g/\kappa = 1.02$; (2) reference line; and (3) $\gamma < 0,\ \xi < 0,\ \delta/\kappa = 0,\ g/\kappa = 0.98$.

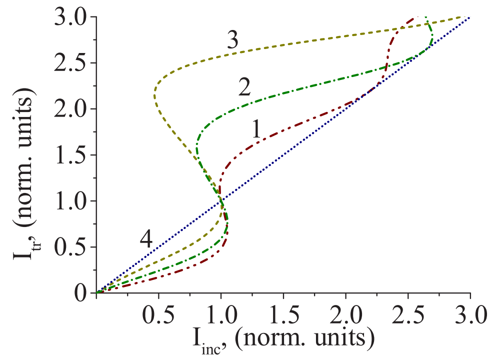

FIG. 6. The transfer characteristics when light wave incidents on the left input surfaces of the $\mathcal{PT}$-symmetric periodic structure with modulated defocusing nonlinearity ($\gamma < 0,\ \xi < 0,\ g/\kappa = 0.98$): (1) $\delta/\kappa = 0.2$, (2) 0.3, (3) 0.4, and (4) reference line.

wave is similar to the transmission through a homogeneous medium. Henceforth, the line $I_{tr} = I_{inc}$ [see Figs. 3(a) and 3(b), line 3] is used as a reference to divide the areas with $g > \kappa$ and $g < \kappa$.

Another important peculiarity of the $\mathcal{PT}$-symmetric state with $g/\kappa = 1$ is a nonreciprocal reflection of the radiation from the different inputs of the periodic medium. The right incidence is accompanied by the reflected wave amplification (see Fig. 4, curve 1). There is no reflected wave when light falls on the left input surface (see Fig. 4, line 2).

The lasing does not allow consideration of the transfer characteristics when the $\mathcal{PT}$ symmetry is broken. Therefore, the bistability is analyzed below the threshold with parameter g assigned in the range $\kappa < g < g_{th}$. In this range, for both input surfaces the gain of an optical radiation prevails over the losses related to the reflection and partially over the transmission losses. As a result, the transfer characteristics are above the reference line [see Figs. 3(a) and 3(b), curve 4] and the feedback between waves reappears. The $\mathcal{PT}$-symmetric periodic structures with the focusing nonlinearity ($\gamma > 0,\ \xi = 0$) have the analogous peculiarities of the transfer characteristics.

At the next step, let us consider the bistability of the periodic structures with the modulated nonlinearity. As it has been noticed earlier, the transitions between the $\mathcal{PT}$-symmetric states induced by an intense optical radiation are characteristic for such structures. The adiabatic increase of a radiation intensity allows one to observe indirectly the change of the $\mathcal{PT}$-symmetry states.

The transfer characteristics for the focusing and defocusing nonlinearity at the left incidence are depicted in Fig. 5. Curve 1 corresponds to the focusing nonlinearity ($\gamma > 0,\ \xi > 0$) and below-threshold mode ($g/\kappa = 1.02$). It lies above the reference line provided there are small values of I_{tr}. With the growth of the power, this curve crosses the reference line and further locates in the area where $g < \kappa$, which indicates the transition between the states of the system. There is an opposite situation when nonlinearity is defocusing ($\gamma < 0,\ \xi < 0$): we observe the transition from the area with $g < \kappa$ to the area with $g > \kappa$ which is induced by intense light radiation (see Fig. 5, curve 3).

In the latter case the lasing at $\delta = 0$ is not supported due to the band gap shift [see Figs. 2(c) and 2(d)]. There is only a convective instability. To reveal the change in the band structure, detuning from the Bragg resonance should be applied. The transfer characteristics of Fig. 6 show the amplification growth of the transmitted optical signal with the increase of δ. Such behavior is expected when one approaches the absolute instability area. Thus, it confirms the $\mathcal{PT}$-symmetry breaking illustrated by Fig. 2.

The transitions between the $\mathcal{PT}$-symmetry states induced by nonlinearity also take place when a light wave falls on the right input surface. These transitions are faint in comparison with the left incidence because of the strong suppression of the feedback.

V. CONCLUSION

In the present paper we considered the propagation of an optical light wave in the $\mathcal{PT}$-symmetric structure with the additional periodic modulation of the Kerr nonlinearity. In such structures the change of a refractive index by an intense optical radiation causes the change of the $\mathcal{PT}$-symmetry-breaking threshold while the gain and loss value is constant. As a result, the system can move from the full $\mathcal{PT}$-symmetry state to the broken $\mathcal{PT}$-symmetry state or vice versa. The direction of the transitions is determined by the Kerr nonlinearity type. When the increase of an incident power is adiabatic, the dependence of transmitted radiation intensity on the incident one allows indirectly observes these transitions and the changes of the band structure. The transfer characteristics were studied in detail with the help of numerical methods. It was shown that the bistability properties depend on the gain and loss value and the input surface. When the $\mathcal{PT}$ symmetry is unbroken, the main effect of the superimposed $\mathcal{PT}$-symmetric refractive index variation on the bistability is the reduction of the feedback between the counterpropagating waves. The feedback forms again if the gain and loss value exceeds the exceptional point. Furthermore, one should note that the $\mathcal{PT}$-symmetry breaking for the periodic structure with finite length takes place when the gain is enough to compensate radiation losses.

The use of the $\mathcal{PT}$-symmetric structure with the modulated Kerr nonlinearity allows one to control the states of $\mathcal{PT}$

symmetry and other related effects: absorption of a reflected signal and lasing. The results of our studies can be used for a fast optical switching.

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