Nonlinear Compton scattering of an ultraintense laser pulse in a plasma

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Laser pulses traveling through a plasma can feature group velocities significantly differing from the speed of light in vacuum. This modifies the well-known Volkov states of an electron inside a strong laser-field from the vacuum case and, consequently, all quantum electrodynamical effects triggered by the electron. Here we present an in-depth study of the basic process of photon emission by an electron scattered from an intense short laser pulse inside a plasma, labeled nonlinear Compton scattering, based on modified Volkov solutions derived from first principles. Consequences of the nonlinear, plasma-dressed laser dispersion on the Compton spectra of emitted photons and implications for high-intensity laser-plasma experiments are pointed out. From a quantitative numerical evaluation we find the plasma to effectively suppress emission of low-frequency photons, whereas the emission of high-frequency photons is enhanced. The emission's angular distribution, on the other hand, is found to remain qualitatively unchanged with respect to the vacuum case.

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I. INTRODUCTION

Exposing matter to lasers of highest electromagnetic field strengths [1] holds promises ranging from technological applications such as compact particle accelerators [2–5] or radiation sources [6–13] to groundbreaking studies of nonlinear quantum electrodynamics (QED) effects. In vacuum, the scattering of an electron (mass m and charge $e < 0$) with initial four-momentum $p^\mu = \varepsilon(1, \beta\boldsymbol{n})/c$, where β is the electron's velocity in units of the speed of light in vacuum c, from a plane wave with electric field amplitude E and wave vector $k_L^\mu = \omega_L(1, \boldsymbol{n}_L)/c$ becomes nonlinear in the regime $\xi = |e|E/cm\omega_L \gtrsim 1$, and quantum effects dominate for $\chi = [(p_\mu k_L^\mu)/mc\omega_L]E/E_{\text{cr}} \gtrsim 1$, where $E_{\text{cr}} = m^2c^3/\hbar|e|$ is the critical field of QED [14]. Such nonlinear QED effects are conventionally accounted for by including them in numerical simulations of laser-matter interactions. While many theoretical efforts continue to further improve these QED-laser-plasma simulation schemes [15–17], they are all based on approximating the QED rates by incoherent single-particle rates in a constant plane wave, since for high particle energies and laser intensities any electromagnetic field can be approximated by a plane wave, constant on the short timescales of QED processes [14,18]. The corresponding QED calculations, however, assume the plane wave to propagate through vacuum, i.e., its group velocity to be c. In a realistic experiment on ultraintense laser-matter interaction, on the other hand, there will be a background of massive particles present, quickly ionized to a plasma of density n_e, introducing the plasma frequency $\omega_p = \sqrt{4\pi n_e e^2/m\gamma_L}$ with $\gamma_L = \sqrt{1 + \xi^2/2}$ as new timescale. It is, however, not known what precise plasma conditions will be present at the exact QED interaction points of plasma electrons and a high-intensity laser. Albeit, it was shown that part of the plasma electron population remains in the paths of high-intensity lasers propagating through the plasma and trigger indisputable QED effects [19,20]. The present manuscript explores how an unperturbed plasma would change photon emission dynamics, as one of the major QED effects in laser-plasma interactions and indicates that the effect of the background plasma cannot always be neglected. In underdense plasmas $\omega_p \leqslant \omega_L$ ultrashort laser pulses propagate over macroscopic distances [2] almost entirely within the plasma and trigger rich dynamics in it only on timescales longer than the pulse duration [21,22]. Also the theory of plasmon provides a straightforward example of a massive Yang-Mills vector boson with the plasma frequency being equivalent to its mass, as argued by P. W. Anderson [23].

The most basic nonlinear QED effect conventionally considered in ultraintense laser-matter interactions is the emission of quantized radiation or Compton scattering. Linear Compton scattering, i.e., the absorption and emission of only one photon by an electron has been studied in the presence of a plasma background already some time ago [24,25]. However, novel laser pulses feature unprecedentedly high photon fluxes ($\xi \gg 1$), leading to an altered effect, labeled nonlinear Compton scattering, commonly approximated as an incoherent sequence of emissions of single high-energy photons on coherent absorption of photon from a laser propagating through vacuum [26]. This process can no longer be described by conventional linear QED, indicating the need for a nonlinear theory of laser-electron interactions. In the presence of a background plasma such a theory is thus far

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missing. On the other hand, the importance of understanding the emission originating from the interaction of intense laser pulses with the electron population of a dilute plasma is also signified by earlier, classical analyses of nonlinear laser-driven particle dynamics [27], later refined to also include the emitted power spectrum [28]. Preliminary studies of this influence were performed at low laser intensities in a classical framework [29], similarly to studies of laser-driven electrons in vacuum [30,31], however, with no QED effects taken into account.

In this work, we provide a first-order or leading-order analysis of the impact of a nonlinear photon dispersion on nonlinear Compton scattering in a QED framework. This study's main goal is to investigate the validity of the hitherto used QED phenomenology model based on scatterings in vacuum. Due to the complexity of the underlying nonperturbative QED framework, we perform a leading-order analysis of the plasma's influence. By identifying deviations of the corresponding perturbative plasma effects from the vacuum theory, we put bounds on the latter's applicability. We thus do not take into account detailed plasma dynamics such as instability excitations. Much rather, we make the simplifying assumption of the background plasma to be cold and collisionless. This assumption was demonstrated to be reasonable for relativistic laser-matter interactions [32] provided ion motion is negligible and the electron temperature T_e is small compared to the energy gain from the laser field $mc^2\xi \gg k_B T_e$, where k_B is the Boltzmann constant [33]. The plasma ions (mass $m_i \gg m$), however, can be heated only on timescales corresponding to several ion plasma periods $t_i = 2\pi/\omega_{p,i} \propto \omega_p^{-1}(m \to m_i) \gg \omega_p^{-1}$ which we ensured to be much longer than the laser pulse durations considered in our work, indicating that the ions also remain cold. As a result, we neglect finite-temperature effects of the plasma and our calculations will be independent of its thermal distribution function, which only extends to energies well below typical laser-driven energy scales and is hence approximated as a δ peak at zero temperature. Consequently, the plasma's effect on the particle dynamics is negligible and its impact on the electromagnetic fields can be treated by a mean-field approximation, yielding a nonlinear dispersion relation. In accordance with most ultrahigh field facilities operating in the optical regime, we assume the laser pulse to have an optical carrier frequency ω_L connected to the laser's wavelength λ_L via $\omega_L\lambda_L = 2\pi\rho c$, where the background plasma acts as a refractive index $\rho \equiv \rho(\omega_L) = [1-(\omega_p/\omega_L)^2]^{1/2} \neq 1$, resulting in a changed wave vector $k_L^\mu = \omega_L(1, \rho\boldsymbol{n}_L)/c$ with $k_L^2 \neq 0$ [34]. Since, on the other hand, when scattered from an ultraintense laser pulse ($\xi \gg 1$) an electron mainly emits photons of frequencies $\omega_1 \sim \xi^3\omega_L \gg \omega_L$, i.e., high-energy harmonics of the laser's base frequency, we may assume the plasma's refractive index experienced by the emitted photons to be $\rho(\omega_1) = [1-(\omega_p/\omega_1)^2]^{1/2} \approx 1$ to leading order and, consequently, model the emitted photons to be unaffected by the background plasma. The basis for nonperturbative QED are solutions of the Dirac equation in the field under consideration, which, for a laser propagating through a medium with nonlinear dispersion relation, have been a long-standing issue and several solutions were communicated [35–40], discriminating between lightlike and spacelike photon fields. Many of these solutions, however, were of exploratory nature [41,42] until recently a more quantitative study was put forward [40]. It was shown that in scattering problems involving energy scales far above any binding barrier, such as studied here, a perturbative approach for the wave function is satisfactory. However, transitions between these electron states have not yet been investigated. In QED such transitions are mediated by the emission of photons and have been analyzed for special non-plane-wave geometries recently [43–46], carrying some resemblance to the full problem of Compton scattering in a plasma background.

II. NONPERTURBATIVE QED SCATTERING AMPLITUDE

We thus start our analysis from the squared Dirac equation in the presence of a strong, plane-wave laser pulse of amplitude $A_L^\mu(\eta) := A_L^\mu\,\psi(\eta) = \epsilon_L^\mu\psi(\eta)(m\xi/|e|)$, where $\psi(\eta)$ is the potential's shape function, ϵ_L^μ its polarization and $\omega_L = 1.55$ eV the optical carrier frequency [47],

$$[-\partial^2 - 2ie(A_L\partial) + e^2A^2 - m^2 - ie\not{k}_L\not{A}'_L]\Psi = 0. \quad (1)$$

Here units $\hbar = c = 1$ are used, and the prime denotes differentiation with respect to the invariant laser phase $\eta = k_L^\mu x_\mu = \omega_L(t - \rho x^\parallel)$, where $x^\parallel$ is the laser's spatial propagation direction. Inserting the usual ansatz for the wave function

$$\Psi = e^{-ipx}F(\eta),$$

where the four-vector p^μ reduces to the electron's momentum in absence of a laser wave, fulfilling $p^2 = m^2$, into Eq. (1) the resulting differential equation for the determination of $F(\eta)$ reads

$$\begin{aligned} 0 &= -k_L^2F''(\eta) + 2i(k_Lp)F'(\eta) + f(\eta)F(\eta) \\ f(\eta) &= 2(k_Lp)\sigma(\eta) - ie\not{k}_L\not{A}', \end{aligned} \quad (2)$$

where we defined $\sigma_p(\eta) = e^2A_L^2(\eta)/2(k_Lp) - e[pA_L(\eta)]/(k_Lp)$. We are going to base the following derivation on multiple-scale perturbation theory [48] (see Ref. [40] for an analogous derivation). In this framework we introduce a second dynamic variable, $t = \eta/k_L^2$. We are going to interpret this as an independent variable that the function $F(\eta) = F(\eta, t)$ depends on. The replacement has to be carried out also in the derivatives, resulting in $\partial F/\partial\eta = k_L^{-2}\partial F/\partial t$ and $\partial^2F/\partial\eta^2 = k_L^{-4}\partial^2F/\partial t^2$ and correspondingly for higher-order derivatives. We are going to look for a solution of the form $F(\eta, t) = F_0(\eta, t) + k_L^2F_1(\eta, t) + k_L^4F_2(\eta, t) + \mathcal{O}(k_L^6)$, which, on insertion into Eq. (1), yields a determining equation of the form

$$\begin{aligned} 0 = {} & \frac{\partial^2F_0}{\partial t^2} - 2i(k_Lp)\frac{\partial F_0}{\partial t} + k_L^2\left(2\frac{\partial^2F_0}{\partial\eta\partial t} + \frac{\partial^2F_1}{\partial t^2}\right) \\ & - 2i(k_Lp)k_L^2\left(\frac{\partial F_0}{\partial\eta} + \frac{\partial F_1}{\partial t}\right) - k_L^2f(\eta)F_0(t,\eta) \\ & + k_L^4\left(\frac{\partial^2F_0}{\partial\eta^2} + 2\frac{\partial^2F_1}{\partial\eta\partial t} + \frac{\partial^2F_2}{\partial t^2}\right) \\ & - 2i(k_Lp)k_L^4\left(\frac{\partial F_1}{\partial\eta} + \frac{\partial F_2}{\partial t}\right) - k_L^4f(\eta)F_1(t,\eta). \end{aligned} \quad (3)$$

Solving this equation order by order and fixing integration constant by invoking physical boundary conditions and exclusion of secular points from the solutions we obtain the results

$$F_0 = \exp\left[i\int d\phi \frac{f(\phi)}{2(k_L p)}\right], \tag{4a}$$

$$\frac{F_1}{F_0} = \frac{f(\phi)}{4(k_L p)^2} + i\int d\phi \frac{\sigma^2(\phi)}{2(k_L p)} + e\frac{\not{k}_L\not{A}}{4(k_L p)^2}\sigma(\phi) - \frac{1}{3}e^2A^2\left[\frac{e\not{k}_L\not{A}}{8(k_L p)^3}\right]. \tag{4b}$$

The spin structure of the found solutions Ψ of Eq. (1) is then fixed by requiring that in the field-free limit $A_L^\mu \to 0$ they obey $[\not{p} - m]\Psi = 0$. This procedure furthermore reduces them to physical solution of the linear Dirac equation. We can then simplify the resulting solution to the form

$$\Psi_p(x) = \left[\Phi_{V,p} + \frac{k_L^2}{2(k_L p)}\delta\Phi_p\right]e^{-ipx+i\Sigma(\eta)}\frac{u_p}{\sqrt{2\varepsilon V}}$$
$$\Sigma(\eta) = \int_{-\infty}^{\eta} d\phi\left[\sigma_p(\phi) + \frac{k_L^2}{2(k_L p)}\sigma_p^2(\phi)\right]$$
$$\delta\Phi_p = \sigma_p(\eta)\left[1 + e\frac{\not{k}_L\not{A}_L(\eta)}{(k_L p)}\right] - \frac{e^2A_L^2(\eta)}{4(k_L p)}\Phi_{V,p} - ie\frac{\not{k}_L\not{A}'_L(\eta)}{2(k_L p)}, \tag{5}$$

where $\Phi_{V,p} = 1 + e\not{k}_L\not{A}_L(\eta)/2(k_L p)$ and u_p is the bispinor for a free electron of momentum p^μ and the Feynman slash notation $\not{a} = \gamma^\mu a_\mu$ with the Dirac matrices γ^μ used. It can be shown that the above solution is formally equivalent to the solution resulting from a perturbative expansion of the full Dirac equation in orders of k_L^2/ω_L^2. We then use these wave functions (5) as basis set for a strong field expansion of QED in a laser field propagating through a plasma to compute the probability of the emission of a photon with wave vector $k_1^\mu = \omega_1(1, \boldsymbol{n}_1)$ toward $\boldsymbol{n}_1 = [\cos(\phi_1)\sin(\theta_1), \sin(\phi_1)\sin(\theta_1), \cos(\theta_1)]$ from an electron changing its initial momentum p_i^μ to a final momentum p_f^μ. The scattering matrix element of this process is given by [14,49]

$$S_{fi} = -ie\sqrt{\frac{4\pi}{2\omega_1 V}}\int d^4x\overline{\Psi}_{p_f}(x)\not{\epsilon}_1^* e^{ik_1x}\Psi_{p_i}(x), \tag{6}$$

where $\overline{\Psi}_p(x) = \Psi_p^*(x)\gamma_0$ is the wave function's Dirac conjugate and $\epsilon_1^{\mu*}$ the emitted photon's polarization vector's complex conjugate. We note that, by changing the integration variables $(t, x^\parallel) \to (\eta, t + \rho x^\parallel)$, three of the four space-time coordinates appear in this expression only linearly in the exponent multiplying momentum sums. Hence, by virtue of the integral representation of the Dirac δ function $\delta(p - q) = 1/2\pi\int_{-\infty}^{\infty} dx\, \exp[i(p-q)x]$ they can be integrated to yield energy-momentum conserving δ functions like in the vacuum analysis [50–53] while, unlike in the vacuum case, the nontrivial integration is in the plasma dressed laser phase η. This phase variable will no longer be a Lorentz invariant, as the refractive index depends on the spatial plasma density, whence we limit our discussion to the experimentally most relevant reference frame in which the plasma is on average at rest. From Eq. (6) one can now obtain the emitted energy according to $d\mathcal{E} = \omega_1 d\Gamma_1 d\Gamma_f \sum_{\sigma,\lambda}|S_{fi}|^2$, where $\sum_{\sigma,\lambda}$ indicates summing (averaging) over final (initial) state spins and polarizations and $d\Gamma_1 = d\boldsymbol{k}_1/(2\pi)^3$ is the emitted photon's phase space and we use energy-momentum conservation to collapse the integrals over the final-state electron's phase space $d\Gamma_f$. We ensured several analytical limits of the resulting QED radiation probability: We found that in the limit $\xi \to 0$, indicating that the total scattering is dominated by a process in which the electron absorbs only one single photon from the laser, which, on the other hand, still sets the dominant energy scale of the scattering, the expression reduces to the perturbative QED amplitude of a dispersive photon undergoing Compton scattering off an electron, i.e., the ordinary linear Compton scattering diagram containing two elementary electron-photon vertices. We also found that in the vacuum limit $n_e \to 0$ it reduces to the well-known expressions of nonlinear Compton scattering in vacuum [50–53]. Finally, we also confirmed that in the classical limit $\chi \to 0$ the QED current $j^\mu = \bar{\Psi}_p(x)\gamma^\mu\Psi_p(x)$ resulting from the used wave functions reduces to its classical counterpart. In order to obtain this classical current, necessary to compute classical emission spectra, we solved the classical equations of motion inside a laser field propagating through a background plasma, mediated by a modification of the laser photons' dispersion relation. To obtain this solution we observe that for $k_L^2 \neq 0$ the classical equations of motion become (see also Ref. [40])

$$\frac{dp^\mu(\eta)}{d\eta} = \frac{e}{[p(s)k_L]}\left[k_L^\mu(A_L p(s)] - A_L^\mu[p(s)k_L)\right]\partial_\eta\psi_{\mathcal{A}}(\eta). \tag{7}$$

This equation is a complete integral and solved by the electron momentum $p^\mu(\eta) = p_i^\mu - eA_L^\mu\psi_{\mathcal{A}}(\eta) + k_L^\mu\{[p(\eta)k_L] - (p_ik_L)\}/k_L^2$. As a consequence, the emission probability obtained from the modulus square of Eq. (6) agrees with the classical radiation power, obtained from inserting the above derived classical electron current into the Liénard-Wiechert potentials, up to terms of order k_L^4 in agreement with the here-used order of the wave function's expansion. There is, however, a discrepancy between the classical and quantum emission inside a plasma: The modulus square of the classical current, entering the emission probability, for an electron initially at rest in a plasma is corrected by $j_{\text{plas}}^2 = j_{\text{vac}}^2[1 - \xi^2k_L^2/\omega_L^2]$. Consequently, we see that classically a plasma suppresses radiation. In the quantum case, e.g., for high emitted frequencies, the emission occurs close to the stationary points η_0 of the rapidly oscillating exponential phase, here distinguished by $\sigma_{p_i}(\eta_0) - \sigma_{p_f}(\eta_0) + k_L^2/2[\sigma_{p_i}^2(\eta_0)/(k_Lp_i) - \sigma_{p_f}^2(\eta_0)/(k_Lp_i)] = 0$. In the vacuum case for an electron initially at rest, as studied below, these stationary points are distinguished by the potential's shape function assuming a value $\psi(\eta_0^{\text{vac}}) = -e(p_fA)(k_Lp_i)/e^2A_L^2(k_Lk') + i\kappa$, where the imaginary part is small $\kappa \sim 1/\xi$ and the real part determines the emission's angular distribution [54]. Inserting this solution back into the original equation, we find the stationary points in the presence of a background plasma to be solutions of the equation $\psi(\eta_0^{\text{plas}}) = \psi(\eta_0^{\text{vac}}) + i\mathcal{C}$, with some complicated real factor $\mathcal{C}$, i.e., the plasma induces a purely imaginary correction to the stationary point and the angular emission range, distinguished by the latter's real part is expected to remain unchanged. On the

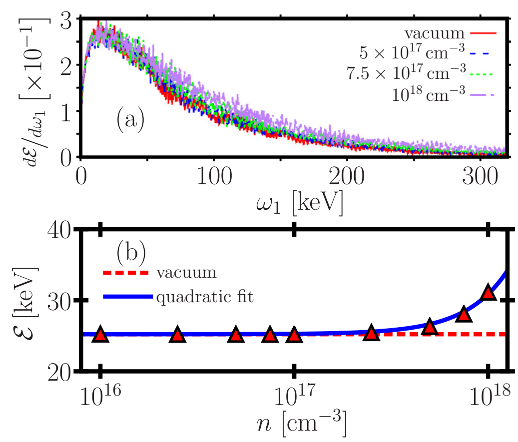

FIG. 1. (a) Integrated spectra for $I_L = 10^{22}$ W/cm^2 for different plasma densities. (b) Total emitted energy compared to the vacuum limit (dashed red) with a quadratic fit (solid blue).

other hand, close to the real stationary points the exponential phase is corrected in the background plasma by a factor $\delta\Sigma(\eta_0) \approx -k_L^2(k_Lp_i)^3(p_fA)^4/4A_L^4(k_Lp_f)^3(k_Lk_1)^3[1 + (k_Lk_1)/(k_Lp_i) - (k_Lp_i)/(k_Lk_1)]\Delta\eta_{\text{coh}} < 0$, where $\Delta\eta_{\text{coh}}$ indicates the process' coherence length. In the high-frequency regime, $(k_Lk_1) \sim (k_Lp_i)$, this correction is negative, i.e., the exponential oscillations are reduced and the radiation probability enhanced. Physically, this seems to imply that while classically the plasma suppresses the radiating charge current, if quantum effects are important, then the suppression concerns quantum phase oscillations, in fact enhancing the emission.

Next, we quantify the above discussion in the context of the ongoing and planned high-intensity laser-plasma interaction experiments [1] by integrating Eq. (6) numerically. We are going to study a two-cycle laser pulse with $\psi(\eta) = \sin^4(\eta/4)\sin(\eta)$ if $\eta \in [0, 4\pi]$ and zero elsewhere scattering an electron initially at rest $p_i^\mu = (m, \mathbf{0})$. In accordance with the cold, collisionless plasma approximation any plasma electron can be approximated by this initial state, as its random thermal motion is negligible compared to its laser-driven dynamics. We begin by considering a laser of intensity $I_L = 10^{22}$ W/cm^2 ($\xi \approx 70$) and visualize the full radiation process by integrating the emission probability over all directions of the emitted photon to obtain the energy emitted per unit frequency $d\mathcal{E}/d\omega_1$ for different plasma densities [see Fig. 1(a)]. While the spectral peak at low ω_1 is unaffected by the plasma, the collective effect of the optical photons accumulates into a higher yield of high-energy photons, even though high-energy photons do not see the plasma as a medium. Integrating the spectra over all frequency components, we obtain the total emitted energy $\mathcal{E}$ as a function of plasma density [see Fig. 1(b)]. The numerical data are well reproduced by a quadratic fit $\mathcal{E} = \mathcal{E}(n_e = 0) + \delta\mathcal{E}\, n_e^2$, with $\delta\mathcal{E} \approx 6 \times 10^{-33}$ eV cm^6, demonstrating an increasing plasma density to lead to a nonlinear increase of emitted energy with respect to the vacuum result, which reduces to the Larmor formula for $\chi \to 0$ [55]. Next to this spectral analysis, the emission's angular distribution is of interest, which in vacuum was shown to be confined to $\theta_1 \geqslant \theta_1^{\text{vac}} := 2\varepsilon/m\xi\psi^{\max}$ [54,56],

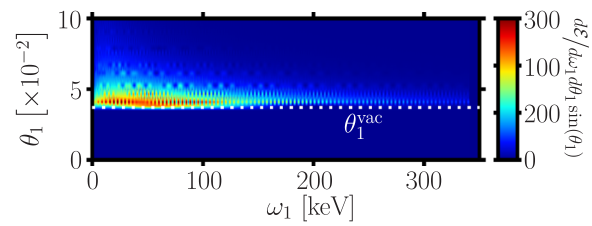

FIG. 2. Angular spectra of the Compton scattered signal for a laser pulse with $I_L = 10^{22}$ W/cm^2 ($\xi \approx 70$) and a plasma density of $n_e = 10^{18}$ cm^{-3} compared to the vacuum boundary angle θ_1^{vac} (white dotted line).

where $\psi^{\max}$ is the shape function's maximal value, i.e., in the present case $\psi^{\max} \approx 0.78$. Integrating Eq. (6) over ϕ_1 we obtain the angular spectrum which even for the largest plasma density studied above exhibits the same angular confinement (see Fig. 2). Thus, the emission's angular distribution is not influenced by the background plasma, unlike its spectrum.

III. IMPLICATIONS FOR THE PLASMA SCATTERING OF THE LASER PULSE

In order to clearly articulate the consequences of the results in the context of ongoing or planned experiments on ultra-high-intensity laser plasma experiments, it is instructive to briefly talk about the scattering of an electromagnetic radiation in a plasma. Broadly speaking, scattering of an electron by an electromagnetic wave in a plasma depends on the parameter $k\lambda_D$, where k is the wave vector of the plasma density fluctuation and $\lambda_D = \sqrt{m_eT_e/\omega_p^2}$ is the Debye length of the plasma. For $k\lambda_D \ll 1$, the scattering is termed as coherent, which properly accounts for the collective plasma effects [57]. In the opposite limit, *viz.*, $k\lambda_D \gg 1$, it is described as the incoherent scattering. In this limit, the scattering process is formally identical to the scattering of the electromagnetic wave by an electron in vacuum. In plasma literature, the incoherent scattering is usually referred as the incoherent Thomson scattering or sometimes as the stimulated Compton scattering, while the plasma instabilities associated with laser propagation are grouped into coherent scatterings. It is known that for plasma electron density $n_e \gg 10^{16}$cm^{-3}, the Thomson scattering of a laser light can strongly reflect collective effects of the plasma especially in the forward direction of the laser pulse [57]. Thus, keeping in mind the plasma densities considered in this paper and the Compton spectrum in Fig. 3, it is important to compare the Compton scattering with the coherent scattering manifested in plasma instabilities of a laser pulse. One of such instabilities is the stimulated Raman scattering (SRS) instability which one of the most important parametric instabilities in a plasma [58–62]. Indeed, the nonresonant Raman scattering is often refereed to as the stimulated Compton scattering in a plasma [32]. The SRS of a laser pulse can occur in any direction, but the forward Raman scattering (FRS) branch of the SRS propagates collinearly with the laser pulse and affects the low-energy laser photons in the laser's propagation direction, i.e., inside the central dip of the emission's angular distribution (see Fig. 2). Comparison of the FRS with the Compton scattering has few salient

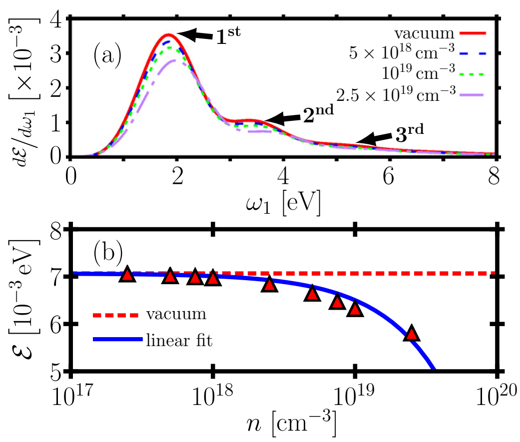

FIG. 3. (a) Integrated spectra for $I_L = 10^{22}\,\mathrm{W/cm^2}$ for forward emission ($\theta_1 \leqslant \theta_1^{\mathrm{vac}}/2$) for different plasma densities. (b) Total emitted energy compared to the vacuum limit (dashed red) with a linear fit (solid blue). The arrows denote the position of the Compton peaks which are compared with the harmonics of the anti-Stokes mode in Fig. 5.

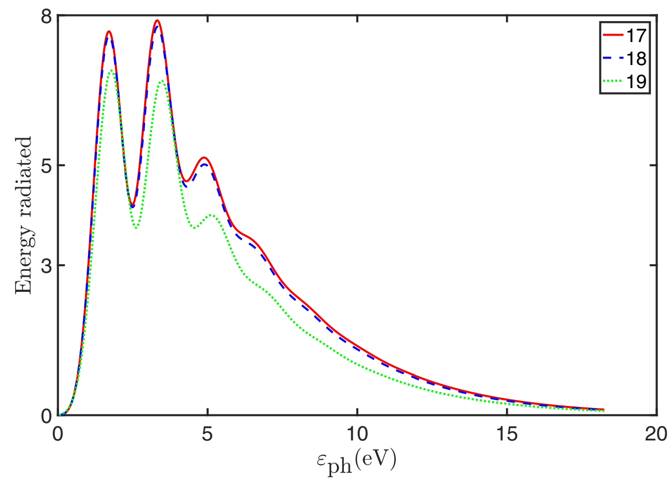

FIG. 4. The photon energy spectrum ($d\mathcal{E}/d\omega_1$, in eV) with photon energy ($\epsilon_{\mathrm{ph}} = \omega_1$) at $\xi = 100$. The spectrum is similar to Fig. 3(a) but is plotted at a fixed angle $\theta = 1/\xi$ instead of being integrated over a cone $\theta_1 \leqslant \theta_1^{\mathrm{vac}}/2$ as in Fig. 3(a). Legends show the plasma densities (in cm^{-3}) on a $\log_{10}$ scale.

points that needs to be highlighted before proceeding further. First, difference in the spectra between the FRS and the Compton scattering can validate the assumption of treating plasma as a passive medium in our calculations. Second, it also facilitates the investigation of the Compton scattering of low-energy photons. Though it may seem counterintuitive at first, it is important as the high-energy photons do not experience the plasma's refractive index but low-energy photons do, satisfying a nonlinear dispersion relation. Since the modified Volkov solution employed in our calculation arises due to plasma refractive index, the influence of the plasma, as a medium, on the Compton scattering can be different for low-energy photons. Moreover, the SRS of a laser pulse is usually probed by interferometric analyses on the laser pulse itself [63] while the high-energy photons from Compton scattering are recorded on a detector. Thus, the detection of the Compton scattering of a laser pulse in a plasma has twofold possibilities both in low-photon energy (eV) and high-photon energy (MeV) regimes. These detection possibilities, instead of being intrusive, are complementary to each other and their simultaneous observations can further affirm the theoretical predictions presented here.

In order to facilitate this comparison, we integrate Eq. (6) over all ϕ_1 but only over $\theta_1 \leqslant \theta_1^{\mathrm{vac}}/2$. In this direction the integrated spectra feature only a few peaks [see Fig. 3(a)], similarly to the vacuum case where for $\theta_1 = 0$ only the first harmonic $\omega_1 \equiv \omega_L$ can be emitted. As argued above, this low-energy emission is reduced at higher plasma densities [see Fig. 3(b)] and the data are well reproduced by a linear fit $\mathcal{E} = \mathcal{E}(n_e = 0) - \delta\mathcal{E}\, n_e$, with $\delta\mathcal{E} \approx 6\times10^{-23}$ eV cm^3, demonstrating that the reduction with respect to the vacuum Larmor result is due to linear plasma effects. Furthermore, the peak positions shift to larger ω_1. Figure 4 shows the same spectra as in Fig. 3(a) but at a fixed angle, $\theta = 1/\xi$, depicting clearly all three Compton peaks as well the dependence on the plasma densities as observed in Fig. 3(a). Since in a plasma $k_L^2 = \omega_p^2$, one can interpret the effective laser photon energy in a plasma as $\omega_L^{\mathrm{plas}} = \omega_L + \omega_p$, equivalent to the quantum of the anti-Stokes mode of the Raman scattering. In the ultrarelativistic regime, the growth rate of the Raman scattering is low despite the enhancement caused by the radiation reaction force [62]. Nevertheless, it can be adequate for the comparison with the Compton signal in terms of the spectral density. Thus one can compare the emission frequencies of nonlinear Compton scattering with those of the anti-Stokes mode of Raman scattering. This comparison is vis-á-vis positions of the peaks in the Compton spectrum (Fig. 4) with the corresponding peaks of the anti-Stokes modes in the Raman spectrum.

Like Compton scattering, Raman scattering of linearly polarized light can exhibit harmonics of the anti-Stokes quanta of Raman scattering [61]. Figure 5 depicts the difference (in eV) between the Compton peaks from Fig. 4 [also in

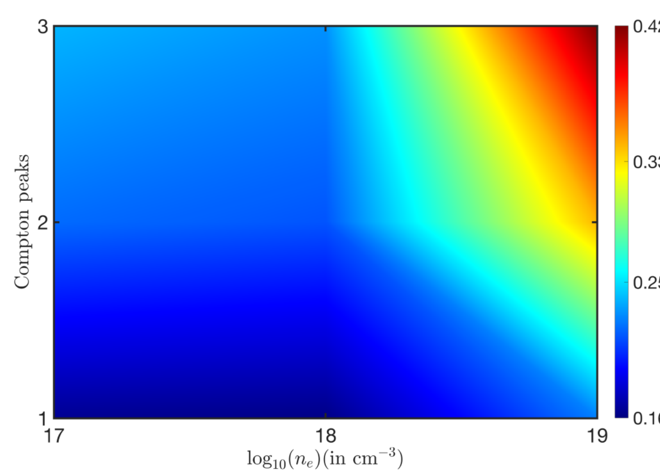

FIG. 5. Difference (color bar in eV) between the Compton spectrum peaks (marked by arrows in Fig. 3) and the corresponding harmonics of the anti-Stokes mode (also in eV) $\omega_n^+ = n(\omega_L + \omega_p)$, with $n = 1, 2, 3$ (for different Compton peaks) at different plasma densities and $\xi = 100$.

Fig. 3(a)] and the harmonics of the anti-Stokes mode which are defined as $\omega_n^+ = n(\omega_L + \omega_p)$, with $n = 1, 2, 3$. For the fundamental quanta, i.e., $\omega_1^+ = \omega_L + \omega_p$, one can see a small difference between the two quanta. Moreover, for all plasma densities one can expect a significant interference between these two fundamental quanta. This strongly suggests that one should also expect possible quantum interference effects on the Raman spectra. These interferences can cause broadening of the Raman signal, which is sometimes also observed in experiments due to other origins. Moreover, at high plasma densities and for higher Compton peaks, the difference gets larger. In this parameter regime it is possible to differentiate between the Compton and the Raman signals, and our assumption of treating plasma as a passive medium can be justified. These predictions can be readily verified for a linearly polarized laser pulse of few femtoseconds duration with intensity $I_L \sim 10^{22}$ W/cm^2 and plasma densities $n_e \sim 10^{16-19}$ cm^{-3}, which are already available. According to Fig. 1, not only the total photon yields are increased but also the maximum photon energy at higher plasma densities. Also the broadening of the SRS signal with respect to plasma density should be easily detected with the current state-of-the-art interferometric analysis such as, e.g., the spectral phase interferometry for direct electric-field reconstruction (SPIDER) and frequency-resolved optical gating (FROG) techniques [64,65].

IV. SUMMARY

To summarize, we have studied nonlinear Compton scattering of an electron in the presence of a strong few-cycle laser field modified in a background plasma based on modified Volkov states. By numerically integrating the complex nonlinear QED scattering matrix element to obtain quantitative predictions for the probability of photon emission we found the background plasma to have different effects on low- and high-frequency photons, respectively. While the emission of the former is suppressed, the emission of the latter is enhanced by denser plasmas. Extrapolating the found scaling laws for the total emitted energy [see Figs. 1 and 3(b)], one can expect the plasma to have strong, possibly dominant, effects at higher plasma densities. In contrast to these alterations with respect to the vacuum case, the angular distribution of the emitted photons was found to remain qualitatively unchanged with respect to that case. The found modifications, however, arise due to the laser dispersion in a plasma and suggest an impact of the plasma on conventional quantum interference effects. We discussed the implications of our results in the context of intense short-pulse laser-plasma interaction experiments and identified the quantum interference effects as an additional mechanism for the broadening of the Raman signals. We wish to state that the plasma densities considered in QED calculations here do not account for the relativistic transparency. Thus, the results presented here can also be applicable to the interaction of an ultraintense laser pulse with a solid target which becomes relativistically transparent due to nonlinear effects. In this scenario, the relativistically adjusted solid plasma density can correspond to the choice of plasma densities in our case and an initially opaque solid target can provide substantial number of plasma electrons to be Compton scattered from the laser pulse.

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