Quenching and revival of oscillations induced by coupling through adaptive variables

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(Received 28 January 2019; published 18 March 2019)

An adaptive coupling based on a low-pass filter (LPF) is proposed to manipulate dynamic activity of diffusively coupled dynamical systems. A theoretical analysis shows that tracking either the external or internal signal in the coupling via a LPF gives rise to distinctly different ways of regulating the rhythmicity of the coupled systems. When the external signals of the coupling are attenuated by a LPF, the macroscopic oscillations of the coupled system are quenched due to the emergence of amplitude or oscillation death. If the internal signals of the coupling are further filtered by a LPF, amplitude and oscillation deaths are effectively revoked to restore dynamic behaviors. The applicability of this approach is demonstrated in laboratory experiments of coupled oscillatory electrochemical reactions by inducing coupling through LPFs. Our study provides additional insight into (ar)rhythmogenesis in diffusively coupled systems.

DOI: 10.1103/PhysRevE.99.032214

I. INTRODUCTION

A model of coupled nonlinear oscillators serves as an excellent framework to unravel and understand the intricacies involved in various complex collective dynamical behaviors and patterns that mimic real-world phenomena in diverse areas of science and technology [1–3]. The intrinsic dynamics of the individual units and the coupling architecture plays a vital role in determining the onset of a rich variety of nonlinear phenomena [4]. In particular, it was revealed that strong couplings could give rise to the Bar-Eli effect [5,6], where the natural rhythm of coupled dynamical systems is lost via two distinct processes, namely amplitude death (AD) and oscillation death (OD) [7–9]. Generally, AD refers to the phenomenon of the quenching of oscillations through the stabilization of one of the existing unstable homogeneous steady states (HSSs), whereas OD is manifested as a stable inhomogeneous steady state (IHSS). AD and OD have their own advantages. In particular, AD serves as a control mechanism in several physical systems, whereas OD provides a mathematical background for cellular differentiation [10].

During the past few decades, there has been a major interest in revealing the emergence of AD and OD along with the transition from AD to OD [11–21]. On the other hand, a few recent investigations have unveiled the counterintuitive phenomenon of reviving oscillations from AD and OD [22–27], as AD and OD favor the onset of static states from evolutionary dynamical states, thereby hampering the functional activity of a large class of real-world networks. Quenching and revival of oscillations have been investigated as two different dynamical entities in coupled systems. Hitherto, a simple unifying scheme to switch between both dynamical behaviors in the same dynamical network is still lacking, which opens up an interesting possibility of research from the viewpoint of engineering rhythmic dynamics of coupled dynamical networks.

We propose an adaptive coupling based on a conventional low-pass filter (LPF), which provides a unifying and efficient approach to manipulate rhythmicity of coupled dynamical systems. A LPF disperses and attenuates the high-frequency signals. Previously, it was reported that the presence of a LPF has a variety of important applications, which modify the dynamics of systems in a nontrivial way [28–32]. For example, a LPF was used to separate a local field potential from the action potential in the electrode recordings of the neural spiking activity in an electroencephalogram (EEG) [28]. A LPF was an essential building block to regulate spatiotemporal dynamics of coupled digital phase-locked loops [29]. The topological limitation concerning an odd number of real positive eigenvalues of the steady state was overcome by using an unstable LPF, which offers an additional unstable degree of freedom [30–32].

In this paper, our study reveals that incorporating LPFs in the coupling may serve as a powerful candidate to engineer the rhythmic dynamics of diffusively coupled dynamical systems. Specifically, we systematically establish that tracking the external or the internal signal of the coupling by a LPF has the complete opposite dynamical effects. Filtering the external signal through a LPF facilitates the onset of AD and OD to suppress the intrinsic rhythmic activities of coupled systems.

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In strong contrast, if the internal signal is further filtered via a LPF, the quenched dynamics can be well revived by switching the stability of the stable homogeneous/inhomogeneous steady states. The theoretical and numerical findings are confirmed in laboratory experiments with two coupled electrochemical reactions.

II. RESULTS AND DISCUSSIONS

A. Effect of LPFs on AD with Stuart-Landau oscillators

We start with an analysis of the following paradigmatic model of two coupled Stuart-Landau limit-cycle oscillators:

$$\dot{Z}_j = (1 + iw_j - |Z_j|^2)Z_j + K[u_k(t) - v_j(t)], \quad (1)$$

where $Z_j = x_j + iy_j$ is a complex variable, w_j is the intrinsic frequency of the jth uncoupled Stuart-Landau oscillator ($j, k = 1, 2,\ j \neq k$), and K is the strength of coupling. For $K = 0$, each uncoupled Stuart-Landau oscillator has a stable limit cycle $Z_j = e^{iw_j t}$ accompanied by an unstable origin $Z_j = 0$. In the coupling, u_k and v_j are governed by two linear ordinary differential equations (ODEs) relevant to Z_k and Z_j as

$$\alpha \dot{u}_k = -u_k + Z_k, \quad (2)$$

$$\beta \dot{v}_j = -v_j + Z_j, \quad (3)$$

which in fact describe two LPFs to attenuate the signals of Z_k and Z_j with the cutoff frequencies $1/\alpha$ ($\alpha > 0$) and $1/\beta$ ($\beta > 0$), respectively. The larger the values α and β, the stronger the Z_k and Z_j are attenuated. In the limit of $\alpha \to 0$ and $\beta \to 0$, u_k and v_j are exactly Z_k and Z_j. Then the coupling form is reduced to the standard diffusive one. Here, we will unveil that by implementing the two LPFs, the external and internal terms of the coupling have two opposite roles on the dynamic activities of the coupled system.

The detailed dynamics of system (1) with $\alpha = 0$ and $\beta = 0$ have been well addressed by Aronson *et al.* [11], who revealed that AD occurs for $1 < K < (1 + \Delta^2/4)/2$ if $\Delta = |w_1 - w_2| > 2$, implying that the limit-cycle oscillations of coupled systems (1) collapse to the origin only if both frequencies are sufficiently disparate and the coupling strength is large enough. Interestingly, incorporating the external LPF associated with the incoming signal Z_k strongly alleviates the restrictions for the onset of AD. Let us first examine AD in system (1) with $\alpha > 0$ and $\beta = 0$, whose stability can be identified from a standard linear stability analysis of system (1) with the LPF (2) around $Z_1 = Z_2 = 0$ and $u_1 = u_2 = 0$ [33]. Figure 1(a) illustrates the stable AD region for $\alpha = 0$ and $\alpha = 0.165$ on the (K, Δ) space. $w_1 = 5 - \Delta/2$ and $w_2 = 5 + \Delta/2$ are used. Clearly, the AD region sharply expands and even extends toward the $\Delta = 0$ axis for a finite interval of coupling strength. The external LPF of Eq. (2) in the coupling induces AD in coupled system (1) even with zero-frequency mismatch. Figure 2(b) further depicts the stable AD interval as a function of α for $w_1 = w_2 = 5$. Identical oscillators suffer AD once $\alpha > \alpha_c = 0.16$. Thus, when the incoming signals are attenuated by LPF in the coupling, AD is facilitated by suppressing the rhythmic activity of the coupled system.

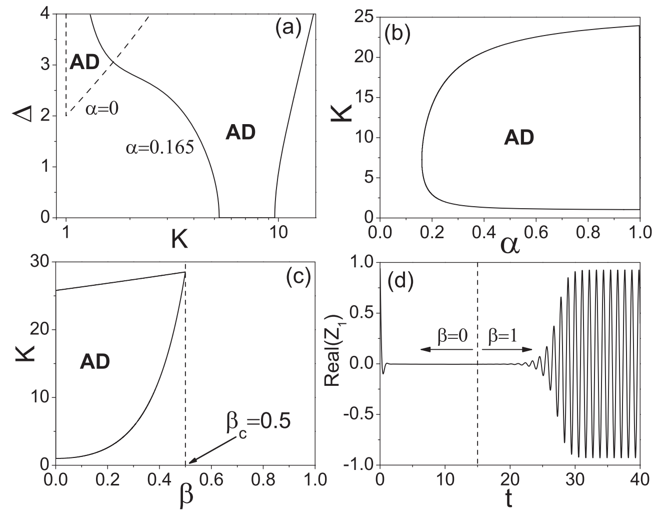

FIG. 1. Quenching and revival of oscillations in two coupled Stuart-Landau oscillators (1). (a) The AD regions in the parameter space of (Δ, K) for $\alpha = 0$ (bounded by the dashed lines) and $\alpha = 0.165$ (bounded by the solid lines). $w_1 = 5 - \Delta/2$, $w_2 = 5 + \Delta/2$, and $\beta = 0$. (b) The AD interval vs α for $w_1 = w_2 = 5$ and $\beta = 0$. AD emerges for $\alpha > \alpha_c = 0.16$ implying that stable limit-cycle oscillations are quenched in identical oscillators. (c) The AD interval vs β for $w_1 = w_2 = 5$ and $\alpha = 10$. AD is completely revoked as $\beta > \beta_c = 0.5$. (d) The plot of time series of real(Z_1). Limit-cycle oscillation is lost for $\alpha = 10$ when $\beta = 0$, which is regained as β switched from 0 to 1 at $t = 15$. $K = 3$ and $w_1 = w_2 = 5$.

In contrast, if the internal signals of the coupling are filtered by the LPF, a distinctly different effect arises. Next, we incorporate the two LPFs of (2) and (3) simultaneously into system (1), where the stability of AD is then determined from the characteristic equation of the coupled system (1) around $Z_1 = Z_2 = 0$ and $u_1 = u_2 = v_1 = v_2 = 0$ [34]. Figure 1(c)

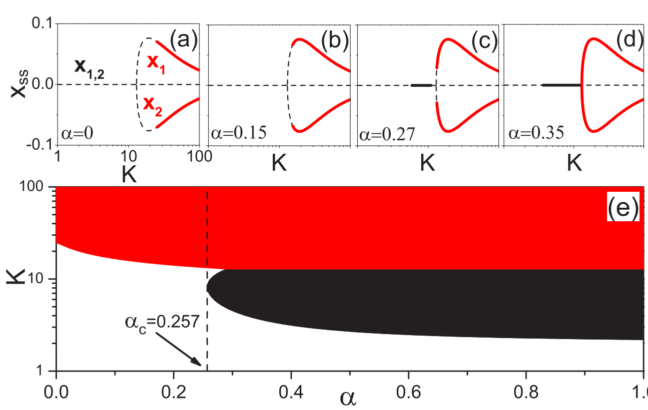

FIG. 2. Quenching oscillations in the coupled system (4) by implementing the LPF associated with only the external term of the coupling. $\beta = 0$ and $w = 5$ are fixed. (a)–(d) The bifurcation diagrams of the steady-state solutions for $\alpha = 0$, 0.15, 0.27, and 0.35, respectively. Solid black (dark gray) and solid red (light gray) lines denote the stable HSS (AD) and the stable IHSS (OD). (b) The stable interval of AD (black region) and OD (red region) vs α for $\beta = 0$. The external LPF facilitates the onset of OD at small values of coupling strength, and further even induces AD as $\alpha > \alpha_c = 0.257$.

depicts the stable interval of AD versus β for $\alpha = 10$, which shows that the stable AD interval monotonically decreases and completely vanishes for $\beta > \beta_c = 0.5$. Thus the internal LPF can provoke oscillations from AD. To visualize this more clearly, we plot the time series for the real part of Z_1 of system (1) with $K = 3$ and $\alpha = 10$ in Fig. 1(d), where β is switched from 0 to 1 at $t = 15$. Clearly, the limit-cycle oscillation that is quenched by the external LPF is restored by implementing the internal LPF. Thus, the external and internal LPFs have two drastically different dynamic effects: the external LPF facilitates AD to suppress oscillations, whereas the internal LPF revokes AD to revive rhythmic behavior.

B. Effect of LPFs on OD with Stuart-Landau oscillators

To further distinguish the distinct roles of the two LPFs in engineering rhythmic behaviors of a coupled system, we study the following system of two Stuart-Landau oscillators now with symmetry-breaking coupling:

$$\dot{Z}_j = (1 + iw_j - |Z_j|^2)Z_j + K[u_k(t) - v_j(t)], \quad (4)$$

$$\alpha\dot{u}_k = -u_j + \mathrm{Re}(Z_k), \quad (5)$$

$$\beta\dot{v}_j = -v_j + \mathrm{Re}(Z_j), \quad (6)$$

where $j, k = 1, 2$ and $j \neq k$. Here the coupling involving only the real parts breaks the rotational symmetry of the coupled system, which is deemed to be a necessary condition for OD in the coupled Stuart-Landau oscillators [8,9]. System (4) with $\alpha = \beta = 0$ reduces to the same case studied by Koseska *et al.* [8], where AD, OD, and the transition from AD to OD were reported. To unveil the distinct roles of both LPFs on AD and OD, we assume here the two oscillators having identical frequencies $w_1 = w_2 = w$. The coupled system (4) has a HSS $Z_1 = Z_2 = 0$ and an IHSS $P(x_1^*, y_1^*, -x_1^*, -y_1^*)$ with $x_1^* = -wy_1^*/(w^2 + 2Ky_1^{*2})$ and $y_1^* = \pm\sqrt{(K - w^2 + \sqrt{K^2 - w^2})/2K}$, where the IHSS appears via a pitchfork bifurcation at $K = (w^2 + 1)/2$. Note that the steady-state solutions of the coupled system are not affected either by the external LPF or by the internal LPF in the coupling, but their stability may be switched. We will illustrate that the presence of a LPF in the external and internal signals of the coupling has a large impact on both AD and OD of system (4), whose stability can be obtained from their corresponding characteristic equations [35].

For $\alpha = \beta = 0$, i.e., without any LPFs in the coupling, system (4) experiences only OD for $K > w^2 + 1/4$, where AD is unstable for all $K > 0$ [8]. Interestingly, by implementing only the external LPF in the coupling, we find that it can facilitate the onset of OD and even induce the occurrence of AD. Figures 2(a)–2(d) depict the four typical bifurcation diagrams of the steady states of system (4) for different values of α. ($w = 5$ and $\beta = 0$ are fixed.) These bifurcation diagrams are generated by depicting the solutions of both HSS and IHSS as a function of K, where the stable steady states are marked by the solid red lines and the unstable ones by the dashed black lines. For $\alpha = 0.15$ in Fig. 2(b), OD occurs at a smaller value of coupling strength compared with that of $\alpha = 0$ in Fig. 2(a). The presence of the external LPF causes the coupled system to exhibit OD even at lower coupling strengths. For a larger value of $\alpha = 0.27$ in Fig. 2(c), AD is found to be stabilized within a pronounced interval of coupling strength. The AD to OD transition is established for $\alpha = 0.35$ in Fig. 2(d). Figure 2(e) elucidates the stable interval of AD and OD as a function of α and K. It is evident that the external LPF facilitates the onset of OD for smaller values of K, then it induces AD in identical systems for $\alpha > 0.257$, and finally it establishes the AD to OD transition when $\alpha > 0.29$. Thus, the external LFP has a strong tendency to induce both AD and OD to quench oscillations in the coupled system.

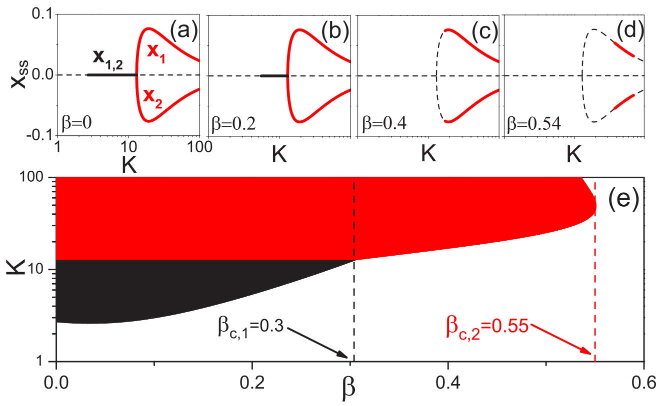

FIG. 3. Reviving oscillations from AD and OD in the coupled system (4) with $\alpha = 0.5$ by implementing LPF in the internal term of the coupling. $w = 5$ is fixed. (a)–(d) The bifurcation diagrams of the steady-state solutions for $\beta = 0$, 0.2, 0.4, and 0.54, respectively. (b) The stable interval of AD (black region) and OD (red region) vs β for $\alpha = 0.5$. The internal LPF destabilizes AD from the lower coupling strength, which disappears at $\beta_{c,1} = 0.3$. For $\beta > \beta_{c,1}$, OD begins to be destabilized and is completely dismissed as $\beta > \beta_{c,2} = 0.55$.

In sharp contrast to the above uncovered role of the external LPF, the internal LPF can revive oscillations from AD and OD. Figures 3(a)–3(d) provide four bifurcation diagrams to display how the presence of β influences the stability of the steady-state solutions of system (4), where the external LFP is fixed as $\alpha = 0.5$ and $w = 5$ is used. Increasing β from zero first destabilizes AD from its lower bound, e.g., Fig. 3(b) for $\beta = 0.2$. Then AD disappears for $\beta = 0.4$ in Fig. 3(c), while OD is destabilized from the lower coupling strength. Upon further increasing β to 0.54 in Fig. 3(d), the coupled system only experiences OD. To systematically characterize the impact of β in destabilizing both AD and OD, Fig. 3(e) plots the stable coupling intervals of AD and OD as a function of β. Increasing β revokes AD from small values of K at first, whereas the stable OD interval remains unchanged until that AD is completely destabilized at $\beta_{c,1} = 0.3$. For $\beta > \beta_{c,1}$, OD starts to be destabilized and vanishes at $\beta_{c,1} = 0.55$. The internal LPF prefers to revive oscillations from deaths, in strong contrast to the external LPF with the tendency to quench oscillations. Therefore, implementing the LPF associated with different terms in the coupling has two completely distinct effects in manipulating the rhythmic activity of coupled system: the external LPF in the coupling hampers the rhythmicity, while the internal LPF facilitates the revival of oscillations from death states.

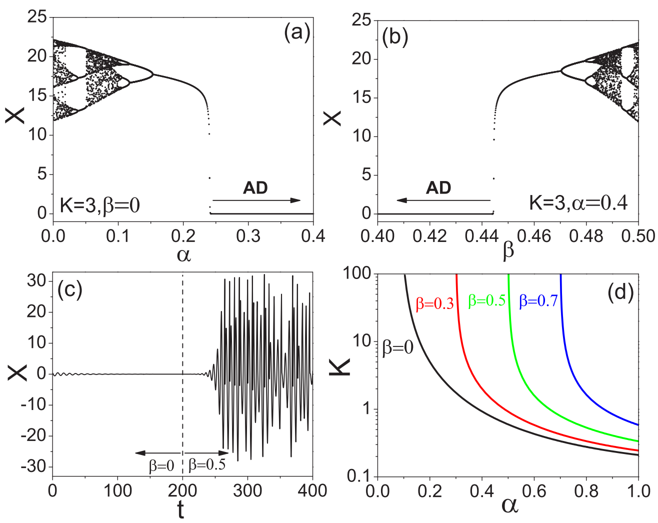

FIG. 4. Quenching and revival of oscillations in two coupled chaotic Rössler oscillators (7). (a) Bifurcation diagram obtained by plotting the local maxima of $X = \frac{x_1+x_2}{2}$ as a function of α for $\beta = 0$ and $K = 3$. The coupled system (7) undergoes a reverse period-doubling bifurcation from chaos to one cycle, and then experiences AD for $\alpha > \alpha_c = 0.24$. (b) Bifurcation diagram as a function of β for $\alpha = 0.4$ and $K = 3$. AD is destabilized at $\beta = \beta_c = 0.444$. For $\beta > \beta_c = 0.444$, the coupled system (7) experiences a period-doubling bifurcation from period-1 oscillation to chaos. (c) The plot of time series of x_1. Chaotic oscillation is regained from AD as β is switched from 0 to 0.5 at $t = 200$. $K = 3$ and $\alpha = 0.4$ are fixed. (d) K vs α for the onset of AD for $\beta = 0$, 0.3, 0.5, and 0.7.

C. Effect of LPFs on AD with Rössler oscillators

Implementing LPFs in the coupling serves as a very generic technique to manipulate dynamic activity of a diffusively coupled system. To validate the generality, our study is extended to the following system of coupled chaotic Rössler oscillators with LPFs in the coupling:

$$\dot{x}_j = -y_j - z_j,$$
$$\dot{y}_j = x_j + ay_j + K[u_k(t) - v_j(t)], \tag{7}$$
$$\dot{z}_j = b + z_j(x_j - c),$$
$$\alpha\dot{u}_k = -u_k + y_k, \tag{8}$$
$$\beta\dot{v}_j = -v_j + y_j, \tag{9}$$

where $a = b = 0.1$, $c = 14$, $j, k = 1, 2$, and $j \neq k$. For $K = 0$, each uncoupled Rössler oscillator exhibits a phase-coherent chaotic motion, and it has an unstable focus $P = (x^*, y^*, z^*)$ with $x^* = -ay^*$, $y^* = -z^*$, and $z^* = \frac{c-\sqrt{c^2-4ab}}{2a}$. System (7) suffers AD via the manifestation of suppression of the chaotic oscillations due to a stabilization of P. The emergence of AD can be identified from the characteristic equation of system (7) with the LPFs (8) and (9) at P [36].

Figure 4(a) shows the bifurcation diagrams by plotting the local maximum of the centroid $X = \frac{x_1+x_2}{2}$ as a function of α for the coupled system (7) with $K = 3$ and $\beta = 0$. As α is increased, a reverse period-doubling cascade from chaos takes place, leading to period-1 oscillation at $\alpha = 0.153$; the oscillation collapses to the fixed point P at $\alpha = 0.244$. This observation confirms that AD is indeed induced by imposing the external LPF in coupled chaotic Rössler oscillators, where the bifurcation route leading to AD is much richer than that in coupled Stuart-Landau oscillators. As the internal LPF is incorporated, the AD is stable only if $\beta < 0.444$ in Fig. 4(b) with $K = 3$ and $\alpha = 0.4$ fixed. Upon further increasing β, the period-doubling bifurcation from period-1 oscillation to chaos is revived from AD. The chaotic oscillations suppressed by the external LPF can be regained by implementing the internal LPF, which can be directly seen from the plot of the time series of X in Fig. 4(c) with β switching from 0 to 0.5 at $t = 200$. To gain an overall view of the distinct roles of the two LPFs, Fig. 4(d) depicts the dependence of K on α for the onset of AD for $\beta = 0$, 0.3, 0.5, and 0.7. It is evident that the emergence of AD critically depends on both α and β: the threshold of coupling strength K for AD monotonically decreases for increasing α and rapidly grows for increasing β. Therefore, we corroborate the generic nature of the external LPF in quenching oscillations and the internal LPF in sustaining rhythmicity.

D. Effect of LPFs on AD with electrochemical oscillators

The effects of LPFs in the coupling on the oscillatory dynamics are further experimentally explored with two coupled electrochemical oscillators. The nickel electrodissolution in

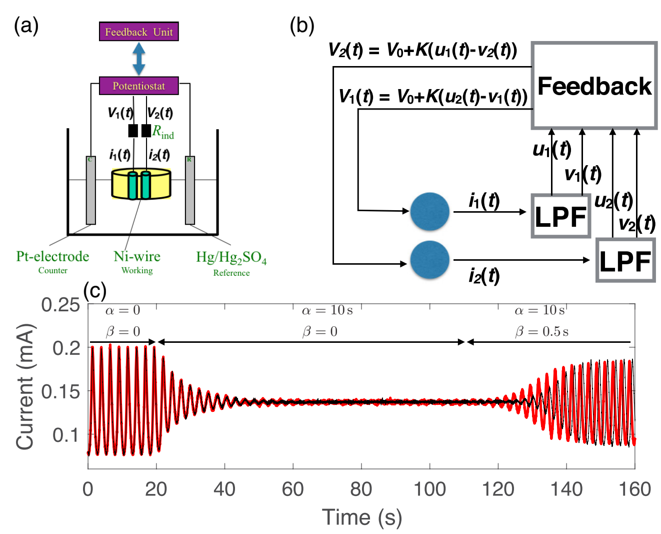

FIG. 5. Inducing AD and reviving the electrochemical oscillations with coupling through LPFs. (a) Experimental setup. $R_{\rm ind}$: Individual resistances attached to the Ni wires. $V_k(t)$ and $i_k(t)$ are the applied circuit potentials and the measured currents for electrodes $k = 1, 2$, respectively. (b) Experimental implementation of feedback scheme. The measured currents $[i_k(t)]$ are filtered with coefficients α and β in Eqs. (2) and (3) to obtain the corresponding $u_k(t)$ and $v_k(t)$ variables. The potentiostat (GILL IK64) applies feedback with set potential V_0 and gain K. (c) Synchronized oscillations, AD, and regained oscillations with coupling ($K = -0.3$ V/mA) without LPF ($t < 20$ s, $\alpha = 0$, $\beta = 0$), LPF in external signal ($20\,\text{s} \leqslant t < 110$ s, $\alpha = 10$ s, $\beta = 0$), and LPFs in both external and internal signals ($t \geqslant 110$ s, $\alpha = 10$ s, $\beta = 0.5$ s), respectively. $V_0 = 1100$ mV, $R_{\rm ind} = 5$ kOhm.

3 mol/L sulfuric acid electrolyte at 10 °C exhibits current oscillations at constant circuit potential and individual resistance attached to the wires [24]. We use two such wires, as shown in Fig. 5(a); the currents (proportional to the metal dissolution rate) are filtered with a LPF, and the circuit potential of each wire is adjusted according to the coupling scheme in Fig. 5(b). The coupling is induced by small adjustments of the circuit potential, proportional to the difference between the external and internal coupling variables. As shown in Fig. 5(c), without any filters the oscillations exhibit in-phase synchronization. With a LPF only in the external variable ($\alpha = 10$ s, $\beta = 0$), AD is observed. The addition of a LPF in the internal variable ($\alpha = 10$ s, $\beta = 0.5$ s) revives oscillations from AD. The electrochemical experiments well confirm that introducing coupling through the LPFs serves as a powerful technique to control oscillatory behaviors.

III. CONCLUSIONS

To conclude, we have unraveled two distinctly opposite effects of a LPF in manipulating rhythmic dynamics by examining its role in the external and internal signals of the coupling. Using numerical examples, we showed that the limit-cycle oscillations of coupled Stuart-Landau oscillators are quenched by the external LPF, and they are regained by the internal LPF. The chaotic dynamics of coupled Rössler oscillators is annihilated to achieve AD through a reverse period-doubling cascade by implementing the external LPF, which is restored via a typical period-doubling bifurcation when imposing the internal LPF. Unlike conventional diffusive coupling, in the proposed coupling scheme the output signals of all the oscillators are independently filtered by LPFs. When the internal signal of the coupling is attenuated via a LPF, the macroscopic rhythmic activities of the coupled system could be weakened seriously or even completely lost due to the high possibility of the occurrence of AD (OD). The external LPF of the coupling has a strong tendency to induce (facilitate) AD (OD) to quench oscillations of the coupled system, while the filtered internal coupling can restore oscillations from both AD and OD.

We have experimentally demonstrated the ability to control the rhythmogenesis of two electrochemical reactions coupled with LPFs. The linear LPFs of the coupling were shown to be vital and nontrivial factors in manipulating rhythmic behaviors of coupled systems. In realistic circumstances, signals often suffer inevitable dispersion and attenuation during their transmissions in diverse communication channels. The coupling scheme with LPFs is thus of practical importance. Our study may expand the understanding of the roots of emergence of rhythmicity in populations of real-world systems, especially in biology, such as in the context of genetic regulatory networks [37], where diffusion of autoinducer (AI) molecules between the cell membranes is commonly governed by a quorum sensing with dynamical evolution quite similar to the LPF coupling.

Finally, we would like to emphasize that although only one experimental (electrochemical) situation is described in the present work, the proposed coupling technique is plausible in many other experimental systems, where oscillatory units in the networks can be reconnected by incorporating LPFs before they are coupled, such as in networks of unijunction transistors [38], Mackey-Glass analog circuits [39], semiconductor lasers [40], and photochemically coupled catalytic micro-oscillators [41]. In fact, the filtered internal coupling has recently been experimentally demonstrated to revive oscillations from AD and OD in two mean-field coupled nonlinear circuits [27], where the competing role of the filtered external coupling in inducing death is ignored. As LPFs are quite relevant and omnipresent in biological and physical systems [42–44], our results are expected to be of widespread experimental applicability.

ACKNOWLEDGMENTS

W.Z. acknowledges support from Research Starting grants from South China Normal University (8S0340) and a project supported by Guangdong Province Universities and Colleges Pearl River Scholar Funded Scheme (2018). D.V.S. is supported by the CSIR EMR Grant No. 03(1400)/17/EMR-II. I.Z.K. acknowledges support from National Science Foundation Grant No. CHE-1465013. M.Z. acknowledges support from the National Natural Science Foundation of China under Grant No. 11475253 and the International (Regional) Cooperation and Exchange Program of the National Natural Science Foundation of China (Research on Inter-organizational Cooperation: NSFC-DFG) (Grant No. 11861131011).

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[33] The characteristic equation determining the stability of AD of coupled system (1) with $\alpha > 0$ and $\beta = 0$ is given by

$$\begin{vmatrix} A-K-\lambda & 0 & 0 & K \\ \frac{1}{\alpha} & -\frac{1}{\alpha}-\lambda & 0 & 0 \\ 0 & K & B-K-\lambda & 0 \\ 0 & 0 & \frac{1}{\alpha} & -\frac{1}{\alpha}-\lambda \end{vmatrix} = 0,$$

where $A = 1 + iw_1$ and $B = 1 + iw_2$. AD is stable when all the real parts of the eigenvalues are negative, $\mathrm{Re}(\lambda) < 0$.

[34] AD in two identical Stuart-Landau oscillators (1) with $\alpha > 0$ and $\beta > 0$ is determined by the following characteristic equation:

$$\begin{vmatrix} 1+iw-\lambda & \pm K & -K \\ \frac{1}{\alpha} & -\frac{1}{\alpha}-\lambda & 0 \\ \frac{1}{\beta} & 0 & -\frac{1}{\beta}-\lambda \end{vmatrix} = 0$$

.

[35] AD (OD) in coupled Stuart-Landau oscillators (4) with the LPFs (5) and (6) is identified from the characteristic equation

$$\begin{vmatrix} A-\lambda & -w-B & \pm K & -K \\ w-B & C-\lambda & 0 & 0 \\ \frac{1}{\alpha} & 0 & -\frac{1}{\alpha}-\lambda & 0 & 0 \\ \frac{1}{\beta} & 0 & 0 & -\frac{1}{\beta}-\lambda \end{vmatrix} = 0,$$

where $A = 1 - 3x_1^{*2} - y_1^{*2}, B = 2x_1^* y_1^*, C = 1 - x_1^{*2} - 3y_1^{*2}$, and $(x_1^*, y_1^*, -x_1^*, -y_1^*)$ denotes the corresponding solution of AD (OD).

[36] AD in two coupled chaotic Rössler oscillators (7) with the LPFs (8) and (9) is decided by the following characteristic equation:

$$\begin{vmatrix} -\lambda & -1 & -1 & 0 & 0 \\ 1 & a-\lambda & 0 & \pm K & -K \\ z^* & 0 & x^*-c-\lambda & 0 & 0 \\ 0 & 1/\alpha & 0 & -1/\alpha-\lambda & 0 \\ 0 & 1/\beta & 0 & 0 & -1/\beta-\lambda \end{vmatrix} = 0,$$

where $x^* = -ay^*$, $y^* = -z^*$, and $z^* = \frac{c-\sqrt{c^2-4ab}}{2a}$.

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