Electron density fluctuations in collisional dusty plasma with variable grain charge

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The kinetic theory of electric fluctuations in a collisional weakly ionized dusty plasma is formulated with due regard to the grain charging dynamics. The correlation functions of electron and ion density are obtained by considering their collisions with neutrals described within the Bhatnagar-Gross-Krook model. The electron density correlation spectra in isothermal and nonisothermal plasma are calculated for various values of grain density, grain size, and ion collisionality.

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I. INTRODUCTION

Electromagnetic fluctuations are important and often the only source of information about the medium's parameters. They are closely related to electromagnetic, kinetic, and thermodynamic properties of the macroscopic systems. In particular, the correlation functions of the microscopic particle densities determine static and dynamic form factors of the system and thus the spectrum of electromagnetic wave scattering by plasmas [1–3]. The kinetic coefficients in the Fokker-Planck equation [1,3,4] and the collision integrals in kinetic equations are also determined by the microscopic phase density fluctuations. It is also obvious that the fluctuations of the electromagnetic field play the role of the Langevin sources in the Brownian motion of charged particles in plasmas [2,5]. Therefore, the calculation of the electric field fluctuations in a dusty plasma [6] is important for describing particle diffusion, the intensity of which determines, to a considerable extent, the processes of formation and melting of plasma crystals [7,8].

The theory of fluctuations in ordinary collisional plasma is well developed [9–13]. The problem of generalization of this theory to the case of dusty plasma has a number of issues that remain open. The dust grains acquire electric charges due to absorption of electrons and ions from the surrounding plasma, i.e., grains are charged by the plasma currents that flow toward its surface. In the stationary state, the flux of electrons on the grain surface is equal to the flux of ions, thus the total current is equal to zero. The fluctuations of the charging current lead to the fluctuations of the stationary grain charge. Therefore, the problem arises of including such fluctuations into self-consistent calculation of electric field fluctuations. Moreover, the grain charges depend on the electromagnetic field via charging currents and, thus, they generate additional dielectric polarization of the medium, which should be taken into account along with the polarization due to the plasma components.

To solve the problem under consideration two approaches are usually used. The first of them is based on the description of grain charge dynamics using the charging equation and explicit representation of the charging currents in terms of fluctuations of plasma particle distribution functions and charging cross sections [14–17]. The disadvantage of such an approach is that it requires phenomenological description of plasma particle collisions with grains (both elastic and inelastic) and additional calculations of "shadow" and bombardment forces (generated by scattering and absorption of plasma particles by a grain in the presence of another grain) to describe consistently the collective grain-grain interaction, if necessary. In the second approach the grain charge is treated as an independent dynamic variable and thus the grain distribution function depends not only on the coordinate and velocity, but also on the grain charge [18–25]. This approach provides the opportunity to work out consistent kinetic theory of fully ionized dusty plasmas and, in principle, to find collision terms for particles of all species. However, this approach faces serious problems with the generalization to the case of collisional plasmas. The main problem here is that the calculations of the quantities describing the grain dynamics are expressed in terms of the charging cross sections that are known for collisionless plasmas only. At the same time phenomenological approximations for such cross sections (see, for example, [24] and related references cited therein) have a limited range of application. Actually, the same problem arises in the first approach, but in that case one can use semiphenomenological approximations for charging currents [26,27] which take into account the influence of collisions and, at the same time, are in a good agreement with the results of experiments and numerical calculations [28] at arbitrary collision frequencies and other plasma parameters. This means that we can avoid the problem of calculating charging cross sections, if the equations of grain charging dynamics can be formulated in terms of charging currents. Since the appropriate formulation can be easily done, the first approach looks more suitable for the generalization of the theory of electromagnetic fluctuations in collisionless dusty plasmas to the case of collisional plasma background.

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The above-mentioned problems with the unknown charging cross sections for collisional dusty plasmas are the reason why the consistent calculations of fluctuation spectra are related to the collisionless [14–16,19], or weakly collisional regimes [22–25,29]. The purpose of the present paper is to give a consistent linear kinetic description of electric fluctuations in collisional weakly ionized dusty plasma with regard to the absorption of electrons and ions by grains and grain charge variations.

It was shown in [23] that the measurements of density fluctuation spectra in dusty plasmas can constitute a basis for *in situ* diagnostic of invisible submicron dust due to its effects on plasma responses. The authors state that the proposed technique can be applied to various plasma environments in laboratory and space, provided measurements of electrostatic fluctuation spectra are available. Application of the proposed method to the diagnostics of dust in space could allow quiescent plasma environments to be covered.

II. FLUCTUATIONS OF DISTRIBUTION FUNCTION

The consistent description of fluctuations in dusty plasma requires considering the charge density fluctuations related to electrons and ions

$$\delta\rho_\alpha(\mathbf{r},t) = e_\alpha \delta n_\alpha(\mathbf{r},t), \quad \alpha = e, i, \tag{1}$$

as well as the grain charge density fluctuations

$$\delta\rho_g(\mathbf{r},t) = e_g \delta n_g(\mathbf{r},t) + n_g \delta e_g(\mathbf{r},t), \tag{2}$$

where e_g is the stationary grain charge and n_g is the mean number density of grains. Notice, that such representation is valid for fluctuations that satisfy the condition $n_g R^3 \gg 1$. Here R is the spatial scale of perturbation.

The number density fluctuations of charged particles have the form

$$\delta n_\alpha(\mathbf{r},t) = n_\alpha \int d\mathbf{v} \delta f_\alpha(\mathbf{r},\mathbf{v},t), \quad \alpha = e, i, g, \tag{3}$$

where $\delta f_\alpha(\mathbf{r},\mathbf{v},t)$ are the fluctuations of distribution function of the corresponding particle species. In the case of electrons or ions, they can be found in the same way as in ordinary plasma [13], but regarding the collisions of electrons and ions with grains in addition to collisions with neutrals.

Fluctuations of electron and ion distribution functions satisfy the equation [13]

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right\}\delta f_\alpha(X,t) + \nu_\alpha \left\{\delta f_\alpha(X,t) - f_{0\alpha}(\mathbf{v}) \times \int d\mathbf{v} \delta f_\alpha(X,t)\right\} = -\frac{e_\alpha}{m_\alpha}\frac{\partial \delta\phi(\mathbf{r},t)}{\partial \mathbf{r}}\frac{\partial f_{0\alpha}(\mathbf{v})}{\partial \mathbf{v}}, \tag{4}$$

where $X = (\mathbf{r},\mathbf{v})$, $\delta\phi(\mathbf{r},t)$ is the fluctuation of electrostatic potential, $f_{0\alpha}(\mathbf{v})$ is the unperturbed distribution function (usually it is the Maxwell distribution), and $\nu_\alpha = \nu_{\alpha n} + \nu_{\alpha g}$, where $\nu_{\alpha n}$ and $\nu_{\alpha g}$ are the effective collision frequencies between particles of α species with neutrals and grains.

In the case under consideration to justify Eq. (4), we made the following assumptions. The electrons and ions absorbed by the grain recombine on its surface and form the neutral gas atoms (molecules) that evaporate into the surrounding plasma and can be ionized again due to collisions or to external ionization sources. Such assumption makes it possible to use the Bhatnagar-Gross-Krook (BGK) collision integral [30] in Eq. (4).

Further, the equation for the grain charge fluctuations is formulated. The averaging of the equation for microscopic phase density of grains results in a kinetic equation with collision integral that can be expressed in terms of the correlation functions of microscopic quantities [13]. For the sake of simplicity, we use the BGK collision integral as in the case of plasma particles. The linearized equation for the fluctuations of grain distribution function has the form of Eq. (4) with $\alpha = g$, where ν_g is the effective collision frequency between grains and other particles. Such an equation, however, does not take into account the fluctuations of charging collision frequencies.

The formal solution of Eq. (4) is given by

$$\delta f_\alpha(X,t) = \delta f_\alpha^{(0)}(X,t) - \frac{e_\alpha}{m_\alpha}\int_{-\infty}^{t} dt' \int dX' \times W_\alpha(X,X';t-t')\frac{\partial \delta\phi(\mathbf{r}',t')}{\partial \mathbf{r}'}\frac{\partial f_{0\alpha}(\mathbf{v}')}{\partial \mathbf{v}'}, \tag{5}$$

where $\delta f_\alpha^{(0)}(X,t)$ is the general solution of the homogenous Eq. (4), i.e., it is the fluctuation of distribution function in the system without self-consistent interaction through the fluctuation electric field. The second term in (5) is the particular solution of Eq. (4). The function $W_\alpha(X,X';t-t')$ also satisfies the homogenous Eq. (4), but with the initial condition $W_\alpha(X,X';0) = \delta(X-X')$. Hence, it is the probability density of particle transition from phase point X' to the phase point X during the time interval $t-t'$ for particles of α species.

As is easy to see, $\delta f_\alpha^{(0)}(X,t)$ play the role of Langevin sources of electric field fluctuations in the system. Their correlation functions have the form [2,3,11]

$$\left\langle \delta f_\alpha^{(0)}(X,t)\delta f_{\alpha'}^{(0)}(X',t')\right\rangle = \frac{\delta_{\alpha\alpha'}}{n_\alpha}\{f_{\alpha'}(X',t')W_\alpha(X,X';t-t')\theta(t-t') + f_\alpha(X,t)W_\alpha(X',X;t'-t)\theta(t'-t)\}. \tag{6}$$

III. GRAIN CHARGE DYNAMICS EQUATIONS

Further, we need to describe the dynamics of grain charge fluctuations $\delta e_g(\mathbf{r},t)$. Following Ref. [31], we assume that the charging currents are the functions of electron and ion number density, temperature, and the grain charge

$$\frac{\partial e_g(\mathbf{r},t)}{\partial t} = I_{\mathrm{ch}} = \sum_{\alpha=e,i} I_{\mathrm{ch}}^\alpha(n_\alpha(\mathbf{r},t), e_g(\mathbf{r},t)). \tag{7}$$

For the small fluctuations of number density $n_\alpha(\mathbf{r},t) = n_\alpha + \delta n_\alpha(\mathbf{r},t)$ from the average value n_α and small fluctuations of the grain charge $e_g(\mathbf{r},t) = e_g + \delta e_g(\mathbf{r},t)$ from its stationary value, which is determined by the condition of zero total charging current

$$I_{\mathrm{ch}}^e(n_e,e_g) + I_{\mathrm{ch}}^i(n_i,e_g) = 0, \tag{8}$$

one obtains from (7) the equation for $\delta e_g(\mathbf{r},t)$

$$\frac{\partial \delta e_g(\mathbf{r},t)}{\partial t} + \nu_{\rm ch}\delta e_g(\mathbf{r},t) = \sum_{\alpha=e,i} \frac{\partial I^{\alpha}_{\rm ch}(n_\alpha, e_g)}{\partial n_\alpha}\delta n_\alpha(\mathbf{r},t), \quad (9)$$

where the charging frequency $\nu_{\rm ch}$ is given by

$$\nu_{\rm ch} = \sum_{\alpha=e,i} \nu^{\alpha}_{\rm ch}, \quad \nu^{\alpha}_{\rm ch} = -\frac{\partial I^{\alpha}_{\rm ch}(n_\alpha, e_g)}{\partial e_g}. \quad (10)$$

Now, the explicit forms of charging currents $I^{\alpha}_{\rm ch}(n_\alpha, e_g)$ in collisional plasma are needed. Since the mean free path of electrons l_e is, usually, about two orders higher than l_i in gas discharge plasma, we use the expression

$$I^{e}_{\rm ch} = e_e n_e \sqrt{8\pi} a^2 v_{Te} \exp{(-\alpha)}, \quad (11)$$

which is obtained in an orbit motion limited (OML) approximation, i.e., the collisions of electrons with neutrals are neglected.

For an ionic charging current we use the interpolation formula [26], which reproduces with high accuracy the results of kinetic calculations [28]

$$I^{i}_{\rm ch} = e_i n_i \sqrt{8\pi} a^2 v_{Ti} \frac{I^{\rm WC} I^{\rm SC}}{I^{\rm WC} + I^{\rm SC}}, \quad (12)$$

where

$$I^{\rm WC} = 1 + \alpha\tau + 0.1(\alpha\tau)^2 \lambda_D / l_i, \quad (13)$$

$$I^{\rm SC} = \sqrt{2\pi}\alpha\tau l_i / a, \quad \tau = T_e/T_i. \quad (14)$$

Here $\alpha = e_e\phi_s/T_e$ (not to be confused with subscript α that denotes the plasma particle species), ϕ_s is the surface potential, a is the grain radius, $k_D^2 = k_{De}^2 + k_{Di}^2$, $k_{D\alpha}^2 = 4\pi e_\alpha^2 n_\alpha / T_\alpha$, $\lambda_D = 1/k_D$ is the Debye length, $v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$ is the plasma particle thermal velocity, $l_i = v_{Ti}/\nu_i$ is the ion mean free path, and ν_i is the collision frequency of ions with other particles. WC stands for weakly collisional and SC is for strongly collisional.

It is reasonable to assume [28,32–35] that the electrostatic potential near the grain is described by the Derjaguin-Landau-Verwey- Overbeek (DLVO) potential

$$\phi(r) = \frac{e_g}{r} \frac{\exp[-k_D(r-a)]}{(1+ak_D)}, \quad (15)$$

then

$$\alpha = \frac{e_e e_g}{a T_e (1 + a k_D)}. \quad (16)$$

The space-time Fourier transform (FT)

$$f_{\mathbf{k}\omega} = \int_{-\infty}^{\infty} dt \int d\mathbf{r} f(\mathbf{r},t) \exp(i\omega t - i\mathbf{k}\mathbf{r}) \quad (17)$$

of Eq. (2) for the grain charge fluctuations along with (9) gives

$$\delta\rho_{g\mathbf{k}\omega} = e_g \delta n_{g\mathbf{k}\omega} + \frac{i n_g}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \frac{I^{\alpha}_{\rm ch}}{n_\alpha} \delta n_{\alpha\mathbf{k}\omega}. \quad (18)$$

It was taken into account in (18) that according to (11) and (12)

$$\frac{\partial I^{\alpha}_{\rm ch}(n_\alpha, e_g)}{\partial n_\alpha} = \frac{I^{\alpha}_{\rm ch}}{n_\alpha}. \quad (19)$$

We substitute Eq. (5) in (3) and after FT obtain

$$\delta n_{\alpha\mathbf{k}\omega} = n_\alpha \int d\mathbf{v} \delta f^{(0)}_{\alpha\mathbf{k}\omega}(\mathbf{v}) + i\frac{e_\alpha n_\alpha}{m_\alpha} \int d\mathbf{v} \int d\mathbf{v}' \times W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\mathbf{k} \frac{\partial f_{0\alpha}(\mathbf{v}')}{\partial \mathbf{v}'} \delta\phi_{\mathbf{k}\omega}$$
$$= \delta n^{(0)}_{\alpha\mathbf{k}\omega} - \frac{k^2}{4\pi e_\alpha} \chi_\alpha(\mathbf{k},\omega) \delta\phi_{\mathbf{k}\omega}, \quad \alpha = e, i, g, \quad (20)$$

where $\chi_\alpha(\mathbf{k},\omega)$ is the dielectric susceptibility of the plasma particle subsystem.

Further, we substitute formula (20) with $\alpha = g$ in the first term of (18) and formula (20) with $\alpha = e, i$ in the second one. Thus we obtain

$$\delta\rho_{g\mathbf{k}\omega} = \delta\rho^{(0)}_{g\mathbf{k}\omega} - \frac{k^2}{4\pi}\chi_g(\mathbf{k},\omega)\delta\phi_{\mathbf{k}\omega} + \frac{i}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \nu_{\alpha g} \delta\rho^{(0)}_{\alpha\mathbf{k}\omega}$$
$$- \frac{k^2}{4\pi} \frac{i}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \nu_{\alpha g} \chi_\alpha(\mathbf{k},\omega)\delta\phi_{\mathbf{k}\omega}, \quad (21)$$

where

$$\nu_{\alpha g} = \frac{n_g I^{\alpha}_{\rm ch}}{e_\alpha n_\alpha} \quad (22)$$

is the frequency of plasma particle collisions with grains.

Equation (21) can be rewritten to be similar to (20):

$$\delta\rho_{g\mathbf{k}\omega} = \delta\tilde{\rho}^{(0)}_{g\mathbf{k}\omega} - \frac{k^2}{4\pi}\tilde{\chi}_g(\mathbf{k},\omega)\delta\phi_{\mathbf{k}\omega}, \quad (23)$$

where

$$\delta\tilde{\rho}^{(0)}_{g\mathbf{k}\omega} = \delta\rho^{(0)}_{g\mathbf{k}\omega} + \frac{i}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \nu_{\alpha g} \delta\rho^{(0)}_{\alpha\mathbf{k}\omega}, \quad (24)$$

$$\tilde{\chi}_g(\mathbf{k},\omega) = \chi_g(\mathbf{k},\omega) + \frac{i}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \nu_{\alpha g}\chi_\alpha(\mathbf{k},\omega). \quad (25)$$

Thus,

$$\delta\phi_{\mathbf{k}\omega} = \frac{4\pi\delta\rho^{(0)}_{\mathbf{k}\omega}}{k^2 \varepsilon(\mathbf{k},\omega)}, \quad (26)$$

where

$$\delta\rho^{(0)}_{\mathbf{k}\omega} = \sum_{\alpha=e,i,g} \delta\rho^{(0)}_{\alpha\mathbf{k}\omega} + \frac{i}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \nu_{\alpha g}\delta\rho^{(0)}_{\alpha\mathbf{k}\omega}, \quad (27)$$

$$\varepsilon(\mathbf{k},\omega) = 1 + \sum_{\alpha=e,i,g} \chi_\alpha(\mathbf{k},\omega) + \frac{i}{\omega + i\nu_{\rm ch}} \sum_{\alpha=e,i} \nu_{\alpha g}\chi_\alpha(\mathbf{k},\omega). \quad (28)$$

We see that the dielectric permittivity of dusty plasma in the present description differs from the dielectric permittivity given by the multicomponent model

$$\varepsilon_{\rm mc}(\mathbf{k},\omega) = 1 + \sum_{\alpha=e,i,g} \chi_\alpha(\mathbf{k},\omega) \quad (29)$$

by the presence of the last term in (28), which is the renormalized susceptibility of grains generated by the charging processes. Furthermore, in calculations of plasma particle dielectric response, collisions with grains should be taken into account on an equal footing with the collisions with neutrals, i.e., $\nu_\alpha = \nu_{\alpha n} + \nu_{\alpha g}$.

IV. FLUCTUATION SPECTRA

In this section we calculate the electron density correlation function $\langle \delta\rho_e^2 \rangle_{\mathbf{k}\omega}$. We start from the electron density fluctuations and it follows from Eqs. (20) and (26) that

$$\delta\rho_{e\mathbf{k}\omega} = \delta\rho^{(0)}_{e\mathbf{k}\omega} - \frac{\chi_e(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}\delta\rho^{(0)}_{\mathbf{k}\omega}, \quad (30)$$

and using (27) we get

$$\delta\rho_{e\mathbf{k}\omega} = \delta\rho^{(0)}_{e\mathbf{k}\omega} - \frac{\chi_e(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}\left[\sum_{\alpha=e,i}\left(1 + \frac{i\nu_{\alpha g}}{\omega + i\nu_{\text{ch}}}\right)\delta\rho^{(0)}_{\alpha\mathbf{k}\omega} + \delta\rho^{(0)}_{g\mathbf{k}\omega}\right]. \quad (31)$$

Finally,

$$\langle \delta\rho_e^2 \rangle_{\mathbf{k}\omega} = \left|1 - \frac{\chi_e(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}\left(1 + \frac{i\nu_{eg}}{\omega + i\nu_{\text{ch}}}\right)\right|^2 \langle \delta\rho_e^{(0)2} \rangle_{\mathbf{k}\omega} + \left|\frac{\chi_e(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}\left(1 + \frac{i\nu_{ig}}{\omega + i\nu_{\text{ch}}}\right)\right|^2 \langle \delta\rho_i^{(0)2} \rangle_{\mathbf{k}\omega} + \left|\frac{\chi_e(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}\right|^2 \langle \delta\rho_g^{(0)2} \rangle_{\mathbf{k}\omega} \quad (32)$$

the electron density correlation function is expressed in terms of $\chi_\alpha(\mathbf{k},\omega)$ and $\langle \delta\rho_\alpha^{(0)2} \rangle_{\mathbf{k}\omega}$

$$\langle \delta\rho_\alpha^{(0)2} \rangle_{\mathbf{k}\omega} = e_\alpha^2 n_\alpha \int d\mathbf{v} \int d\mathbf{v}' W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') f_{0\alpha}(\mathbf{v}') + \text{c.c.} \quad (33)$$

in the equilibrium state [2,3,11]

$$\langle \delta\rho_\alpha^{(0)2} \rangle_{\mathbf{k}\omega} = \frac{T_\alpha k^2}{2\pi\omega}\text{Im}\,\chi_\alpha(\mathbf{k},\omega). \quad (34)$$

Neglecting the grain charge variations, i.e., putting the frequencies ν_{ch} and $\nu_{\alpha g}$ equal to zero in (32), one obtains

$$\langle \delta\rho_e^2 \rangle_{\mathbf{k}\omega} = \left|1 - \frac{\chi_e(\mathbf{k},\omega)}{\varepsilon_{\text{mc}}(\mathbf{k},\omega)}\right|^2 \langle \delta\rho_e^{(0)2} \rangle_{\mathbf{k}\omega} + \left|\frac{\chi_e(\mathbf{k},\omega)}{\varepsilon_{\text{mc}}(\mathbf{k},\omega)}\right|^2 \langle \delta\rho_i^{(0)2} \rangle_{\mathbf{k}\omega} + \left|\frac{\chi_e(\mathbf{k},\omega)}{\varepsilon_{\text{mc}}(\mathbf{k},\omega)}\right|^2 \langle \delta\rho_g^{(0)2} \rangle_{\mathbf{k}\omega}, \quad (35)$$

the electron density correlation function in the multicomponent model. The ion density correlation function is found in the same way.

For the dielectric susceptibility of collisional plasma we use the results obtained [36] on the basis of kinetic equations with the BGK collision integral

$$\chi_\alpha(\mathbf{k},\omega) = \frac{k_{D\alpha}^2}{k^2}\frac{(\omega + i\nu_\alpha)W(z_\alpha)}{\omega + i\nu_\alpha W(z_\alpha)}, \quad (36)$$

where $z_\alpha = (\omega + i\nu_\alpha)/kv_{T\alpha}$ and $W(z)$ is the plasma dispersion function [2]

$$W(z) = 1 - ze^{-z^2/2}\int_0^z dy\, e^{y^2/2} + i\sqrt{\frac{\pi}{2}}ze^{-z^2/2}. \quad (37)$$

V. RESULTS OF NUMERICAL CALCULATIONS

Now let us consider the results of numerical calculations of electron density fluctuation spectra obtained on the basis of formula (32). There are several parameters in (32) we should discuss in more detail. The charging frequency ν_{ch} [Eq. (10)] is the sum of electron and ion charging frequencies that are the derivatives of corresponding charging currents given by Eqs. (11) and (12). Their explicit expressions and dependencies on ion mean free path are presented in Ref. [31]. It was found there that in a nonisothermal plasma charging frequency ν^α_{ch} is of the order of the ion plasma frequency $\omega_{pi} = 4\pi e^2 n_i/T_i$, hence the grain charge fluctuations can influence the electron density fluctuations in this frequency domain. The electron and ion charging frequencies vs ν_{in}/ω_{pi} in isothermal ($\tau = 1$) plasma are shown in Fig. 1. One can see that the charging frequency strongly depends on the grain size and is higher for the bigger grains.

The charging currents are the functions of parameter α [Eq. (16)], which can be referred to as the normalized grain charge, which, in turn, is determined by the condition of zero total current on the grain surface [Eq. (8)]. Thus, the grain charge is found from the equation

$$\frac{n_e}{n_i}\mu\exp(-\alpha) = \frac{I^{\text{WC}}I^{\text{SC}}}{I^{\text{WC}} + I^{\text{SC}}}, \quad (38)$$

where $\mu = v_{Te}/v_{Ti} = \sqrt{\tau m_i/m_e}$.

The ratio n_e/n_i in Eq. (38) is defined by a quasineutrality condition, which in the case of dusty plasma has the form

$$e_e n_e + e_i n_i + e_g n_g = 0. \quad (39)$$

For singly charged ions

$$\frac{n_e}{n_i} = 1 - P, \quad P = \frac{e_g n_g}{e_e n_i}, \quad (40)$$

where P is the Havnes parameter, which describes the part of the electron charge collected by dust. The Havnes parameter is

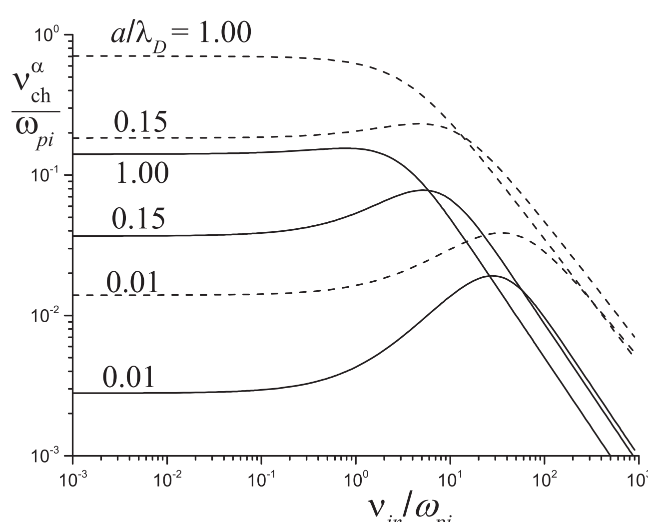

FIG. 1. Electron (dashed lines) and ion (solid lines) charging frequencies $\nu^\alpha_{\text{ch}}/\omega_{pi}$ vs ion-neutral collision frequency ν_{in} in isothermal ($\tau = 1$) argon plasma for $a/\lambda_D = 1, 0.15, 0.01$.

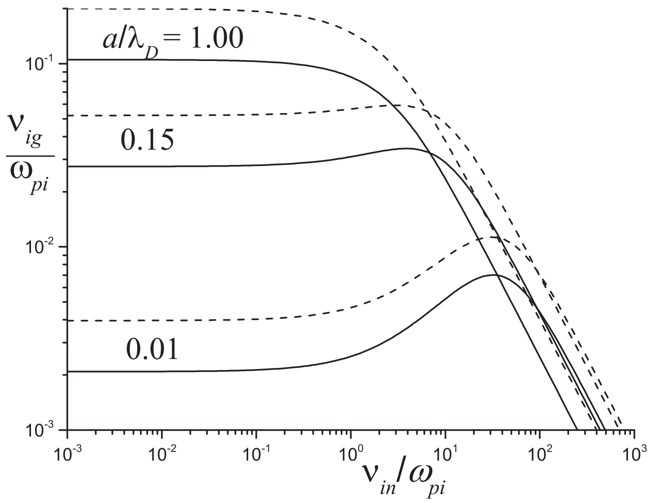

FIG. 2. Ion-grain collision frequency ν_{ig}/ω_{pi} vs ion-neutral collision frequency in isothermal ($\tau = 1$) argon plasma for $P = 0.5$ (solid lines) and $P = 0.8$ (dashed lines), $a/\lambda_D = 1, 0.15, 0.01$.

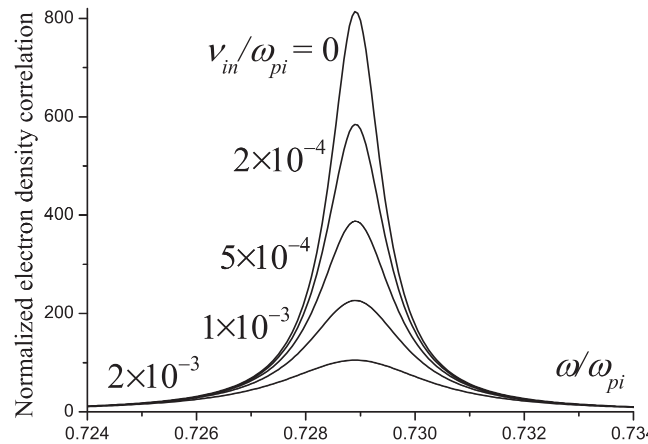

FIG. 3. Normalized electron density correlation spectra $\langle\delta n_\alpha^2\rangle_{\mathbf{k}\omega}\omega_{pi}/n_i$ in ordinary nonisothermal ($\tau = 100$) argon plasma for $\nu_{in}/\omega_{pi} = 0,\ 2\times10^{-4},\ 5\times10^{-4},\ 1\times10^{-3},\ 2\times10^{-3}$; $k/k_{Di} = 0.1$.

found in the normalized expression for electron susceptibility

$$\chi_e(\mathbf{k},\omega) = \frac{1-P}{\tau\tilde{k}^2}\frac{(\tilde{\omega}+i\tilde{\nu}_e)W(z_e)}{\tilde{\omega}+i\tilde{\nu}_e W(z_e)}, \quad (41)$$

where

$$\tau = \frac{T_e}{T_i},\ \tilde{k} = \frac{k}{k_{Di}},\ \tilde{\omega} = \frac{\omega}{\omega_{pi}},\ \tilde{\nu}_e = \frac{\nu_e}{\omega_{pe}},\ z_e = \frac{\tilde{\omega}+i\tilde{\nu}_e}{\mu\tilde{k}}. \quad (42)$$

The frequency of collisions between ions (electrons) and grains $\nu_{\alpha n}$, which is the part of effective collision frequency $\nu_\alpha = \nu_{\alpha n} + \nu_{\alpha g}$, is also defined by the charging currents [Eq. (22)]. The value of ν_{ig} depends considerably on the grain size and can be dominant in the effective collision frequency as well as negligible (see Fig. 2). The relation $\nu_{ig} = (1-P)\nu_{eg}$ between electron and ion frequencies follows from (22) in the stationary state.

We performed the calculations for argon dusty plasma with various values of grain size ak_D, Havnes parameter P, and ion-neutral collision frequency ν_{in} for both isothermal $\tau = 1$ and nonisothermal $\tau > 1$ plasmas. Since the plasma frequency of grains is much smaller than ion plasma frequency, the motion of grains does not affect the fluctuations in this frequency domain and the last terms in (32) and (35) are neglected as well as $\chi_g(\mathbf{k},\omega)$ in (28) and (29). The electron-neutral collision frequency ν_{en} can be expressed via ion frequency. The simple relation $\nu_{\alpha n} = v_{T\alpha}\sigma_\alpha n_n$, where σ_α is the scattering cross section, is valid for a weakly ionized plasma. Thus, the electron collision frequency is related to ion frequency through the expression $\nu_e = \nu_i\mu(\sigma_e/\sigma_i)$. The scattering cross section of Ar^+ ions with an energy of 0.1 eV on Ar atoms is about 157 $Å^2$ and it decreases with the increase of energy (Table 7 in Ref. [37]). The scattering cross section of electrons on argon atoms has the minimum of $\approx$0.1 $Å^2$ for the energy of $\approx$0.2 eV, and for $\approx$2.5 eV it is $\approx$3 $Å^2$ (Fig. 4 in Ref. [38]). Therefore, we can assume that $\sigma_e/\sigma_i \approx 0.02$ for nonisothermal plasma and this value was used in our computations.

Summarizing, we can highlight the main factors included in our description that influence the fluctuation spectra of electron density in dusty plasma:

(i) The grain charge variations are described by the last term in (28) and corresponding terms in (32) that contain ν_{ch} and $\nu_{\alpha g}$.

(ii) The plasma particle collisions with neutrals and grains define the plasma particle effective collision frequencies $\nu_\alpha = \nu_{\alpha n} + \nu_{\alpha g}$, which influence the susceptibility of electrons and ions [Eq. (36)].

(iii) The decrease of the electron to ion density ratio in dusty plasma influences the stationary grain charge α [see Eq. (38)] and electron susceptibility (41).

In order to clarify separately the input of the second factor, we start from considering the electron density correlation function given by (35) and $\nu_\alpha = \nu_{\alpha n}$ in (36), i.e., we start by studying the influence of ion-neutral collisions on electron density fluctuations in ordinary plasma. Figure 3 shows that the intensity of fluctuations in nonisothermal ($\tau = 100$) plasma is sensitive to ion collisions with neutrals. Even for small values of ν_{in}/ω_{pi} the fluctuations are considerably suppressed.

Concerning the last factor, it is known that the decrease of n_e/n_i results in the increase of dust ion-acoustic wave eigenfrequency and therefore the fluctuation maxima are also displaced in higher frequencies [31,39]. This effect is illustrated by Fig. 4, where the fluctuation spectra are obtained using the expressions for ordinary collisional plasma, but with $n_e/n_i = 1-P$. Besides the fluctuation maximum shift, its decrease is also observed. It can be explained by the decrease of $\langle\delta\rho_e^{(0)2}\rangle_{\mathbf{k}\omega}$ via decrease of electron susceptibility [Eq. (41)], since it is proportional to $1-P$.

In order to see the isolated effect of the dust charge variability (first factor), we, initially, "turn on" both second and third factors. Namely, the collisions of plasma particles with grains are included into effective collision frequency $\nu_\alpha = \nu_{\alpha n} + \nu_{\alpha g}$ in the (36), but for electron density correlation function formula (35) is still used (dashed line in Fig. 4). For $a/\lambda_D = 0.15$ and $P = 0.2, 0.5$ the collision frequencies of

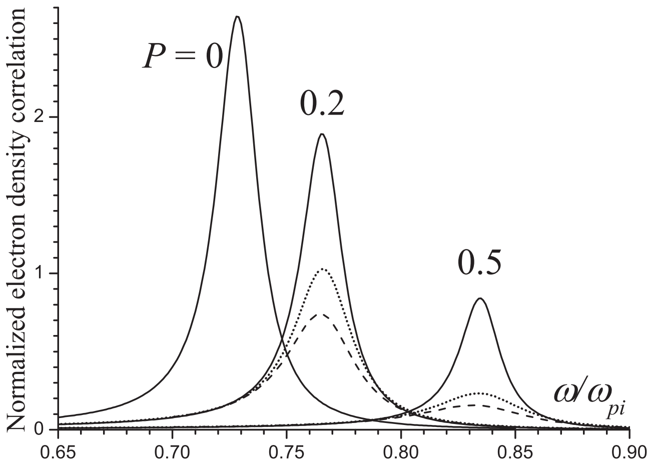

FIG. 4. Normalized electron density correlation spectra $\langle \delta n_\alpha^2 \rangle_{\mathbf{k}\omega} \omega_{pi}/n_i$ in ordinary nonisothermal ($\tau = 100$) argon plasma for $\nu_{in}/\omega_{pi} = 0.02$, $k/k_{Di} = 0.1$, $P = 0, 0.2, 0.5$. Dashed line corresponds to Eq. (35) and plasma particle susceptibility (36) including collisions with grains $\nu_\alpha = \nu_{\alpha n} + \nu_{\alpha g}$, $a/\lambda_D = 0.15$. Dotted line corresponds to Eq. (32).

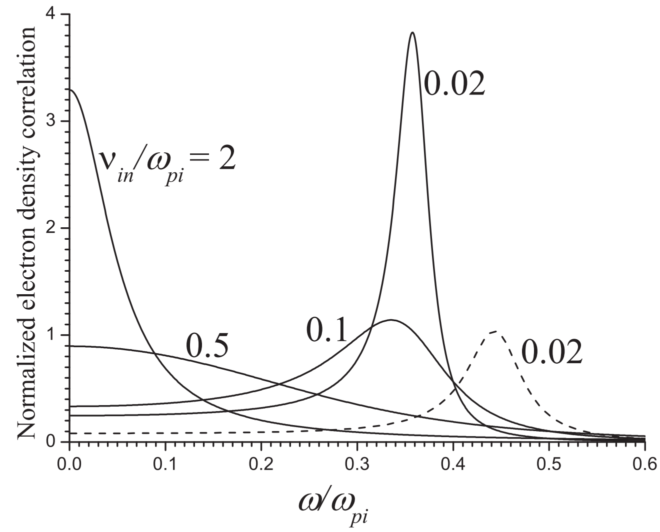

FIG. 6. Normalized electron density correlation spectra $\langle \delta n_\alpha^2 \rangle_{\mathbf{k}\omega} \omega_{pi}/n_i$ in nonisothermal ($\tau = 10$) argon plasma for $k/k_{Di} = 0.1$, $\nu_{in}/\omega_{pi} = 0.02, 0.1, 0.5, 2$, $P = 0$ (solid lines), and $P = 0.5$; $a/\lambda_D = 0.15$ (dashed line).

ions with grains equal to $\nu_{ig}/\omega_{pi} \approx 0.015$ and 0.035 corresponding, thus they are of the order of ion-neutral collision frequency $\nu_{in}/\omega_{pi} = 0.02$. As expected, the increase of ion collisionality leads to decrease of fluctuation intensity. Finally, we include into consideration the grain charge variation (dotted line in Fig. 4), which means that we calculate the electron density correlation function using formula (32) with dielectric permittivity (28). One can conclude that the variations of the grain charge leads to enhancement of the electron density fluctuations as compared to the dashed line, but they are considerably suppressed as compared to multicomponent description (solid line).

The fluctuation spectra in strongly nonisothermal ($\tau = 100$) plasma, which are presented in Fig. 5, show that positions and intensities of maxima depend on the wave number k/k_{Di} and coincide with the eigenfrequency of ion-acoustic waves in collisional dusty plasma (see Fig. 7 in Ref. [31]). The presence of grains leads to the shift of fluctuation maxima toward higher frequencies and to the decrease of fluctuation intensity. It was already mentioned that the increase of the eigenfrequency of the ion-acoustic wave is caused by the decrease of electron to ion density ratio n_e/n_i. This assertion is confirmed by the curves in Fig. 5 corresponding to $k/k_{Di} =$

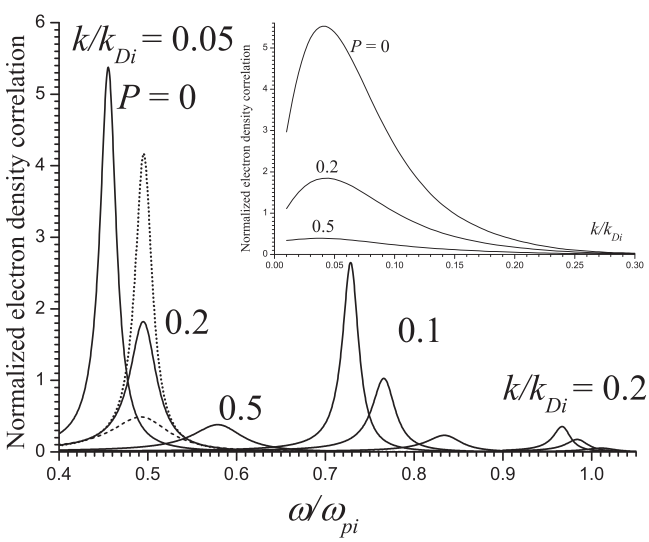

FIG. 5. Normalized electron density correlation spectra $\langle \delta n_\alpha^2 \rangle_{\mathbf{k}\omega} \omega_{pi}/n_i$ in nonisothermal ($\tau = 100$) argon plasma for $\nu_{in} = 0.02\omega_{pi}$, $a/\lambda_D = 0.15$, $k/k_{Di} = 0.05, 0.1, 0.2$, $P = 0, 0.2, 0.5$; $a/\lambda_D = 0.01$ (dotted line) and $a/\lambda_D = 1$ (dashed line). Insert is the maximum value of normalized electron correlation function vs k/k_{Di}.

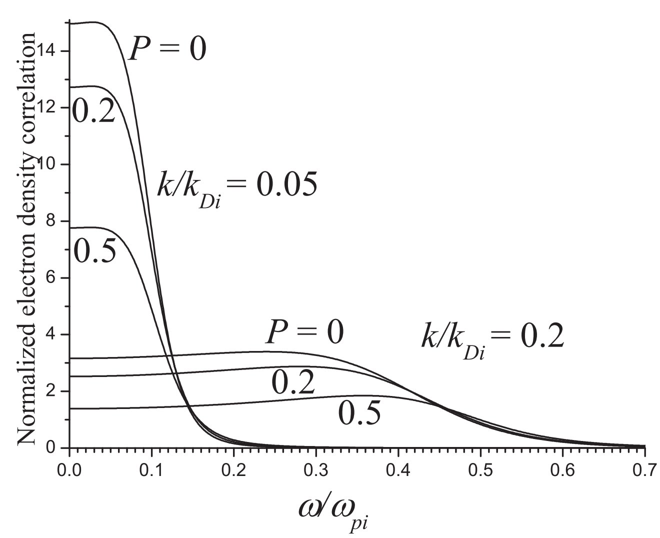

FIG. 7. Normalized electron density correlation spectra $\langle \delta n_\alpha^2 \rangle_{\mathbf{k}\omega} \omega_{pi}/n_i$ in isothermal ($\tau = 1$) argon plasma for $\nu_{in} = 0.02\omega_{pi}$, $a/\lambda_D = 0.15$, $k/k_{Di} = 0.05, 0.1, 0.2$, and $P = 0, 0.2, 0.5$.

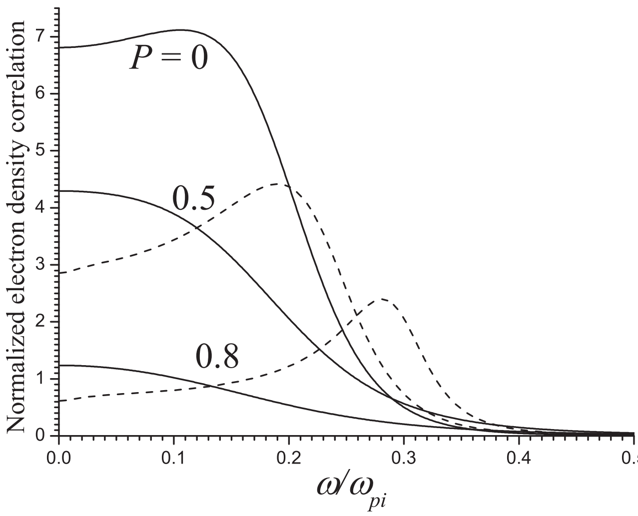

FIG. 8. Normalized electron density correlation spectra $\langle \delta n_\alpha^2 \rangle_{\mathbf{k}\omega} \omega_{pi}/n_i$ in isothermal ($\tau = 1$) argon plasma for $\nu_{in} = 0.02\omega_{pi}$, $k/k_{Di} = 0.1$, $P = 0, 0.5, 0.8$, $a/\lambda_D = 0.01$ (dashed lines), and $a/\lambda_D = 1$ (solid lines).

0.05, $P = 0.2$, and different values of grain size $a/\lambda_D = 0.01, 0.15, 1$. The values of collision ν_{ig} and charging ν_{ch}^α frequencies for $a/\lambda_D = 0.01$ are much less than for $a/\lambda_D = 1$ (see Figs. 1 and 2; or see Figs. 2 and 5 in Ref. [31]). As a consequence the fluctuations are less suppressed in the presence of small grains than of a big one, but the value of the shift depends almost entirely on the Havnes parameter.

The insert in Fig. 5 shows that the highest intensity of fluctuations is at $k/k_{Di} \approx 0.05$ and the increase of the Havnes parameter leads to a decrease of the fluctuation intensity in all wave-number domains under consideration.

The transformation of fluctuation spectra in an ordinary nonisothermal plasma ($\tau = 10$) with the increase of ion collisionality is shown in Fig. 6. With growing of ν_{in}/ω_{pi}, the fluctuations decrease, but then the fluctuation maxima grow at $\omega = 0$. The presence of dust grains also increases the ion effective collision frequency $\nu_i = \nu_{in} + \nu_{ig}$ due to collisions between ions and grains. For example, $\nu_{ig}/\omega_{pi} = 0.028$ for $P = 0.5$, $ak_D = 0.15$ and $\nu_{in}/\omega_{pi} = 0.02$, thus $\nu_i/\omega_{pi} = 0.048$ and fluctuations are also suppressed. But the maximum value of the electron density correlation in dusty plasma is approximately equal to that in ordinary plasma with $\nu_{in} = 0.1$. It means that the influence of dust on fluctuation spectra cannot be described only by the increase of ion collisionality. Also, the maximum is shifted to a higher frequency. As already mentioned, the increase of fluctuation frequency is provided by the decrease of n_e/n_i.

The fluctuation spectra in isothermal plasma (see Fig. 7) differ from that in nonisothermal one: the maxima are broader and situated at the lower frequencies. The presence of grains suppresses the fluctuations but not so efficiently as in the case of nonisothermal plasma. Figure 8 illustrates the influence of the grain size on the fluctuations in isothermal plasma. Since ν_{ig} and ν_{ch} for $a/\lambda_D = 0.01$ are much less than for $a/\lambda_D = 1$ (see Figs. 1 and 2), the presence of grains of different sizes change the fluctuation spectra differently, even if the Havnes parameter is the same.

As it was mentioned above, we have omitted the last term in (32) and $\chi_g(\mathbf{k}, \omega)$ in (28) in our calculations. It means we considered immovable grains. That is the reason why the maxima corresponding to collective fluctuations of the grains (dust acoustic resonances) are not observed in the region of grain plasma frequency $\omega_{pg} \ll \omega_{pi}$ in Figs. 6–8.

VI. CONCLUSIONS

Electron density correlation spectra are strongly affected by the presence of grains with variable charge. The main factors of this influence are the decrease of the electron to ion density ratio, increase of ion collisionality due to collisions with grains, and grain charge variations.

In the case of nonisothermal plasmas the positions of the ion-acoustic resonances and their intensities depend on the grain density (Havnes parameter). The decrease of the electron to ion density ratio n_e/n_i in dusty plasma leads to the shift of fluctuation maxima to higher frequencies and to the decrease of fluctuations due to decrease of electron susceptibility. The increase of ion effective collision frequency additionally suppresses the electron density correlations. The variations of grain charges themselves enhance the electron density fluctuations, but, in summary, the fluctuations in dusty plasma are considerably suppressed. This effect depends on the grain size and is more pronounced for bigger grains since the ion-grain collision frequency is proportional to the square of the grain radius.

The presence of grains increases the effective ion collision frequency, but the resulting influence on the fluctuation spectra is different from that in ordinary plasma with the same ion-neutral collision frequency. Particularly, the increase of ion-neutral collisionality in ordinary plasma can lead to suppression of ion-acoustic resonance and growth of the maximum near the zero frequency that is not observed in dusty plasma. In the case of isothermal plasmas the presence of grains results in even more crucial changes. At large densities (the Havnes parameter exceeds 0.5) of small ($a/\lambda_D = 0.01$) grains the ion-acoustic maxima become visible.

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