High-temperature ratchets driven by deterministic and stochastic fluctuations

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We consider the overdamped dynamics of a Brownian particle in an arbitrary spatial periodic and time-dependent potential on the basis of an exact solution for the probability density in the form of a power series in the inverse friction coefficient. The expression for the average velocity of a Brownian ratchet is simplified in the high-temperature consideration when only the first terms of the series can be used. For the potential of an additive-multiplicative form (a sum of a time-independent contribution and a time-dependent multiplicative perturbation), general explicit expressions are obtained which allow comparative analysis of frequency dependencies of the average velocity, implying deterministic and stochastic potential energy fluctuations. For qualitative and quantitative analysis of these dependences, we choose illustrative examples for spatial harmonic fluctuations: with deterministic time dependences of a relaxation type and stochastic time dependences describing Markovian dichotomous and harmonic noise processes. We explore the influence of fluctuation types on the ratchet effect and demonstrate its enhancement in the case of harmonic noise.

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I. INTRODUCTION

The ability of Brownian particles to move directionally subjected to a ratchet mechanism can appear under the action of external processes of different nature [1–5]. Most of human-created processes used in ratchets, as a source of driving, are deterministic ones; they are cyclically repeated in time and described by periodic functions [2,3,5–11]. Stochastic processes usually govern operation of ratchetlike objects in biological systems (e.g., protein motors in cells) [1,5,12–17]. It is important that average values of driving forces acting on a particle due to these processes are zero (unbiased perturbations), but asymmetry of a system and effects (induced by the perturbations) that are nonlinear in potential energy fluctuations lead to a directed particle current. Among the first examples of such effect is the appearance of a constant electric current under the action of a high-frequency electromagnetic field in media without a center of symmetry (photovoltaic effect) [18].

Speaking of deterministic fluctuations in ratchet systems, it is necessary to single out a special class of harmonic (in time) fluctuations [19–25]; they are easily realizable and can be described by the Fourier transform method. Biharmonic fluctuations can also be a source of a ratchet driving force [26]. Deterministic dichotomic processes are no less popular in theoretical studies and experimental realizations of ratchets. These processes can be described as a two-state model with the two states alternating in time (each of which has time-invariant characteristics) [27–31]. At the same time, there exist a number of ratchet systems characterized by the relaxation delay on the deterministic dichotomic process, e.g., on periodic rectangular laser pulses [32]. In this case, a theory must be expanded to describe an arbitrary periodic process [28,32–34].

The presence of stochastic fluctuations usually assumes that there is a set of discrete states (describing, for example, conformational states of a protein, modeled by a Brownian ratchet) between which transitions occur with certain rate constants [1,35]. Since, in each conformational state, a particle (say, a protein) interacts with an environment in different ways, we have the following problem statement: particle potential energy is a function of a state's number, and particle dynamics is governed by equations containing rate constants of transitions between the states. If the states are well defined and durations of transitions between them are essentially shorter than lifetimes of the states, one can consider that the rate constants are independent of the process history, that is, the process is Markovian [36]. For the convenience of description, it is usually limited to a small number of states [37–39], most often two [27,28,33,39–48]; in the latter case, one speaks of a stochastic dichotomous process. A somewhat more general process is harmonic noise [49–53], the results of which are interesting to compare with those obtained for the stochastic dichotomous process. Along with Markov processes that can control the ratchet operation, there exist various models of anomalous molecular motors with non-Markovian diffusion,

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subdiffusing rocking [54–57] and flashing [58–60] ratchets (see also [4,61,62] and references therein).

A vast majority of ratchet models suggest overdamped dynamics in which friction dominates inertia and hence inertial effects can be neglected. On the other hand, there are ratchets operating due to dry (Coulombic) friction [63–67]. Such systems include unidirectional rotation of granular motors [63,65], unidirectional motion of a solid object along a surface [64], or the asymmetric Rayleigh piston [66]. The description of these systems rejects the overdamped approximation and requires taking into account inertial effects on the basis of a Boltzmann-Lorentz equation [63] or Kramers equation [67] in which velocity-dependent friction terms can be included. Such ratchets are beyond the scope of our article.

The influence of fluctuations of different types on ratchet characteristics can be compared effectively in case one has explicit analytical expressions for quantities of interest. Such an opportunity can be provided, for example, by the representation of solutions of equations (namely, Smoluchowski equation) in the form of a power series in the inverse friction coefficient [28,33]. In the present paper, we generalize this approach to obtain analytical expressions for the average velocity of high-temperature ratchets driven by fluctuations of different nature. The structure of the paper is as follows. In Sec. II, we give the solution of the ratchet problem in a series form. In Sec. III, we simplify the solution supposing the high-temperature regime of ratchet operation with potential energy fluctuations of different nature, including deterministic periodic processes and stochastic Markovian processes with the special emphasis on dichotomous and harmonic noises. Further we restrict our consideration to a spatial harmonic signal (Sec. IV), which essentially simplifies the final expression for the average particle velocity and, hence, the analysis of different cases of stationary potentials and fluctuation types. The results are summarized in Sec. V.

II. THE SOLUTION OF THE RATCHET PROBLEM IN A SERIES FORM

Following the approach proposed in Refs. [28,33], consider the overdamped dynamics of a Brownian particle in a viscous medium, which can be statistically described by its distribution function $\rho(x,t)$ that satisfies the Smoluchowski equation [68],

$$\frac{\partial}{\partial t}\rho(x,t) = -\frac{\partial}{\partial x}J(x,t),\qquad J(x,t) = \zeta^{-1}\left[F(x,t)\rho(x,t) - k_B T\frac{\partial}{\partial x}\rho(x,t)\right], \tag{1}$$

and the normalization condition $\int_0^L \rho(x,t)dx = 1$. Here, ζ is the friction coefficient, k_B is the Boltzmann constant, T is the equilibrium absolute temperature, and $F(x,t) = -\partial U(x,t)/\partial x$ is a spatially periodic (with the period L) and time-dependent applied force that corresponds to the potential energy $U(x,t)$. The instantaneous particle velocity is determined through the flux $J(x,t)$ as

$$v(t) = \int_0^L dx\, J(x,t). \tag{2}$$

Spatial periodicity of the driving $F(x,t)$ and the consideration of the steady-state process allow the Fourier representation for any spatial function $f(x,t) = f(x+L,t)$ appearing in our model:

$$f(x,t) = \sum_q f_q(t)\exp(ik_q x),\qquad k_q = (2\pi/L)q,\quad q = 0,\pm1,\ldots. \tag{3}$$

On the basis of this, the integral form of differential equation (1) can be written as

$$\rho_q(t) = L^{-1}\delta_{q,0} - i\zeta^{-1}k_q\exp\left(-Dk_q^2 t\right) \times \sum_{q'}\int_{-\infty}^{t} dt' F_{q'}(t')\rho_{q-q'}(t')\exp\left(Dk_q^2 t'\right), \tag{4}$$

where $D = k_B T/\zeta$ is the diffusion coefficient, $\delta_{q,0} = 1$ at $q = 0$, and $\delta_{q,0} = 0$ at $q \neq 0$. The initial time $t = -\infty$ has been chosen here to eliminate the influence of transient processes which "remember" initial conditions. Using (1) and (2), one can also represent the velocity as a sum,

$$v(t) = \zeta^{-1}L\sum_q F_q(t)\rho_{-q}(t). \tag{5}$$

Substituting an iterative solution of Eq. (4) in the form of a power series in ζ^{-1} into Eq. (5) yields a general expression for the velocity,

$$\begin{aligned} v(t) = {}& \zeta^{-1}F_0(t) + \sum_{n=1}^{\infty} i^n\zeta^{-n-1}\sum_{q_1,\ldots,q_n} k_{q_1}k_{q_1+q_2}\cdots k_{q_1+q_2+\cdots+q_n}\int_0^\infty d\tau_1\int_0^\infty d\tau_2\cdots\int_0^\infty d\tau_n \\ & \times F_{q_1}(t)F_{q_2}(t-\tau_1)\cdots F_{q_n}(t-\tau_1-\cdots-\tau_{n-1})F_{-q_1-q_2-\cdots-q_n}(t-\tau_1-\cdots-\tau_{n-1}-\tau_n) \\ & \times \exp\left(-Dk_{q_1}^2\tau_1 - Dk_{q_1+q_2}^2\tau_2 - \cdots - Dk_{q_1+q_2+\ldots+q_n}^2\tau_n\right). \end{aligned} \tag{6}$$

Next we will use the fact that a broad class of functions of two variables can be represented in the following general additive-multiplicative form:

$$F(x,t) = \sum_{r=1}^{N}\sigma^{(r)}(t)f^{(r)}(x). \tag{7}$$

The functions $\sigma^{(r)}(t)$ govern the time dependence of the driving and describe the features of the fluctuations, which can be either of stochastic or deterministic nature. We will consider the operation of averaging over the fluctuations; hereinafter it will be denoted by the brackets $\langle\cdot\rangle$. It is clear that the definition of the operation depends on the nature of the time-dependent quantities $\sigma^{(r)}(t)$. Following the abstract in [1], directed transport in the case of so-called Brownian motors (ratchets) implies the consideration of "a dissipative dynamics in the presence of thermal noise and some prototypical perturbation that drives the system out of equilibrium without introducing *a priori* an obvious bias into one or the other direction of motion." Thus, one needs to distinguish a nontrivial ratchet effect from the trivial particle motion with an applied bias. It means that we must put $\langle F_0(t)\rangle = \sum_r \langle\sigma^{(r)}(t)\rangle f_0^{(r)} = 0$; it can be realized when either $f_0^{(r)} = 0$ while $\sigma^{(r)}(t)$ is arbitrary or for $f_0^{(r)} \neq 0$ with $\langle\sigma^{(r)}(t)\rangle = 0$. The first and second cases describe, respectively, the so-called flashing (pulsating) and rocking (forced) ratchets, that is, the appearance of a ratchet effect due to fluctuations of periodic potential profiles in the former case and of a tilting force in the latter. Applying the operation of averaging to the instantaneous particle velocity (6), we obtain the desired general formula for the average particle (ratchet) velocity:

$$v \equiv \langle v(t)\rangle = \sum_{n=1}^{\infty} i^n \zeta^{-n-1} \sum_{q_1,\dots,q_n} k_{q_1}k_{q_1+q_2}\cdots k_{q_1+q_2+\dots+q_n} \sum_{r_1,\dots,r_{n+1}} f_{q_1}^{(r_1)} f_{q_2}^{(r_2)}\cdots f_{q_n}^{(r_n)} f_{-q_1-q_2-\dots-q_n}^{(r_{n+1})} \int_0^\infty d\tau_1$$
$$\times \int_0^\infty d\tau_2 \cdots \int_0^\infty d\tau_n \langle\sigma^{(r_1)}(t)\sigma^{(r_2)}(t-\tau_1)\cdots\sigma^{(r_n)}(t-\tau_1-\dots-\tau_{n-1})\sigma^{(r_{n+1})}(t-\tau_1-\dots-\tau_{n-1}-\tau_n)\rangle$$
$$\times \exp\left(-Dk_{q_1}^2\tau_1 - Dk_{q_1+q_2}^2\tau_2 - \dots - Dk_{q_1+q_2+\dots+q_n}^2\tau_n\right). \quad (8)$$

This exact expression shows that the main ratchet characteristic, its average velocity, is determined by the correlation functions of different orders, $\langle\sigma^{(r_1)}(t)\sigma^{(r_2)}(t-\tau_1)\cdots\sigma^{(r_n)}(t-\tau_1-\dots-\tau_{n-1})\sigma^{(r_{n+1})}(t-\tau_1-\dots-\tau_{n-1}-\tau_n)\rangle$. The low-order correlation functions determine the average velocity of high-temperature ratchets (for which a ratio of a spatial amplitude of a ratchet potential to the thermal energy is suggested to be small); in this case, the summation over n is confined, for the flashing ratchet type, to $n = 1$ and 2. Such ratchets will be considered in detail further.

III. HIGH-TEMPERATURE RATCHETS

For the purposes of the analysis in this section, it is enough to consider a flashing ratchet with the potential energy $U(x, t)$ of the simplest additive-multiplicative form,

$$U(x, t) = u(x) + \sigma(t)w(x), \quad (9)$$

which is a particular case of the representation (7) corresponding to $N = 2$ with $\sigma^{(1)}(t) = 1$, $f^{(1)}(x) = -du(x)/dx$, $\sigma^{(2)}(t) = \sigma(t)$, and $f^{(2)}(x) = -dw(x)/dx$. With these assumptions made, the average particle velocity for high-temperature ratchets can be written as

$$\langle v\rangle = \frac{i}{D\zeta^3} \sum_{\substack{qq'(\neq 0)\\(q+q'\neq 0)}} k_q k_{q'} k_{q+q'} w_q w_{q'} \left[u_{-q-q'}\Psi_2\left(Dk_q^2, Dk_{q'}^2\right) - Dk_q k_{q'} w_{-q-q'}\Psi_3\left(Dk_q^2, Dk_{q'}^2\right)\right], \quad (10)$$

where we have introduced the functions

$$\Psi_2(a, b) = \frac{1}{a-b}\int_0^\infty d\tau\, K_2(\tau)(ae^{-a\tau} - be^{-b\tau}),$$
$$\Psi_3(a, b) = \int_0^\infty d\tau \int_0^\infty d\tau' K_3(\tau, \tau')e^{-a\tau - b\tau'}, \quad (11)$$

and

$$K_2(\tau) \equiv \langle\sigma(t)\sigma(t-\tau)\rangle,$$
$$K_3(\tau, \tau') \equiv \langle\sigma(t)\sigma(t-\tau)\sigma(t-\tau-\tau')\rangle \quad (12)$$

are the second- and third-order correlation functions.

The importance of the result (10)–(12) lies in the fact that these relations allow analyzing fluctuations of any nature, stochastic or deterministic, which hence can be described by arbitrary functions of time. One can see that only two correlation functions, of the second and third order, are enough for the description of high-temperature ratchets. Time-symmetric fluctuations are characterized by zero values of the odd correlation functions, so that $\Psi_3(a, b) = 0$ and only one (first) term remains in Eq. (10). In this case, the average particle velocity is an odd functional of the stationary part $u(x)$ of the potential and equals zero at $u(x) = 0$ (as a simple example here, a potential fluctuates in sign). For time-asymmetric fluctuations, the first and the second terms in Eq. (10) can compete. Hence, the direction of motion (the sign of $\langle v\rangle$) in this case will be dictated by the result of the competition between the spatial and time asymmetry of the particle potential energy. Such a competition has been considered in detail in Ref. [47] for potential energy fluctuations described by an arbitrary dichotomous stochastic process. Below we consider various types of processes which can operate time dependences of fluctuations: deterministic and stochastic processes of the general form.

Deterministic periodic processes. They are the simplest and widely used processes. The processes of this type are responsible, for example, for the motion of particles suspended in solution and exposed to a periodic asymmetric potential [69,70] (dielectrophoresis effect [71]), vortices in superconductors [72], atoms in dissipative optical lattices [73], and electrons in organic semiconductors [31]. The potential energy changes are governed by the periodic function $\sigma(t) = \sigma(t+\tau)$ (τ is the period); it can be expanded into the Fourier series

$$\sigma(t) = \sum_j \tilde{\sigma}_j \exp(-i\omega_j t),$$
$$\omega_j = 2\pi j/\tau, \quad j = 0, \pm 1, \dots, \quad (13)$$

with the Fourier components $\tilde{\sigma}_j$. The operation of averaging in the case of periodic functions means the averaging over the period, $\tau : \langle\dots\rangle = \tau^{-1}\int_0^\tau dt\ \dots$; which gives the desired

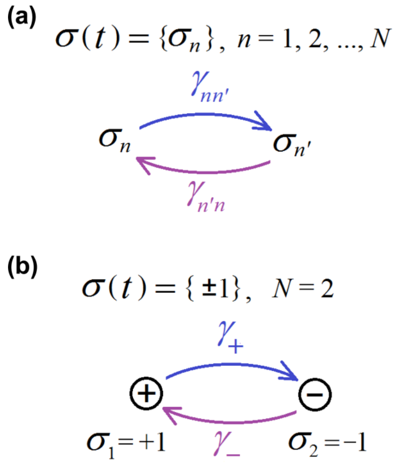

FIG. 1. The scheme of transitions between the states of the Markovian process: (a) general case (N-state model) and (b) dichotomous process.

correlation functions,

$$K_2(t) = \sum_j |\tilde{\sigma}_j|^2 \exp(-i\omega_j t),$$
$$K_3(t,t') = \sum_{jj'} \tilde{\sigma}_j \tilde{\sigma}_{j'} \tilde{\sigma}_{-j-j'} \exp(-i\omega_j t - i\omega_{-j'} t'). \tag{14}$$

By substitution of Eq. (14) to Eq. (11), one can arrive at the following explicit expressions for the $\Psi_{2,3}$ functions:

$$\Psi_2(a,b) = 2(a+b)\sum_{j=1}^{\infty} \frac{\omega_j^2 |\tilde{\sigma}_j|^2}{(\omega_j^2 + a^2)(\omega_j^2 + b^2)},$$
$$\Psi_3(a,b) = \frac{1}{2}\sum_{jj'} \frac{\tilde{\sigma}_j \tilde{\sigma}_{j'} \tilde{\sigma}_{-j-j'} + \tilde{\sigma}_{-j} \tilde{\sigma}_{-j'} \tilde{\sigma}_{j+j'}}{(i\omega_j + a)(i\omega_{-j'} + b)}, \tag{15}$$

determining the average velocity. It can be shown that only second-order correlation functions contribute to the average velocity [$\Psi_3(a,b) = 0$] if $\sigma(t)$ belongs to the shift-symmetric (supersymmetric) or antisymmetric functions [32,74] [for deterministic dichotomous processes, such functions $\sigma(t)$ describe transitions between the two states with equal lifetimes].

Stochastic Markovian processes. Let the function $\sigma(t)$ be a stochastic variable with its discrete values σ_n, $n = 1, 2, \ldots, N$, and the $\sigma_n \rightleftarrows \sigma_{n'}$ transitions are characterized by certain time-independent rate constants $\gamma_{nn'}$ and $\gamma_{n'n}$ [Fig. 1(a)]. The main quantity in the description of a stochastic process is the conditional probability $\rho_{nn'}(t)$ of finding the system in a state n at time t given that it was in a state n' at the previous initial time, $t = 0$; the time dependence of this quantity is governed by the Pauli master equation,

$$\frac{d}{dt}\rho_{nn'}(t) + \sum_{n''} \Gamma_{nn''}\rho_{n''n'}(t) = 0,$$
$$\Gamma_{nn''} = \delta_{nn''}\sum_{n'} \gamma_{nn'} - \gamma_{n''n}, \tag{16}$$

with the initial condition $\rho_{nn'}(0) = \delta_{nn'}$. This equation describes the memoryless Markovian processes: the rate of change of $\rho_{nn'}(t)$ depends on $\rho_{nn'}(t)$ at the same time moment (future depends on the present and not the past). Note that the equilibrium solutions $\rho_n^{(0)}$ of Eq. (16) do not depend on n' and obey the detailed balance principle, $\gamma_{n'n}\rho_{n'}^{(0)} = \gamma_{nn'}\rho_n^{(0)}$, which is valid for all systems in thermal equilibrium.

Following the approach proposed in Ref. [36] (see also Refs. [20,75]), the solution of the differential equation (16) can be represented in the most general form using the eigenvalues Γ_j and corresponding eigenvectors C_{nj} of the matrix $\Gamma_{nn'}$, defined by the equation

$$\sum_{n'} \Gamma_{nn'} C_{n'j} = \Gamma_j C_{nj}. \tag{17}$$

The matrix $\Gamma_{nn'}$ obeys the nonsymmetric condition $\Gamma_{n'n}\rho_n^{(0)} = \Gamma_{nn'}\rho_{n'}^{(0)}$ following from the detailed balance principle and the definition of $\Gamma_{nn'}$ in Eq. (16). Introduce an ancillary symmetric matrix $\tilde{\Gamma}_{nn'} = (\rho_n^{(0)})^{-1/2}\Gamma_{nn'}(\rho_{n'}^{(0)})^{1/2}$ and vectors $\tilde{C}_{nj} = (\rho_n^{(0)})^{-1/2}C_{nj}$ satisfying the equation $\sum_{n'}\tilde{\Gamma}_{nn'}\tilde{C}_{n'j} = \Gamma_j\tilde{C}_{nj}$ similar to Eq. (17). This equation with $\tilde{\Gamma}_{nn'} = \tilde{\Gamma}_{n'n}$ leads to the reality of the eigenvalues Γ_j and to the unitarity of $\tilde{C}_{nj}$, from which the unitarity of C_{nj} accurate to the weight factors $\rho_n^{(0)}$ follows:

$$\sum_n \left(\rho_n^{(0)}\right)^{-1} C_{nj}C_{nj'}^* = \delta_{jj'}, \quad \sum_j C_{nj}^* C_{n'j} = \rho_n^{(0)}\delta_{nn'}. \tag{18}$$

Since $\sum_n \Gamma_{nn'} = 0$, we have $\sum_{nn'}\Gamma_{nn'}C_{n'j} = \Gamma_j\sum_n C_{nj} = 0$. The linear independence of rows C_n suggests that at least one eigenvalue Γ_j is equal to zero. Assume that $j = 0$ corresponds to the zero value, $\Gamma_0 = 0$, and that $\sum_n C_{nj} = 0$ at $j \neq 0$. Using the second Eq. (18) and the normalization condition $\sum_n \rho_n^{(0)} = 1$, we arrive at the following useful relations: $C_{n0} = \rho_n^{(0)}$ and $\sum_n C_{nj} = \delta_{j0}$. With these relations, one can express the matrix $\Gamma_{nn'}$ in terms of its eigenvalues and eigenvectors as $\Gamma_{nn'} = (\rho_{n'}^{(0)})^{-1}\sum_j \Gamma_j C_{nj}C_{n'j}^*$, and hence get the time dependence of the desired conditional probability (the solution of Eq. (16)):

$$\rho_{nn'}(t) = \left(\rho_{n'}^{(0)}\right)^{-1}\sum_j C_{nj}C_{n'j}^* \exp(-\Gamma_j t). \tag{19}$$

Using the definition of $\Gamma_{nn'}$ in Eq. (16), one can show that

$$\Gamma_j = \frac{1}{2}\sum_{nn'}\gamma_{nn'}\rho_n^{(0)}\left|\frac{C_{nj}}{\rho_n^{(0)}} - \frac{C_{n'j}}{\rho_{n'}^{(0)}}\right|^2 \geqslant 0. \tag{20}$$

Thus, from this inequality, we get the non-negativity of the eigenvalues Γ_j, and the equality $\Gamma_0 = 0$ follows for which the corresponding eigenvector $C_{n0} = \rho_n^{(0)}$. Since $\Gamma_j > 0$ at $j > 0$, the equilibrium is established at $t \to \infty$ [$\rho_{nn'}(\infty) = \rho_n^{(0)}$; see Eq. (19)] as it should be.

Using the normalized orthogonal conditions (18) with the weight factors $\rho_n^{(0)}$, it is easy to obtain Kolmogorov-Chapman equation,

$$\sum_{n'} \rho_{nn'}(t)\rho_{n'n''}(t') = \rho_{nn''}(t+t'), \tag{21}$$

which is the basis for the derivation of the Pauli master equation (16) valid for Markovian processes [36].

The operation of averaging of a product of r fluctuations $\sigma(t)$, taken at different times $t_1, t_2, \ldots, t_r$, is defined by means of r-point unconditional probabilities, $p_{n_1 n_2 \ldots n_r}(t_1|t_2|\cdots|t_r)$; the last are calculated using the conditional probabilities,

$$p_{n_1 n_2 \ldots n_r}(t_1|t_2|\cdots|t_r) \underset{t_1>t_2>\cdots>t_r}{=} \rho_{n_1 n_2}(t_1 - t_2) p_{n_2 \ldots n_r}(t_2|\cdots|t_r), \quad p_n(t) = \rho_n^{(0)}, \tag{22}$$

so that for the desired second- and third-order correlation functions, we have

$$K_2(t) = \sum_{nn'} \sigma_n \sigma_{n'} p_{nn'}(t|0) = \sum_{nn'} \sigma_n \sigma_{n'} \rho_{nn'}(t)\rho_{n'}^{(0)},$$
$$K_3(t,t') = \sum_{nn'n''} \sigma_n \sigma_{n'} \sigma_{n''} p_{nn'n''}(t+t'|t'|0) = \sum_{nn'n''} \sigma_n \sigma_{n'} \sigma_{n''} \rho_{nn'}(t)\rho_{n'n''}(t')\rho_{n''}^{(0)}, \tag{23}$$

or, after substituting Eq. (19) to Eq. (23), they become

$$K_2(t) = \sum_j |\tilde{\sigma}_j|^2 \exp(-\Gamma_j t),$$
$$K_3(t,t') = \sum_{jj'} \tilde{\sigma}_j \tilde{\sigma}_{j'}^* \tilde{\sigma}_{jj'} \exp(-\Gamma_j t - \Gamma_{j'} t'), \tag{24}$$

where

$$\tilde{\sigma}_j = \sum_n \sigma_n C_{nj}, \quad \tilde{\sigma}_{jj'} = \sum_n \sigma_n \left(\rho_n^{(0)}\right)^{-1} C_{nj}^* C_{nj'}. \tag{25}$$

Integrals in Eq. (11) with the correlation functions (24) are easily taken, and we come to the expressions for the auxiliary functions,

$$\Psi_2(a,b) = \sum_{j=1}^{N} \frac{\Gamma_j |\tilde{\sigma}_j|^2}{(\Gamma_j + a)(\Gamma_j + b)},$$
$$\Psi_3(a,b) = \frac{1}{2} \sum_{jj'} \frac{\tilde{\sigma}_j \tilde{\sigma}_{j'}^* \tilde{\sigma}_{jj'} + \tilde{\sigma}_j^* \tilde{\sigma}_{j'} \tilde{\sigma}_{jj'}^*}{(\Gamma_j + a)(\Gamma_{j'} + b)}. \tag{26}$$

Stochastic Markovian dichotomous noise. As a simple illustration of the stochastic Markovian processes, here we consider dichotomous noise (there exists a great variety of phenomena caused by dichotomous noise) which corresponds to the particular case of the above model, when $N = 2$, and the function $\sigma(t)$ can take only two values, $+1$ and -1, with the rate constants $\gamma_{12} = \gamma_+$ and $\gamma_{21} = \gamma_-$ [Fig. 1(b)]. The rate constants matrix is readily written as

$$\hat{\Gamma} = \begin{pmatrix} \gamma_+ & -\gamma_- \\ -\gamma_+ & \gamma_- \end{pmatrix}, \tag{27}$$

and it has the following eigenvalues and eigenvectors (columns of the $\hat{C}$ matrix):

$$\Gamma_0 = 0, \quad \Gamma_1 = \gamma_+ + \gamma_-,$$
$$\hat{C} = \frac{1}{\gamma_+ + \gamma_-} \begin{pmatrix} \gamma_- & \gamma_+ \\ -\sqrt{\gamma_+\gamma_-} & \sqrt{\gamma_+\gamma_-} \end{pmatrix}. \tag{28}$$

This result allows obtaining the matrix elements (25),

$$\tilde{\sigma}_0 = \frac{\gamma_- - \gamma_+}{\gamma_+ + \gamma_-}, \quad \tilde{\sigma}_1 = -\frac{2\sqrt{\gamma_+\gamma_-}}{\gamma_+ + \gamma_-}, \quad \tilde{\sigma}_{jj'} = \begin{pmatrix} \tilde{\sigma}_0 & \tilde{\sigma}_1 \\ \tilde{\sigma}_1 & -\tilde{\sigma}_0 \end{pmatrix}, \tag{29}$$

and with them we arrive at the solution of the Pauli master equation (16),

$$\rho_{\sigma\sigma'}(t) = \tfrac{1}{2}\{1 + e^{-\Gamma_1 t}\sigma\sigma' + [1 - e^{-\Gamma_1 t}]\tilde{\sigma}_0 \sigma\}, \quad \sigma, \ \sigma' = \pm 1. \tag{30}$$

It is easy to check validity of the following equalities, which the conditional probability (30) obeys:

$$\rho_{\sigma\sigma'}(0) = \delta_{\sigma\sigma'}, \quad \rho_{\sigma\sigma'}(\infty) = \frac{1}{2}\{1 + \tilde{\sigma}_0 \sigma\} = \rho_\sigma^{(0)},$$
$$\sum_\sigma \rho_{\sigma\sigma'}(t) = 1, \quad \sum_{\sigma'} \rho_{\sigma\sigma'}(t)\rho_{\sigma'\sigma''}(t') = \rho_{\sigma\sigma''}(t+t'). \tag{31}$$

Next, substituting expressions (29) to Eqs. (24) and (26) gives both the desired correlation functions and the auxiliary functions $\Psi_2(a,b)$ and $\Psi_3(a,b)$,

$$K_2(t) = \tilde{\sigma}_0^2 + \tilde{\sigma}_1^2 e^{-\Gamma_1 t},$$
$$K_3(t,t') = \tilde{\sigma}_0\big[\tilde{\sigma}_0^2 + \tilde{\sigma}_1^2(e^{-\Gamma_1 t} + e^{-\Gamma_1 t'} - e^{-\Gamma_1 (t+t')})\big], \tag{32}$$

$$\Psi_2(a,b) = \frac{\Gamma_1 \tilde{\sigma}_1^2}{(\Gamma_1 + a)(\Gamma_1 + b)},$$
$$\Psi_3(a,b) = \frac{\tilde{\sigma}_0^3}{ab} + \tilde{\sigma}_0 \tilde{\sigma}_1^2 \frac{\Gamma_1(a+b) + ab}{ab(\Gamma_1 + a)(\Gamma_1 + b)}, \tag{33}$$

so one can get the final expression (10) for the ratchet average velocity [28],

$$\langle v \rangle = i\Gamma_1 \tilde{\sigma}_1^2 \beta^3 D \sum_{\substack{qq'(\neq 0) \\ (q+q' \neq 0)}} k_{q+q'} w_q w_{q'} \times \frac{D k_q k_{q'} u_{-q-q'} + \Gamma_1 \tilde{\sigma}_0 w_{-q-q'}}{\left(\Gamma_1 + D k_q^2\right)\left(\Gamma_1 + D k_{q'}^2\right)}. \tag{34}$$

Harmonic noise. We can consider a coordinate of a harmonic oscillator driven by white noise as nonequilibrium fluctuations controlling the motion of the ratchet. Such a model was used for a correlation ratchet in Ref. [51]. Here we apply this type of noise, called harmonic noise, to "drive" high-temperature flashing ratchets, the average velocities of which are defined by the lowest-order correlation functions [see Eqs. (10)–(12)]. We will be interested in the second-order correlation function $K_2(t) = \langle \varepsilon(t)\varepsilon(0)\rangle / \langle \varepsilon^2(0)\rangle$ of the harmonic noise process $\varepsilon(t)$ with zero average, $\langle\varepsilon(t)\rangle = 0$, described through the stochastic differential equation

$$\ddot{\varepsilon}(t) + \nu\dot{\varepsilon}(t) + \Omega_0^2\varepsilon(t) = \xi(t). \quad (35)$$

Here, ν and Ω_0 are the damping and frequency parameters, and $\xi(t)$ is zero centered Gaussian white noise of intensity α with the correlation function $\langle\xi(t)\xi(t')\rangle = 2\alpha\delta(t-t')$. It is easy to show that the desired correlation function $K_2(t)$ can be written as [49–53]

$$K_2(t) = \frac{\theta_1 e^{-\theta_2 t} - \theta_2 e^{-\theta_1 t}}{\theta_1 - \theta_2} = \exp\left(-\frac{1}{2}\nu t\right)\left(\cosh\Omega_1 t + \frac{\nu}{2\Omega_1}\sinh\Omega_1 t\right), \quad (36)$$

where θ_1 and θ_2 are the roots of the quadratic equation $\theta^2 - \nu\theta + \Omega_0^2 = 0$, $\theta_{1,2} = \nu/2 \pm \Omega_1$, $\Omega_1 = \sqrt{\nu^2/4 - \Omega_0^2}$.

Introduce the correlation time for harmonic noise, which can be easily evaluated by the relation $\tau = \int_0^\infty K_2(t)dt = \nu/\Omega_0^2$. When $\nu \to \infty$, $\Omega_0 \to \infty$, and τ is fixed, one obtains $K_2(t) = \exp(-t/\tau)$ so that harmonic noise approaches Ornstein-Uhlenbeck noise of correlation time τ. Since the second-order correlation functions of Ornstein-Uhlenbeck and symmetric dichotomous noises coincide [see Eq. (32) with $\Gamma_1 = \tau^{-1}$], we can use the quantity $\Psi_2(a,b)$ to analyze the effect of the damping parameter ν on the ratchet velocity. Substituting Eq. (36) to (11) leads to the expression

$$\Psi_2(a,b) = \Gamma_1\frac{\nu(\nu+a+b)}{[\nu(\Gamma_1+a)+a^2][\nu(\Gamma_1+b)+b^2]}. \quad (37)$$

It is reduced to Eq. (33) for $\Psi_2(a,b)$ with $\tilde{\sigma}_0 = 0$ and $\tilde{\sigma}_1 = 1$ in the limit $\nu \to \infty$, as it should be.

IV. ILLUSTRATIVE EXAMPLES FOR A SPATIAL HARMONIC SIGNAL

We now will illustrate the ratchet behavior for concrete laws of variation of a ratchet potential in time. To emphasize the consideration of different time dependences of the potential, we confine analysis to the simplest case of a spatial harmonic signal which governs the ratchet functioning,

$$w(x) = w\cos 2\pi(x/L - \lambda_0), \quad w_q = (w/2)(e^{-2\pi i\lambda_0}\delta_{q,1} + e^{2\pi i\lambda_0}\delta_{q,-1}), \quad (38)$$

where $2\pi\lambda_0$ $(0 \leqslant \lambda_0 \leqslant 1)$ is the phase shift; it provides the maximal positive signal value (w) to be reached at $x/L = \lambda_0$. Since the products $w_q w_{q'} w_{-q-q'}$ equal zero for such a signal, and summation in Eq. (10) is limited to the values $q,\ q' = \pm 1$, the average velocity is represented in the simple form:

$$\langle v\rangle = k_1 D\tilde{\Psi}_2\beta^3 w^2 \mathrm{Im}\{u_2 e^{4\pi i\lambda_0}\}, \quad (39)$$

where we introduce the dimensionless function

$$\tilde{\Psi}_2 \equiv Dk_1^2\Psi_2(Dk_1^2, Dk_1^2), \quad (40)$$

which is "responsible" for the character of time fluctuations. It is significant that the expression (39) contains the product of functions $\tilde{\Psi}_2$ and $\beta^3 w^2\mathrm{Im}\{u_2 e^{4\pi i\lambda_0}\}$ depending on the time (the former) and spatial (the latter) characteristics of the ratchet potential energy. In this case, one can analyze the contribution of those characteristics independently. So, let us proceed to the examples of such dependences as well as to discussing the role of the fluctuation's nature.

Two-well stationary potential. One can see from Eq. (39) that if a spatial signal $w(x)$ is described by only the first harmonic, only the second harmonic of the stationary potential $u(x)$ contributes to the average ratchet velocity. In this connection, it is not surprising that potentials of the first two harmonics, which provide the asymmetry necessary for ratchet operating, are so popular in ratchet models [1]. An example of a real system with potential energy of this type is a planar dipole rotor in a two-well symmetric potential (of the hindered rotation) placed in an alternating electric field $E(t)$ [20],

$$U(\varphi,t) = \tfrac{1}{2}u[1-\cos 2(\varphi-\Phi)] - \mu E(t)\cos(\varphi-\varphi_0), \quad (41)$$

where μ is the rotor dipole moment, u is the barrier of the hindered rotation, and Φ and φ_0 are the phase shifts of the stationary and fluctuating parts of the potential. If we put $\varphi = 2\pi x/L$, $\Phi = 2\pi\lambda$, and $\varphi_0 = 2\pi\lambda_0$, we have $u(x) = (u/2)[1-\cos 4\pi(x/L-\lambda)]$ so that the second harmonic is $u_2 = -(u/4)e^{-4\pi i\lambda}$, and the imaginary part from Eq. (39) becomes

$$\mathrm{Im}\{u_2 e^{4\pi i\lambda_0}\} = \frac{u}{4}\sin 4\pi(\lambda-\lambda_0). \quad (42)$$

The direction of motion of this ratchet is determined by the phase shifts of the potentials.

Sawtooth stationary potential. This is another practice oriented example. The stationary (unperturbed) part of the potential energy is a piecewise linear function,

$$u(x) = \begin{cases} ux/l, & 0 < x < l, \\ u(L-x)/(L-l), & l < x < L \end{cases} \quad (43)$$

which second harmonic is

$$u_2 = -u\frac{1-e^{-ik_2 l}}{l(L-l)k_2^2}, \quad k_2 = \frac{4\pi}{L}, \quad \lambda = \frac{l}{L}, \quad (44)$$

and we obtain

$$\mathrm{Im}\{u_2 e^{4\pi i\lambda_0}\} = -\frac{u}{8\pi^2}f_2(\lambda,\lambda_0), \quad f_2(\lambda,\lambda_0) = \frac{\sin 2\pi\lambda\sin 2\pi(\lambda-2\lambda_0)}{\lambda(1-\lambda)}. \quad (45)$$

The dependence on the geometrical parameters of potential profiles turns out to be alternating (see Fig. 2). This means that the direction of motion depends not only on the asymmetry of a sawtooth potential, but also on the phase shift of harmonic fluctuations. The inset of Fig. 2 shows the regions of λ and λ_0 values at which the motion in the positive and negative directions is realized. Thus, the application of harmonic fluctuations to an asymmetric sawtooth profile allows controlling the direction of motion of Brownian motors. The experimental realization of that can be, for example, the use of interference of laser beams propagating in opposite directions and forming a spatially periodic potential which is widely used in Brownian motors on optical lattices [73,76,77]. In this case, the phase shift of such a potential with respect to the asymmetric

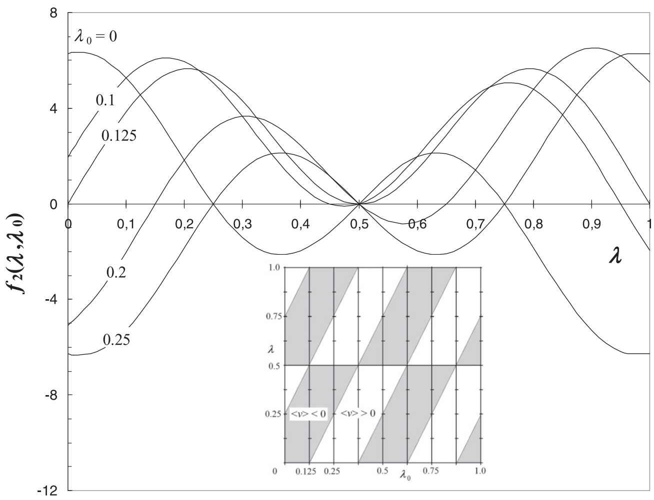

FIG. 2. Dependence of the dimensionless factor $f_2(\lambda, \lambda_0)$ [see Eq. (45)] determining the average velocity of a high-temperature Brownian motor on the geometric parameters of fluctuating and stationary parts of the potential. The inset illustrates the intervals of λ (symmetry parameter) and λ_0 (phase shift) values of the sawtooth potential at which the motor moves to the right or left (light and dark areas, respectively).

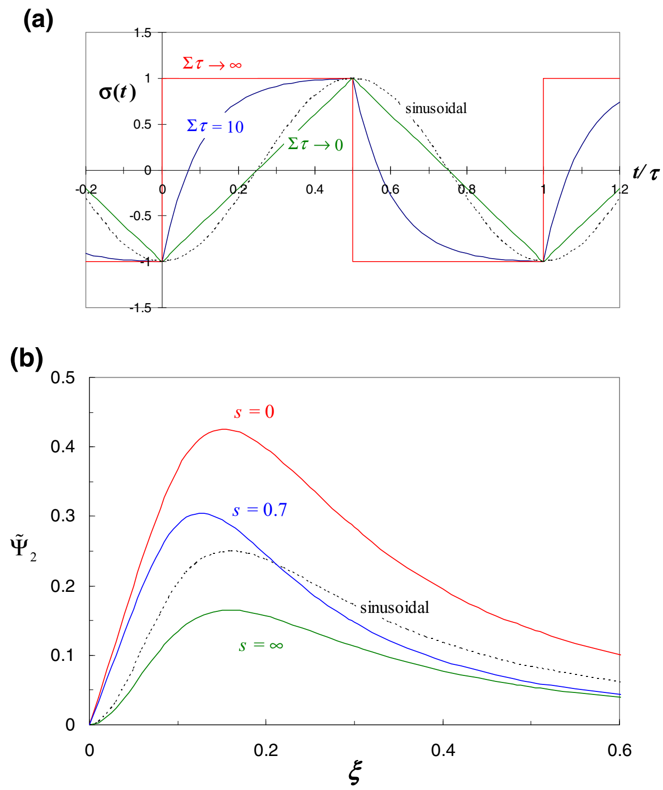

FIG. 3. Different shapes of the relaxation-type periodic time dependence $\sigma(t)$ [see Eq. (46)] for (a) different values of the normalized inverse relaxation time $\Sigma\tau$ and (b) the character of their influence on the frequency dependence [Eq. (47)] of the ratchet average velocity (b); ξ is the normalized frequency, s is the normalized relaxation time defined in Eq. (48). The dotted lines correspond to the sinusoidal time dependence $\sigma(t)$.

sawtooth potential, created, for example, by a polar substrate, is easy to control by changing the settings of the lasers.

The family of deterministic processes of a relaxation type. Among periodic processes governing ratchet operating, relaxation processes play an important role since usually there is some delay in response of a ratchet to any external perturbation. Manifestation of such retardation in characteristics of photoinduced diffusion molecular transport has been theoretically studied in Ref. [32] using solutions of a relaxation equation with periodic boundary conditions which describe a response of a Brownian photomotor on a controlling deterministic dichotomous process. Here, to analyze the effect of a shape of the time dependence, $\sigma(t)$, on the average velocity of a ratchet, governed by a spatial harmonic signal, we will consider the shift-symmetric (supersymmetric) periodic function $\sigma(t) = \sigma(t+\tau) = -\sigma(t+\tau/2)$ of a relaxation type. Unlike the function used in Ref. [32], this function obeys fixed boundary conditions $\sigma(0) = -1$ and $\sigma(\tau/2) = 1$ on the interval $0 < t < \tau/2$:

$$\sigma(t) = -1 + 2(1 - e^{-\Sigma\tau/2})^{-1}(1 - e^{-\Sigma t}), \quad 0 < t < \tau/2,$$
$$\sigma_{2j+1} = -\frac{4\Sigma}{\omega_{2j+1}\tau(\omega_{2j+1} + i\,\Sigma)} \coth\left(\frac{\Sigma\tau}{4}\right), \tag{46}$$

where Σ is the inverse relaxation time. The advantage of this representation consists of the fact that it permits the universal consideration of not only steplike and triangularlike limiting shapes, popular among theoreticians, but also intermediate (close to reality) shapes of $\sigma(t)$ [Fig. 3(a)]. The result of substitution of the Fourier components σ_{2j+1} in Eq. (46) to Eqs. (15) for $\Psi_2(a, a)$ and (40) followed by analytical summation can be written as

$$\tilde{\Psi}_2 = \frac{\coth^2[(4s\xi)^{-1}]}{1 - s^2}\left\{\frac{8s^3\xi}{1 - s^2}\tanh[(4s\xi)^{-1}] + 4\xi\left(1 - \frac{2s^2}{1 - s^2}\right)\tanh[(4\xi)^{-1}] - \text{sech}^2[(4\xi)^{-1}]\right\}, \tag{47}$$

where we have introduced the dimensionless frequency and the relaxation parameter

$$\xi = \left(Dk_1^2\tau\right)^{-1}, \quad s = Dk_1^2/\Sigma. \tag{48}$$

If the process under study is deterministic dichotomic fluctuations ($s \to 0$), the function (47) reduces to

$$\tilde{\Psi}_2^{(\text{step})} = 4\xi \tanh[(4\xi)^{-1}] - \text{sech}^2[(4\xi)^{-1}]. \tag{49}$$

Another limiting case, $s \to \infty$, corresponding to a triangularlike shape of $\sigma(t)$, gives

$$\tilde{\Psi}_2^{(\text{trian})} = 16\xi^2\{2 - 12\xi \tanh[(4\xi)^{-1}] + \text{sech}^2[(4\xi)^{-1}]\}. \tag{50}$$

The frequency dependences of $\tilde{\Psi}_2$ corresponding to different relaxation times s are plotted in Fig. 3(b). The dependence is nonmonotonous with the maximum position, which is also

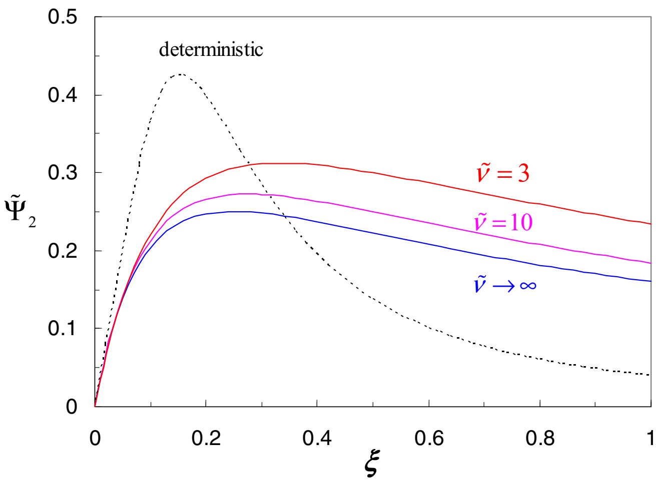

FIG. 4. Comparative frequency behavior of the average velocity of a ratchet controlled by deterministic dichotomous processes (dotted curve) and stochastic processes (solid curves): dichotomous ($\tilde{\nu} \to \infty$) and harmonic noise ($\tilde{\nu} = 3,\ 10$) processes.

nonmonotonically dependent on the duration of the relaxation process. Thus, there is an optimum frequency ξ of a driving which must be chosen in the case when the system permits variation in s values. Figure 3 also shows the influence of sinusoidal time dependence $\sigma(t) = -\cos(2\pi t/\tau)$ on the frequency dependences of the average velocity, with $\tilde{\Psi}_2$ function of the form

$$\tilde{\Psi}_2^{(\text{sinusoidal})} = 4\pi^2\xi^2/(1 + 4\pi^2\xi^2)^2. \quad (51)$$

This dependence is similar to those for large values of the relaxation parameter s.

Effect of damping on the ratchet velocity. Let us compare the behavior of a ratchet with the space harmonic driving (38) in two cases of its time dependence: symmetric dichotomous and harmonic noises. Using Eqs. (33) and (37), we write the frequency dependence of the ratchet average velocity, which is determined by the function $\tilde{\Psi}_2$, for the two cases, respectively, as

$$\tilde{\Psi}_2^{(\text{Dich})} = \frac{4\xi}{(1 + 4\xi)^2}, \quad \tilde{\Psi}_2^{(\text{HN})} = \frac{4\tilde{\nu}(2 + \tilde{\nu})\xi}{[1 + \tilde{\nu}(1 + 4\xi)]^2}, \quad (52)$$

where

$$\xi = \left(Dk_1^2\tau\right)^{-1} = \Gamma_1/\left(4Dk_1^2\right), \quad \tilde{\nu} = \nu/\left(Dk_1^2\right). \quad (53)$$

The corresponding curves are represented in Fig. 4. One can see that the damping reduction results in enhanced values of the ratchet velocity. Additionally, the decrease in the damping parameter $\tilde{\nu}$ is accompanied by an increase in maximum values of $\tilde{\Psi}_2$, and hence of the velocity, as well as by the shift of the maximum position to the right. At this, the frequency region for which the ratchet effect is still large enough becomes larger with the damping reduction. The comparison with the deterministic case (see the dotted curve) shows that the bell-shaped frequency dependence of the average velocity for stochastic driving becomes wider and is shifted to higher-frequency values.

V. CONCLUSIONS

It is well known that the functioning of ratchets presupposes the existence of external processes of a different nature that supply energy to the system, which is then converted into the energy of directed motion. The variety of these processes generates the need to create a variety of models describing the characteristics of corresponding ratchets. As a result, the question arises whether there is an effective way to compare (qualitatively and quantitatively) these characteristics for different processes, that is, for different time dependences of perturbations. To answer it, to solve the problem of comparative description, one must choose a class of systems that can be described analytically. In this paper, such a choice implies consideration of high-temperature ratchets as well as potential energies of the additive-multiplicative form (in particular, a sum of a time-independent contribution and time-dependent perturbations). This allowed us to use only the first terms of the series in powers of the inverse friction coefficient and hence to obtain a general expression for the average ratchet velocity, valid for arbitrary spatial dependences of the stationary and fluctuating contributions to the potential energy. The time dependence of the fluctuating part enters into this expression in terms of the second- and third-order correlation functions, which can be calculated for fluctuations of a different nature. That was the program realized in this article.

As time dependences of fluctuations, we considered deterministic periodic and stochastic Markovian processes. The first can be described analytically through the expansion of periodic functions into Fourier series, and the latter due to the diagonalization of the generalized transition matrix in the Pauli master equation. Expressions (15) and (26) for correlation functions that determine the average velocity have a similar form: they are sums in eigenvalues and eigenfunctions of the problems under consideration. The specification of these expressions has been carried out for the following particular cases.

(a) As a deterministic periodic process, we considered a process of a relaxation type, describing the response of a system to an external stepwise signal. The prototype of such a signal can be a cyclic process of switching a laser on and off, causing directional motion of a Brownian photomotor. A relaxation process in this case is a result of a delayed response of the electronic subsystem of the photomotor to the laser-induced perturbation. By varying the system relaxation time, one can obtain results valid for limiting cases, such as a deterministic dichotomous process and an external triangular signal, as well as for intermediate relaxation responses. A comparison of these results shows that the largest ratchet effect can be achieved in the case of extremely short relaxation times [see Fig. 3(b)].

(b) As stochastic processes, we considered symmetric dichotomous and harmonic noise processes. The Markovian dichotomous process is a special case of a stochastic Markovian process (with the number of states equal to two). It is characterized by the second-order correlation function coinciding with the Ornstein-Uhlenbeck one. On the other hand, harmonic noise is reduced to the Ornstein-Uhlenbeck one when the damping parameter tends to infinity. Thus, with the comparison of ratchet velocities for different damping parameters, we conclude that the damping reduction enhances the ratchet effect.

The main dependence by which one can judge the influence of various fluctuations on the average velocity of motion of a high-temperature ratchet is the dependence of the velocity on the frequency of the fluctuations. To get this dependence, we limited ourselves to the case of spatially harmonic fluctuations, for which it is possible to write the result as a product of two functions: (i) "containing" the spatial shape of the static part of the potential and (ii) describing the time dependence of its fluctuations [see Eq. (39)]. This allowed us to write explicit expressions for the cases of double-well and sawtooth static potential profiles, as well as for the relaxation-type deterministic and Markovian stochastic time dependencies discussed above, which can be practiced for estimating ratchet velocities. It turned out that the phase shift between the nonfluctuating part of the potential and its fluctuating (spatially harmonic) contribution can control the motion direction.

The main conclusion following from the illustrative examples is that deterministic governing (driving) processes lead to a relatively narrow bell-shaped frequency dependence of the average velocity, the maximum value of which corresponds to dichotomous relaxation-free fluctuations. Stochastic driving leads to a wide bell-shaped curve, extending far into the region of high frequencies. In this case, damping reduction of harmonic noise makes the frequency-dependence maximum higher and shifts the maximum to the high-frequency region. Therefore, if the aim is to get the largest ratchet effect in a narrow frequency domain, one should use deterministic relaxation-free dichotomous processes, while to maintain the ratchet effect in a wide frequency range, it is reasonable to choose stochastic dichotomous governing processes.

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