Periodicity of quantum correlations in the quantum kicked top

Udaysinh T. Bhosale[*] and M. S. Santhanam[†]

Indian Institute of Science Education and Research, Dr. Homi Bhabha Road, Pune 411 008, India

Quantum kicked top is a fundamental model for time-dependent, chaotic Hamiltonian system and has been realized in experiments as well. As the quantum kicked top can be represented as a system of qubits, it is also popular as a test bed for the study of measures of quantum correlations, such as entanglement, quantum discord, and other multipartite entanglement measures. Further, earlier studies on kicked top have led to a broad understanding of how these measures are affected by the classical dynamical features. In this work, relying on the invariance of quantum correlation measures under local unitary transformations, it is shown exactly these measures display periodic behavior either as a function of time or as a function of the chaos parameter in this system. As the kicked top has been experimentally realized using cold atoms as well as superconducting qubits, it is pointed out that these periodicities must be factored in while choosing experimental parameters so that repetitions can be avoided.

I. INTRODUCTION

Periodically kicked quantum systems are popular models of Hamiltonian chaos. Their popularity, in part, arises from the relative ease of analysis. The quantum dynamics of such systems can be reduced to a Floquet map, while in the classical limit, the dynamics can be reduced to a set of difference equations. The quantum kicked top is a prominent member of this class and it physically represents a repeating sequence of free precession and state-dependent rotation (kick). For sufficiently large kick strengths, the system displays chaotic classical dynamics. Several approaches to experimental realization of quantum kicked top were suggested [1] and was attained using a cloud of cold Cs atoms in the total hyperfine spin of its ground state interacting with time-dependent magnetic fields [2].

In the past two decades, kicked top was widely used to study the interplay between chaotic dynamics and quantum correlations in the context of continued interest in quantum information and computation. The kicked top has a natural representation in terms of spins or qubits and this makes it a suitable choice for studies on entanglement. In this approach, the number of spins tending to infinity represents the classical limit of kicked top. Hence, this model continues to attract research interest [3–8] for the study of entanglement [9–14] and its relation to classical dynamics [15], signatures of bifurcations on various quantum correlation measures [3], quantum-classical correspondence in the vicinity of periodic orbits [4], and quantum metrology [16]. Measures of quantum correlations have been found to strongly correlate with the qualitative nature of classical phase space, whether it is regular or chaotic [3,7,9,14,17,18]. In general, as demonstrated extensively in a series of papers using kicked tops [2,3,7,9,14,17–19], the

qualitative nature and details of classical dynamics influences entanglement. In addition, classical dynamical features such as the bifurcation also affect the quantum correlation measures with interesting semiclassical consequences [3]. Similar results have been obtained for other measures of quantum correlations such as quantum discord and Mayer-Wallach Q measure.

Unlike the earlier experimental effort [2] involving manipulation of atomic and nuclear spins, recently kicked top was realized in a system of just three superconducting qubits ("spins") examining its behavior in the deep quantum regime [19]. The latter experiment has verified the theoretically predicted connections [11–13] between chaotic dynamics and bipartite entanglement. Quite remarkably, ergodic behavior in this isolated quantum system was demonstrated [19]. Surprisingly, a recent theoretical work has shown that even in the deep quantum limit possible with just two qubits, the system appears to take into account the nature of classical dynamics in the vicinity of the phase-space coordinates where the spin-coherent state is initially placed [7]. Further, this work also hints that the entanglement entropy might display (quasi-)periodic behavior in time and also as a function of kick strength. This observation, if generalized, has important implications for both experimental and theoretical work on kicked tops. Let us consider a kicked top system with j representing the total spins and k its kick strength. This corresponds to $2j$ number of spin-1/2 particles. If a quantum correlation measure, say A, for this kicked top displayed periodic behavior, then for a given initial state we can expect the following functional relations; $A(t;k,j) = A(t+T;k,j)$ or $A(t;k,j) = A(t;k+\kappa,j)$ representing periodic behaviour in time t and kick strength k with periodicities, respectively, T and κ.

This implies that for a fixed number of qubits quantum correlations will repeat after a certain time period T or after certain value kick strength κ. Thus, generally and crucially in an experimental context, the choice of k and j indirectly

*udaybhosale0786@gmail.com
†santh@iiserpune.ac.in

sets the upper limit T and κ before repetitions begin to occur. This argument can be turned around to derive another useful information. If an experimental realization of the kicked top is expected to maintain coherence for timescale τ_{coh}, then the question is about the values of k and j that must be used to explore unique time evolution until time τ_{coh}. The mean coherence time τ_{coh} is generally a function of experimental (and environmental) parameters, and together with values of j and k it will uniquely determine the relevant timescale for the experiment to be $\min(\tau_{\text{coh}}, T)$. Thus, the present study of the periodicities in the kicked top will serve as a crucial guide for experimental efforts to make the appropriate choice of parameters.

In this work, we show exactly that the time variation of quantum correlations of kicked top displays nontrivial periodicity provided the total spin $j = 1$ and kick strength is of the form $k = r\pi/s$, r and s being integers. This includes the special case of two qubits, $j = 1$, already reported in Ref. [7]. Further, it is also shown that for any $j > 1$, though quantum correlations do not show temporal periodicity, they display periodic behavior in kick strength k. Thus, this periodicity holds good in the semiclassical limit of large j as well. The structure of the paper is as follows: In Sec. II the measures of quantum correlations are introduced. In Sec. III the kicked top model is introduced. In Sec. IV analytical results on the periodicity of quantum correlations as a function of chaos parameter k are given. In Sec. V reflection symmetry of phase space in k and its experimental consequences are discussed. In Sec. VI analytical results on time periodicity for the case of a two-qubit kicked top is studied.

II. MEASURES OF QUANTUM CORRELATIONS

A. von Neumann entropy

Let us consider a standard bipartite system $A \otimes B$ composed of two smaller subsystems denoted as A and B, having Hilbert spaces $\mathcal{H}_A{}^{(N)}$ and $\mathcal{H}_B{}^{(M)}$ (with dimensions N and M), respectively. For simplicity, $N \leqslant M$ can be assumed and the full system belongs to the product Hilbert space $\mathcal{H}_{AB}{}^{(MN)} = \mathcal{H}_A{}^{(N)} \otimes \mathcal{H}_B{}^{(M)}$. Consider a normalized pure state $|\psi\rangle = \sum_{i=1}^{N} \sum_{\alpha=1}^{M} c_{i,\alpha} |i\rangle \otimes |\alpha\rangle$ of the full system $A \otimes B$, where $|i\rangle \otimes |\alpha\rangle$ is the orthonormal basis of $\mathcal{H}_{AB}$. Its density matrix is $\rho = |\psi\rangle\langle\psi|$ satisfying the $\text{Tr}[\rho] = 1$ condition. The reduced density matrix of the subsystem A is obtained by tracing out B, i.e., $\rho_A = \text{Tr}_B[\rho] = \sum_{\alpha=1}^{M} \langle\alpha|\rho|\alpha\rangle$. Similarly, the subsystem B is described by $\rho_B = \text{Tr}_A[\rho]$. The singular value decomposition of the matrix $c_{i,\alpha}$ gives the following Schmidt decomposition form:

$$|\psi\rangle = \sum_{i=1}^{N} \sqrt{\lambda_i} \, |u_i^A\rangle \otimes |v_i^B\rangle, \tag{1}$$

where $|u_i^A\rangle$ and $|v_i^B\rangle$ are the eigenvectors of ρ_A and ρ_B, respectively, with the same eigenvalues λ_i. The eigenvalues $\lambda_i \in [0, 1]$ are such that $\sum_{i=1}^{N} \lambda_i = 1$. The remaining $M - N$ eigenvalues of ρ_B are identically equal to zero.

Given the Schmidt eigenvalues λ_i ($i = 1 \ldots N$), entanglement between A and B, where von Neumann entropy is used

as a measure, is given as follows:

$$S_{\text{VN}} = -\text{tr}(\rho_A \log \rho_A) = -\sum_{i=1}^{N} \lambda_i \ln(\lambda_i). \tag{2}$$

This is a good measure of entanglement for a bipartite pure state [20,21]. It satisfies $0 \leqslant S_{\text{VN}} \leqslant \ln(N)$, where zero corresponds to a separable state and $\ln(N)$ corresponds to a maximally entangled state.

B. Quantum discord

Quantum discord measures all possible quantum correlations including and those beyond entanglement in a quantum state [22,23]. This method involves removing the classical correlations from the total correlations of the system. Now the procedure to evaluate discord will be given in detail [3]. For a bipartite quantum system having density matrix ρ_{AB}, total correlations are quantified by the quantum mutual information given by

$$\mathcal{I}(B : A) = \mathcal{H}(B) + \mathcal{H}(A) - \mathcal{H}(B, A). \tag{3}$$

However, the classical mutual information, based on Baye's rule, is given by

$$I(B : A) = H(B) - H(B|A), \tag{4}$$

where $H(B)$ denotes the Shannon entropy of B. The conditional entropy $H(B|A)$ is defined as the average of the Shannon entropies of system B conditioned on the values of A. It can be thought of as the ignorance of B given the information about A [24].

The quantum measurements on the subsystem A are represented by a set of positive-operator valued measure (POVM) $\{\Pi_i\}$, such that the conditioned state of B for given outcome i is equal to

$$\rho_{B|i} = \text{Tr}_A(\Pi_i \rho_{AB})/p_i \quad \text{and} \quad p_i = \text{Tr}_{A,B}(\Pi_i \rho_{AB}), \tag{5}$$

and its entropy is $\tilde{\mathcal{H}}_{\{\Pi_i\}}(B|A) = \sum_i p_i \mathcal{H}(\rho_{B|i})$. In this case, the quantum mutual information is equal to $\mathcal{J}_{\{\Pi_i\}}(B : A) = \mathcal{H}(B) - \tilde{\mathcal{H}}_{\{\Pi_i\}}(B|A)$. Maximizing this over all possible measurement sets $\{\Pi_i\}$ one obtains

$$\mathcal{J}(B : A) = \max_{\{\Pi_i\}} \big[\mathcal{H}(B) - \tilde{\mathcal{H}}_{\{\Pi_i\}}(B|A) \big]$$
$$= \mathcal{H}(B) - \tilde{\mathcal{H}}(B|A), \tag{6}$$

where $\tilde{\mathcal{H}}(B|A) = \min_{\{\Pi_i\}} \tilde{\mathcal{H}}_{\{\Pi_i\}}(B|A)$. The minimum value is achieved using rank-one POVMs due to concave nature of the conditional entropy over the set of convex POVMs [25]. By taking $\{\Pi_i\}$ as rank-one POVMs, the quantum discord is defined as $\mathcal{D}(B : A) = \mathcal{I}(B : A) - \mathcal{J}(B : A)$, such that

$$\mathcal{D}(B : A) = \mathcal{H}(A) - \mathcal{H}(B, A) + \min_{\{\Pi_i\}} \tilde{\mathcal{H}}_{\{\Pi_i\}}(B|A). \tag{7}$$

The quantum discord is shown to be nonnegative for all quantum states [22,25,26] and is subadditive [27]. For the bipartite pure state, the quantum discord is shown to be equal to the von Neumann entropy [22,23].

C. Concurrence and the 3-tangle

Concurrence [28,29] is a measure of entanglement present between two qubits. This measure was used to study phase

transition in the Heisenberg chain [30]. Given two qubit density matrix ρ_{AB}, firstly the spin-flipped state $\tilde{\rho}_{AB} = \sigma_y \otimes \sigma_y \rho_{AB}^* \sigma_y \otimes \sigma_y$ is calculated, where σ_y is the Pauli matrix and the complex conjugation is done in the standard basis. Then the eigenvalues of the non-Hermitian matrix $\rho_{AB}\tilde{\rho}_{AB}$ are obtained, which are all real and nonnegative such that $\lambda_4 \leqslant \lambda_3 \leqslant \lambda_2 \leqslant \lambda_1$. Then, the concurrence $C_{12} = C(\rho_{AB})$ is equal to

$$\max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right), \qquad (8)$$

and $0 \leqslant C_{12} \leqslant 1$. It is zero for separable state and one for maximally entangled state. It is shown that the entanglement of formation [31] of ρ_{AB} is a monotonic function of concurrence [28,32]. For the Bell state, concurrence is equal to one.

The 3-tangle is a pure multipartite entanglement measure for pure as well as mixed three-qubit states [33]. For the case of a three-qubit pure state, it is given by $\tau = C_{1(23)}^2 - C_{12}^2 - C_{13}^2$ [33], where C_{ij} measures the concurrence between ith and jth qubits. The quantity $C_{1(23)}$ is the concurrence between qubit 1 and the pair of qubits 2 and 3. This is because in a three-qubit pure state, the reduced density matrix of qubits 2 and 3 is of rank-2. The 3-tangle τ is permutationally invariant and satisfies $0 \leqslant \tau \leqslant 1$ [33]. For given concurrence C_{12} the maximum 3-tangle τ a three-qubit pure state can have has been calculated [34]. States satisfying these limits have also been evaluated.

D. Meyer and Wallach Q measure

This multipartite entanglement measure [35] was studied earlier in the context of spin Hamiltonians [36–38], system of spin-bosons [39], and how it is affected due to the classical bifurcation in the kicked top model [3]. The geometric multipartite entanglement measure Q is shown to be related to one-qubit purities [40], making its calculation and interpretation straightforward. If ρ_i is the reduced density matrix of the ith spin obtained by tracing out the rest of the spins in a N qubit pure state then the Q measure is defined as follows:

$$Q(\psi) = 2\left[1 - \frac{1}{N}\sum_{i=1}^{N}\text{Tr}\left(\rho_i^2\right)\right]. \qquad (9)$$

The relation in Eq. (9) between Q and the single spin reduced density matrix purities has led to a generalization of Q measure to multiqudit states as well as for various other bipartite splits [41].

III. KICKED TOP

The quantum kicked top is characterized by an angular momentum vector $\mathbf{J} = (J_x, J_y, J_z)$ and its components obey the standard algebra of angular momentum. Here, the Planck's constant has been set to unity. The Hamiltonian governing the dynamics of the top is given by

$$H(t) = pJ_y + \frac{k}{2j}J_z^2 \sum_{n=-\infty}^{+\infty} \delta(t-n). \qquad (10)$$

The first term represents the free precession of the top around y axis with angular frequency p while the second term is periodic δ kicks applied to the top. Each kick gives a torsion
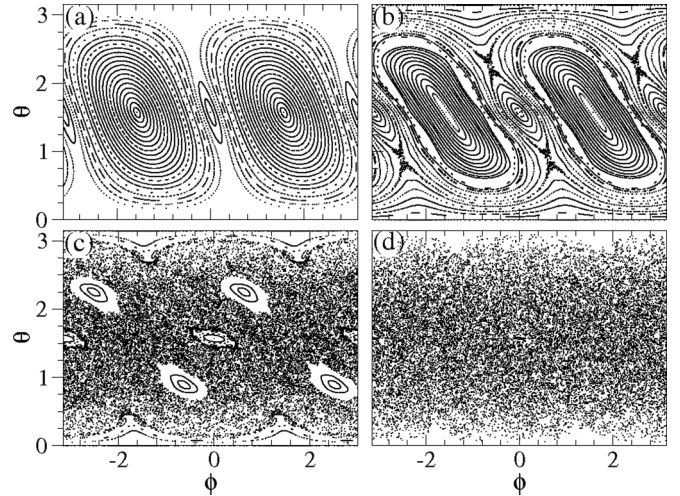

FIG. 1. Phase-space pictures of the classical kicked top for $p = \pi/2$ and (a) $k = 1$, (b) $k = 2$, (c) $k = 3$, and (d) $k = 6$.

about the z axis by an angle $(k/2j)J_z$. Here, k is called as the chaos parameter or the kick strength. For $k = 0$ the classical limit of Eq. (10) is integrable and for $k > 0$ it becomes increasingly chaotic. The corresponding period-one Floquet operator of the Hamiltonian in Eq. (10) is given as follows:

$$U = \exp\left(-i\frac{k}{2j}J_z^2\right)\exp(-ipJ_y). \qquad (11)$$

The Hilbert space dimension is equal to $2j + 1$ implies that the dynamics can be explored without any truncation of the Hilbert space. The kicked top has been realized in various experimental test beds, in hyperfine levels of cold Cs atoms and coupled superconducting qubits [2,19], in which $p = \pi/2$. In Ref. [19], it was found that the time-averaged von Neumann entropy showed the clear resemblance with the corresponding classical phase space.

The quantum kicked top for given angular momentum j can be considered equivalent to a quantum simulation of a collection of $N = 2j$ number of qubits (spin-half particles) whose evolution is restricted to the subspace which is symmetric under the exchange of the qubits. The state vector is restricted to a symmetric subspace spanned by the basis states $\{|j, m\rangle; (m = -j, -j + 1, ..., j)\}$ where $j = N/2$. The basis states satisfy the property $S_z|j, m\rangle = m|j, m\rangle$ and $S_\pm|j, m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}|j, m \pm 1\rangle$, where S_z and $S_\pm$ are collective spin operators [42,43]. The states $\{|j, m\rangle\}$ are also known as Dicke states. Thus, it is a multiqubit system whose collective behavior is governed by the Hamiltonian in Eq. (10) and the quantum correlations between any two qubits can be studied.

The classical phase space is displayed in Fig. 1 as a function of coordinates θ and ϕ. To explore quantum dynamics in the kicked top, spin-coherent states [44–47] pointing along the direction of θ_0 and ϕ_0 are constructed and are evolved under the action of the Floquet operator. The classical map for the kicked top is given as follows [44,48]:

$$X' = (X\cos p + Z\sin p)\cos[k(Z\cos p - X\sin p)]$$
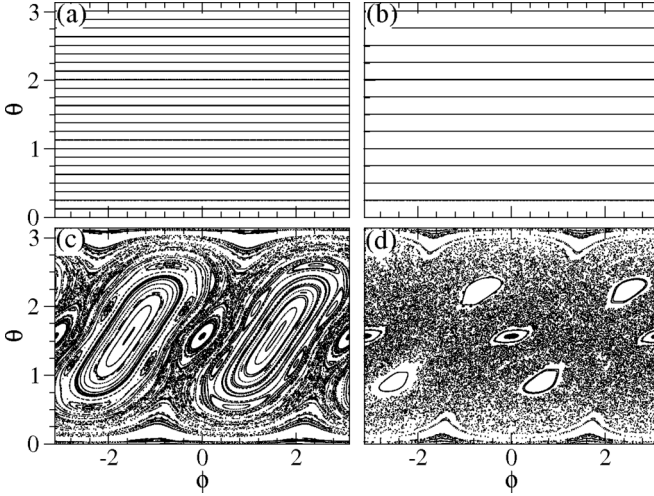$$- Y\sin(k(Z\cos p - X\sin p)), \qquad (12a)$$

FIG. 2. Phase-space pictures of the classical kicked top for $k = 3\pi/5$ and (a) $p = \pi$ and (b) $p = 2\pi$. Same for $p = \pi/2$ and (c) $k = -2$ and (d) $k = -3$.

$$Y' = (X \cos p + Z \sin p) \sin [k(Z \cos p - X \sin p)]$$
$$+ Y \cos [k(Z \cos p - X \sin p)], \qquad (12b)$$
$$Z' = -X \sin p + Z \cos p. \qquad (12c)$$

Here, the dynamical variables (X, Y, Z) satisfy the constraint $X^2 + Y^2 + Z^2 = 1$, i.e., they are restricted to be on the unit sphere. Thus, it is possible to parametrize them in terms of the polar angle θ and the azimuthal angle ϕ as $X = \sin\theta \cos\phi$, $Y = \sin\theta \sin\phi$ and $Z = \cos\theta$. First, the map in Eq. (12) is evolved and then the values of (θ, ϕ) are determined using the inverse relations, which are not shown here.

Another feature of this map is that under the transformation $k \to -k$ the phase space is reflected about $\theta = \pi/2$. This is because $k \to -k$ is equivalent to the transformation $X \to -X$ and $Z \to -Z$ in Eq. (12). This implies $Z' \to -Z'$ which results in $\theta \to \pi - \theta$. Thus, the phase space corresponding to k and $-k$ are isomorphic to each other. This can be seen from Figs. 1(b) and 2(c), as well as from Figs. 1(c) and 2(d). This has experimental implications which will be discussed in later part of the paper.

1. Classical map for various values of p In this work, the model is studied for various values of p. Thus, it will be helpful to study the corresponding map equations and the phase space. First, the case of $p = \pi/2$ is considered. In this case, due to additional symmetries, a simpler classical map can be obtained and was studied in detail in Refs. [2–4,14,17,19,44]. In this case, the map given in Eq. (12) reduces to

$$X' = Z \cos(kX) + Y \sin(kX),$$
$$Y' = Y \cos(kX) - Z \sin(kX), \qquad (13)$$
$$Z' = -X.$$

The phase space obtained using these equations is displayed in Fig. 1. It can be seen that for $k = 1$ and $k = 2$ the phase space is mostly covered by regular orbits. The trivial fixed points at $(\theta, \phi) = (\pi/2, \pm\pi/2)$ can be seen in Figs. 1(a) and 1(b) becomes unstable at $k = 2$. As k is increased further

the chaotic regions are increased. At $k = 6$ the phase space is covered mostly by the chaotic sea with very tiny regular islands.

The map for $p = 3\pi/2$ can be obtained from that of $p = \pi/2$ by the transformation $X' \to -X'$ and $Z' \to -Z'$. This implies $\phi \to -\phi$ and $\theta \to \pi - \theta$ which are reflections about $\phi = 0$ and $\theta = \pi/2$. Thus, the phase space, as well as other properties, can be obtained by taking these reflections.

Now consider the case of $p = \pi$. In this case using Eq. (12) the classical map is obtained as follows:

$$X' = Y \sin(kZ) - X \cos(kZ),$$
$$Y' = Y \cos(kZ) - X \sin(kZ), \qquad (14)$$
$$Z' = -Z.$$

The phase space is plotted in Fig. 2(a). It can be seen that there is no fully developed chaos since for given initial Z the angle θ oscillates between $\cos^{-1} Z$ and $\pi - \cos^{-1} Z$. Both these values are reflection about $\pi/2$ which can also be seen in the figure.

For the case $p = 2\pi$ the map equations are

$$X' = X \cos(kZ) - Y \sin(kZ),$$
$$Y' = X \sin(kZ) + Y \cos(kZ), \qquad (15)$$
$$Z' = Z.$$

The phase space is plotted in Fig. 2(b). In this case too there is no fully developed chaos and for given initial Z the angle θ remains fixed at $\cos^{-1} Z$.

IV. PERIODICITY OF QUANTUM CORRELATIONS AS A FUNCTION OF CHAOS PARAMETER

In this section, it will be shown analytically and through numerical simulations that the quantum correlations display periodicity as a function of kick strength k. In particular, it will be shown that for a fixed value of j and for a given initial state, the quantum correlations are periodic in k, with $\kappa = 2j\pi$ being its periodicity.

1. j = 1 case

Let us consider the simplest case of $j = 1$ which is equivalent to two qubits. Then, the basis states are $|1, -1\rangle$, $|1, 0\rangle$ and $|1, 1\rangle$. The standard two qubit basis states are $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$ (subscripts label qubits) such that $\sigma_z|0\rangle = -|0\rangle$ and $\sigma_z|1\rangle = |1\rangle$. Both the basis states are related to each other by $|1, -1\rangle = |0\rangle_1|0\rangle_2$, $|1, 1\rangle = |1\rangle_1|1\rangle_2$ and $|1, 0\rangle = (|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)/\sqrt{2}$.

Setting $j = 1$ in Eq. (11), the corresponding Floquet operator is

$$U = \exp\left(-i\frac{k}{2}J_z^2\right) \exp(-ipJ_y). \qquad (16)$$

It can be seen that when $k \to k + 2\pi$ one obtains

$$U \to \widehat{O}\, U \quad \text{where} \quad \widehat{O} = \exp\left(-i\pi J_z^2\right). \qquad (17)$$

Thus, $U|\psi_j\rangle \to \widehat{O}\, U|\psi_j\rangle$ where $|\psi_j\rangle$ is any vector in the $|j, m\rangle$ basis. For $j = 1$ case, denoting the vector $U|\psi_1\rangle = [a, b, c]^T$. Operator $\widehat{O}$ is diagonal in $\{|j = 1, m\rangle\}$ basis, i.e.,
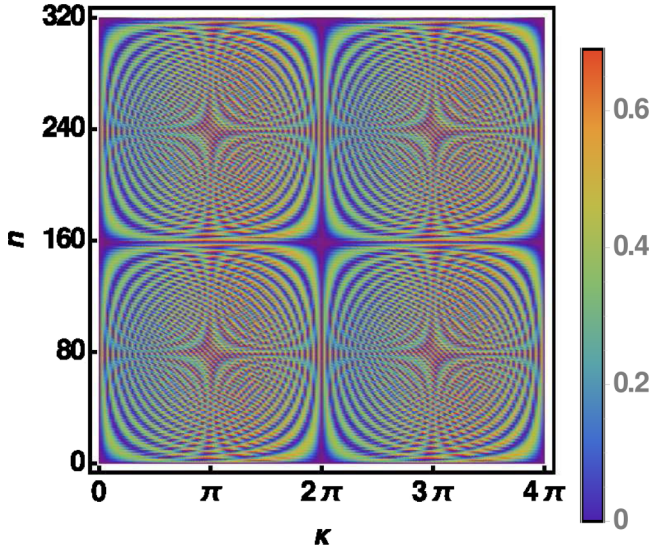
FIG. 3. The von Neumann entropy (S_{VN}) is displayed for a two-qubit ($j = 1$) kicked top with parameters $p = \pi/2$ and $k = r\pi/40$ ($r = 0 \dots 160$). The color bar by the side represents the color code for the von Neumann entropy values. Note the 2π periodicity in kick strength k as well as the periodicity in time evident in this picture.

$\widehat{O} = \text{diag}[-1, 1, -1]$. However, in the standard two-qubit basis it becomes

$$\widehat{O} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{18}$$

Thus, it can be seen that even though $\widehat{O}$ is unitary in $\{|j = 1, m\rangle\}$ basis, it is not so in the standard two-qubit basis. This implies that $\widehat{O}$ is not a local unitary but it will seen now that its action on any state in $\{|j = 1, m\rangle\}$ basis does not change the quantum correlations among the qubits. Thus, in $\{|j = 1, m\rangle\}$ basis $[a, b, c]^T \rightarrow \widehat{O}[a, b, c]^T = [-a, b, -c]^T$. It can be shown easily that in the standard two qubit basis states, $[a, b, c]^T$ becomes $|\chi_1\rangle = [a, b/\sqrt{2}, b/\sqrt{2}, c]^T$, whereas $[-a, b, -c]^T$ becomes $|\chi_1\rangle' = [-a, b/\sqrt{2}, b/\sqrt{2}, -c]^T$. Thus, we have

$$|\chi_1\rangle = a|1\rangle_1|1\rangle_2 + (b/\sqrt{2})(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2) + c|0\rangle_1|0\rangle_2 \quad \text{and}$$

$$|\chi_1\rangle' = -a|1\rangle_1|1\rangle_2 + (b/\sqrt{2})(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2) - c|0\rangle_1|0\rangle_2. \tag{19}$$

It is seen that $|\chi_1\rangle$ and $|\chi_1\rangle'$ are related to each other by a local unitary transformation, i.e., $|\chi_1\rangle' = -\sigma_z \otimes \sigma_z|\chi_1\rangle$. Quantum correlation measures by definition are invariant under local unitary operations [49]. Using concurrence for two-qubit pure state [29] it can be seen to be equal to $2|b^2/2 - ac|$ for both the states. These imply that the correlations are invariant under the transformation $k \rightarrow k + 2\pi$. This can be seen in Fig. 3 where von Neumann entropy shows a periodicity of 2π as a function of chaos parameter k.

2. General j case

Let us consider the case of general j, beginning with even integer value for j. Here, the corresponding operator $\widehat{O} = \exp(-i\pi J_z^2)$ is diagonal matrix of order $2j + 1$ in $\{|j, m\rangle\}$ basis, i.e., $\widehat{O} = \text{diag}[1, -1, \dots, -1, 1]$. The transformation $k \rightarrow k + 2j\pi$ gives $U \rightarrow \widehat{O}U$. The operator $\widehat{O}$ is diagonal matrix of dimension $2j + 1$ in $\{|j, m\rangle\}$ basis, i.e., $\widehat{O} = \text{diag}[1, -1, \dots, -1, 1]$. Now, the basis $\{|j, m\rangle\}$ will be written in the standard basis of qubits. For given value of m there are $\binom{2j}{j+m}$ basis states superposed equally to form $|j, m\rangle$ where each of the basis state is such that $j + m/2$ qubits are in up-state $|1\rangle$ and remaining $j - m/2$ qubits are in down-state $|0\rangle$. In this paper, such a basis state will be called as m-particle state since it is an eigenvector of the total spin operator S_z with eigenvalue m. Thus, there are $\binom{2j}{j+m}$ m-particle states and the normalization constant after superposing all such m-particle states is $1/\sqrt{\binom{2j}{j+m}}$. For example, $|j, 1\rangle = (|1\rangle_1|0\rangle_2 \dots |0\rangle_{2j} + |0\rangle_1|1\rangle_2 \dots |0\rangle_{2j} + \dots + |0\rangle_1|0\rangle_2 \dots |1\rangle_{2j})/\sqrt{\binom{2j}{1}}$.

It is easily evident that $\widehat{O}$ is a block-diagonal matrix in $\{|j, m\rangle\}$ basis and can be denoted as $\text{diag}[\widehat{O}_0, \widehat{O}_1, \dots, \widehat{O}_{2j}]$. Similar to the $j = 1$ case, $\widehat{O}$ is unitary in $\{|j, m\rangle\}$ basis but it is no longer unitary when written in the standard $2j + 1$ qubit basis. Thus, $\widehat{O}$ is not a local unitary. But, we will now show that the quantum correlations remains invariant after $\widehat{O}$ acts on any state in the $\{|j, m\rangle\}$ basis. Here, each $\widehat{O}_n$ ($n = 0, 1, \dots 2j$) is a square matrix of dimension $\binom{2j}{n}$ and each element in it is equal to $\exp(-i\pi n^2)/\binom{2j}{n}$, where $n = j + m$ takes values in the range $0 \dots 2j$. It should be noted that each $\widehat{O}_n$ is written in the set of all n-particle states. The vector $U|\psi_j\rangle$, in the $\{|j, m\rangle\}$ basis, is denoted as $[c_0, c_1, c_2, \dots, c_{2j-1}, c_{2j}]^T$. The same vector in the m-particle basis, $m = -j$ to j, becomes $|\chi_j\rangle = [c_0', c_1', c_1', \dots, c_{2j-1}', c_{2j-1}', c_{2j}']^T$. In this, $c_n' = c_n/\sqrt{\binom{2j}{j+m}}$ and each c_n' occurs $\binom{2j}{n}$ times in a sequence. Thus, $\widehat{O}|\chi_j\rangle = \text{diag}[\widehat{O}_0, \widehat{O}_1, \dots, \widehat{O}_{2j}][c_0', c_1', c_1', \dots, c_{2j}']^T$.

Thus, it is seen that the matrix $\widehat{O}_0$ having dimension one gets multiplied by the column vector of dimention one containing c_0', the matrix $\widehat{O}_1$ having dimention $\binom{2j}{1}$ gets multiplied by the column vector of dimention $\binom{2j}{1}$ having c_1' as its element at all the rows and so on. Thus, in general the matrix $\widehat{O}_n$ of order $\binom{2j}{n}$ gets multiplied by the column vector of length $\binom{2j}{n}$ having c_n' as its element at all the rows.

Let us denote this (unnormalized) column vector by $|\xi_j^n\rangle = [c_n', c_n', \dots, c_n']^T$. As pointed out earlier, $\widehat{O}_n$ is square matrix of order $\binom{2j}{n}$ with matrix elements $\exp(-i\pi n^2)/\binom{2j}{n}$. This leads to

$$\widehat{O}_n|\xi_j^n\rangle = \exp(-i\pi n^2)[c_n', c_n', \dots, c_n']^T. \tag{20}$$

Thus, the final product becomes

$$\widehat{O}|\chi_j\rangle = [c_0', -c_1', -c_1', \dots, -c_{2j-1}', -c_{2j-1}', c_{2j}']^T. \tag{21}$$

When transformed to $\{|j, m\rangle\}$ basis, it becomes $[c_0', -c_1', c_2', \ldots, -c_{2j-1}', c_{2j}']^T$. It can also be written as $\sum_{n=0}^{2j}(-1)^n c_n'|j, j-n\rangle$. Here j is even and using the properties of $|j, j-n\rangle$ it becomes

$$\left(\prod_{i=1}^{2j}\otimes\sigma_z^i\right)|j, j-n\rangle = (-1)^n|j, j-n\rangle, \qquad (22)$$

where the superscript denotes the qubit position. Thus,

$$\sum_{n=0}^{2j}(-1)^n c_n'|j, j-n\rangle = \left(\prod_{i=1}^{2j}\otimes\sigma_z^i\right)\sum_{n=0}^{2j} c_n'|j, j-n\rangle$$

$$= \left(\prod_{i=1}^{2j}\otimes\sigma_z^i\right)[c_0', c_1', c_1', \ldots, c_{2j-1}', c_{2j-1}', c_{2j}']^T$$

$$= \left(\prod_{i=1}^{2j}\otimes\sigma_z^i\right)|\chi_j\rangle.$$

Hence,

$$\widehat{O}|\chi_j\rangle = \left(\prod_{i=1}^{2j}\otimes\sigma_z^i\right)|\chi_j\rangle, \qquad (23)$$

which implies

$$\widehat{O}U|\psi_j\rangle = \left(\prod_{i=1}^{2j}\otimes\sigma_z^i\right)U|\psi_j\rangle. \qquad (24)$$

Clearly, for the case of even j as well, the two states are related to each other by local unitary operations. Relying on the invariance of the quantum correlation measures under local unitary operations [49], which in this context implies invariance under $k \to k + 2j\pi$, it is inferred that the quantum correlations are periodic as a function of k with period $2j\pi$. It must emphasized that the quantum correlations are periodic in k even for large value j, i.e., in the semiclassical limit as well. Similar result can be proved for the case of odd and half-integer values of j. This can be seen in the simulation results displayed in Figs. 4, 5, 6, and 7, where various quantum correlations show periodicity of $2j\pi$ as a function of chaos parameter k. Here, the initial coherent state is positioned at $\theta = 2.5$ and $\phi = 1.1$ for all values of k. It should be emphasized here that this result is valid only for any initial state $|\psi_j\rangle$ in the symmetric subspace spanned by the basis states $\{|j, m\rangle\}$ which may or may not be an eigenstate of J_z. It should also be noticed from Eq. (23) that the operator $\widehat{O}$ is nonunitary in the qubit basis while $\prod_{i=1}^{2j}\otimes\sigma_z^i$ a local unitary operator in the same basis. However, the results of their actions on the state $|\psi_j\rangle$ are equal.

V. REFLECTION SYMMETRY IN k AND EXPERIMENTAL CONSEQUENCES

Now, consider two different values of chaos parameters k_1 and k_2 such that $0 \leqslant k_1 \leqslant j\pi$ and $j\pi \leqslant k_2 \leqslant 2j\pi$. Further, they are related by $k_2 = 2j\pi - k_1$ representing a reflection symmetry about $j\pi$. As the quantum correlations are periodic in k with a period of $2j\pi$, the time evolution of quantum correlations at $k = k_2$ is identical to that at $k = -k_1$. As mentioned in Sec. III, the phase space for k and $-k$ are isomorphic to each other and are related by the transformation
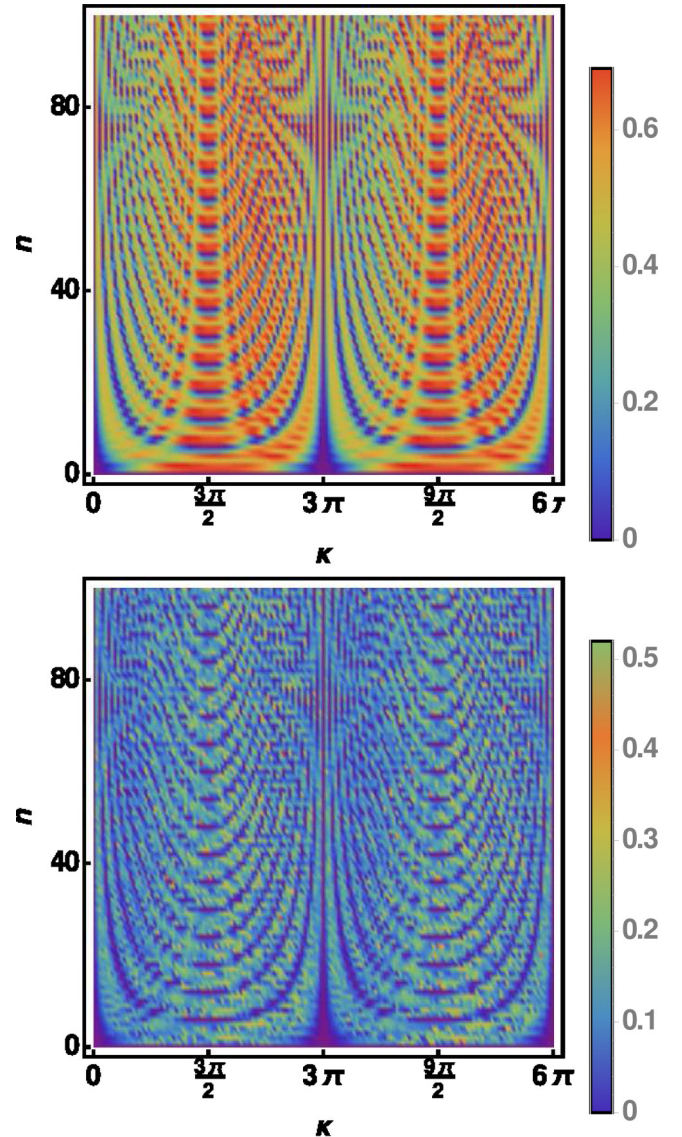

FIG. 4. (top) von Neumann entropy ($S_{\rm VN}$) of kicked top which is partitioned as a single qubit and two qubits, (bottom) quantum discord ($\mathcal{D}$) between any two qubits. Both are plotted as function of kick strength k and time. In this, $j = 3/2$. The values of von Neumann entropy and discord are color coded using the color map shown by the side.

$\theta \to \pi - \theta$. This implies that if an initial state is evolved for $k = k_2$ then it is equivalent to the evolution of initial state for $k_1 = 2j\pi - k_2$ provided the initial positions of both the coherent states are related by $\theta \to \pi - \theta$. We will call this a signature of phase space.

Thus, the combination of $2j\pi$ periodicity and symmetry in k results in quantum correlations that are symmetric about $k = j\pi$. In other words, for fixed value of j, the maximum value of chaos parameter $k_{\max}$ for which the phase-space effects are unique is $j\pi$. Beyond $k = k_{\max}$, the observed structure repeats itself. The maximum chaos parameter $k = k_{\max}$ for the given number of qubits in the top is shown in Fig. 8. This result has implications for kicked top experiments. If two qubits are used to represent the kicked top, i.e. $j = 1$, then one
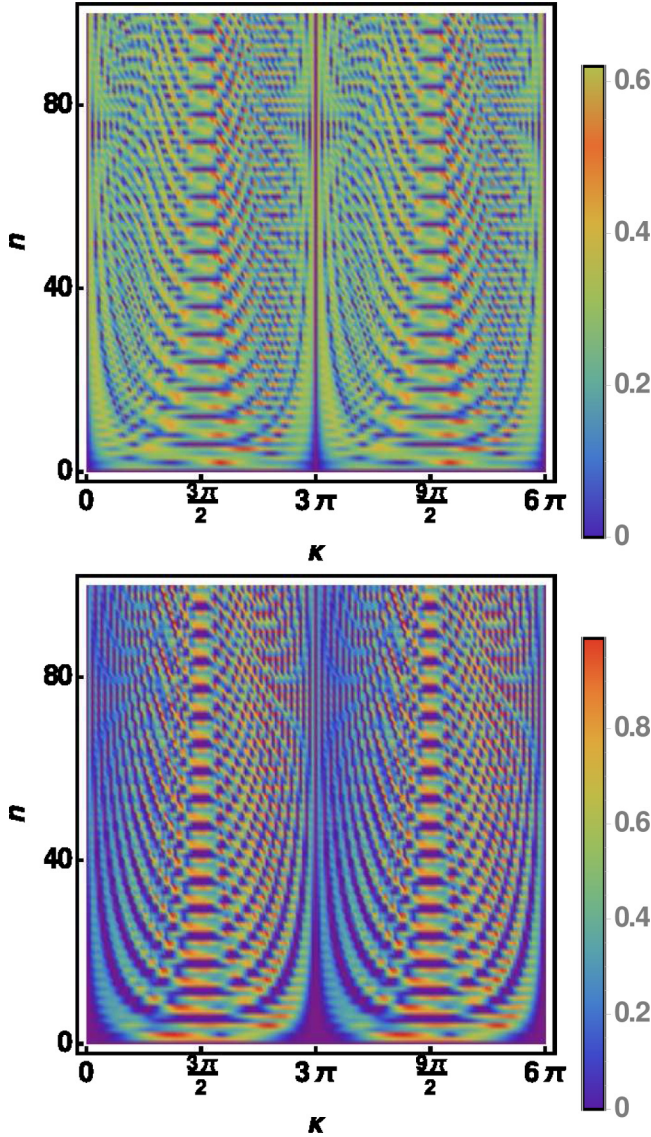
FIG. 6. Same as Fig. 4 for $j = 2$.

FIG. 5. (top) Concurrence (C_{12}) between any two qubits, (bottom) 3-tangle (τ). Both are plotted as function of kick strength k and time. In this, $j = 3/2$. The concurrence and 3-tangle values are color coded using the color map shown by the side.

can observe the unique signatures of the phase space only up to $k = \pi$. If three qubits are used, as done in the case of a recent experimental realization reported in Ref. [19], one can observe the unique signatures of the phase space only up to $k = 3\pi/2 \approx 4.71$ and so on.

VI. TIME PERIODICITY OF QUANTUM CORRELATIONS FOR $j = 1$

It can be seen from Fig. 3 that the von Neumann entropy also exhibits periodicity in time for certain values of k. A similar effect, quasiperiodicity of entanglement, was also observed in Refs. [7,10]. The quantum discord between any two qubits was numerically shown to display quasi-periodic modulations for initial states localized in the regular regions [17]. It was also pointed out that all the quantum expectation
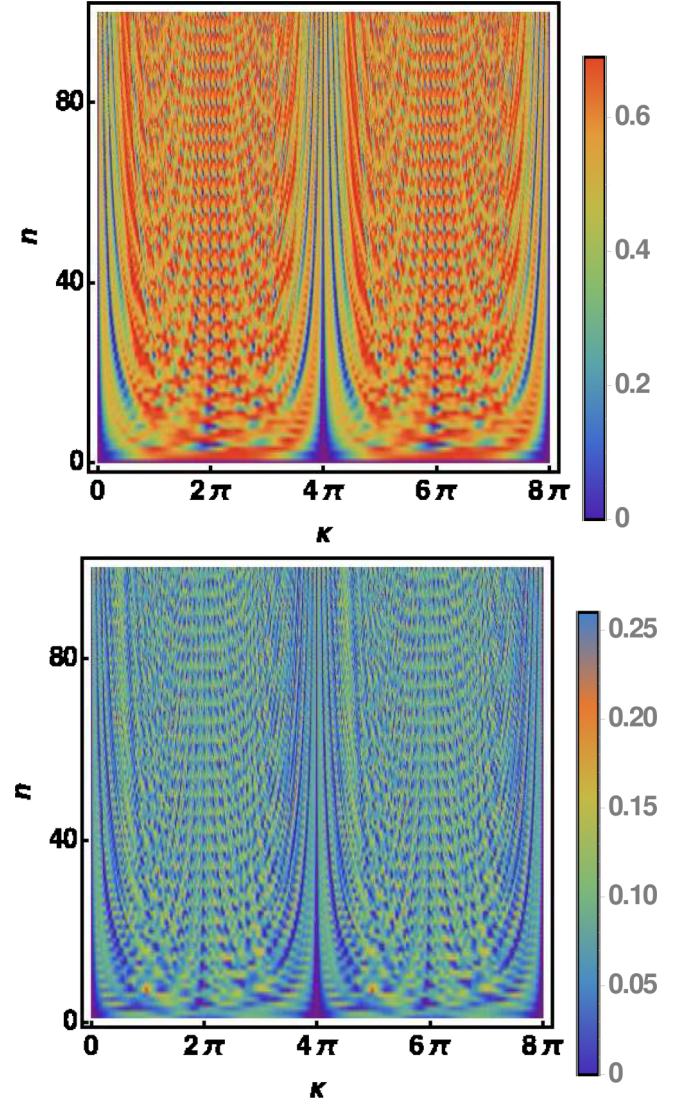
values are quasiperiodic in in time due to the discreteness of the spectrum of Floquet operator [48].

In this section, the $j = 1$ case is considered and it is shown analytically that when k is a rational multiple of π, and p takes value from the set $\{0, \pi/2, \pi, 3\pi/2, 2\pi\}$, the quantum correlations show periodic nature. We note that in the experiments reported in Refs. [2,19], $p = \pi/2$ is used. In Fig. 3 the von Neumann entropy is plotted for $p = \pi/2$ and $k = r\pi/40$ such that $r = 0, 1, \ldots, 160$. This gives the time period as 160. This section is devoted to explaining this observation. Starting from Eq. (16) the matrix elements of the corresponding Floquet operator can be determined and assembled in matrix form.

A. Case of $p = \pi/2$

If $p = \pi/2$, then the Floquet operator reduces to

$$U = \begin{pmatrix} \frac{e^{-ik/2}}{2} & \frac{-e^{-ik/2}}{\sqrt{2}} & \frac{e^{-ik/2}}{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ \frac{e^{-ik/2}}{2} & \frac{e^{-ik/2}}{\sqrt{2}} & \frac{e^{-ik/2}}{2} \end{pmatrix}. \tag{25}$$

Its eigenvalues are $\{e^{-ik/2}, -i\, e^{-ik/4}, i\, e^{-ik/4}\}$ and the corresponding eigenvectors are $[1/\sqrt{2}, 0, 1/\sqrt{2}]^T$, $[-1/2, -i\, e^{ik/4}/\sqrt{2}, 1/2]^T$, and $[-1/2, i\, e^{ik/4}/\sqrt{2}, 1/2]^T$,

respectively. Using these the Floquet operator for nth time can be obtained, which is given as follows:

$$
U^n = \frac{1}{4}\begin{pmatrix} 2\,e^{-ikn/2} + (-ie^{-ik/4})^n + (ie^{-ik/4})^n & \frac{(-ie^{-ik/4})^n - (ie^{-ik/4})^n}{i\,2^{-1/2}\,e^{ik/4}} & 2\,e^{-ikn/2} - (-ie^{-ik/4})^n - (ie^{-ik/4})^n \\ \frac{(-ie^{-ik/4})^n - (ie^{-ik/4})^n}{-i\,2^{-1/2}\,e^{-ik/4}} & 2[(-ie^{-ik/4})^n - (ie^{-ik/4})^n] & \frac{(-ie^{-ik/4})^n - (ie^{-ik/4})^n}{i\,2^{-1/2}\,e^{-ik/4}} \\ 2\,e^{-ikn/2} - (-ie^{-ik/4})^n - (ie^{-ik/4})^n & \frac{(-ie^{-ik/4})^n - (ie^{-ik/4})^n}{-i\,2^{-1/2}\,e^{ik/4}} & 2\,e^{-ikn/2} + (-ie^{-ik/4})^n + (ie^{-ik/4})^n \end{pmatrix}. \quad (26)
$$

Now, we will consider the case of $k = r\pi/s$, for various choices of integral values of r and s. It will be proved that if r is odd then the time period of quantum correlations is $T = 4s$, otherwise it is $T = 2s$.

Odd r: If r is odd integer and time $n = 4s$, Eq. (26) simplifies to

$$
U^{4s} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (27)
$$

Thus, $U^{4s}[a, b, c]^T = [c, -b, a]^T$. In the two-qubit basis, this becomes

$$
c|1\rangle_1|1\rangle_2 - (b/\sqrt{2})(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2) + a|0\rangle_1|0\rangle_2. \quad (28)
$$

Now, this can be rewritten in the following form:

$$
(\sigma_z \otimes \sigma_z)(\sigma_x \otimes \sigma_x)(a|1\rangle_1|1\rangle_2
$$
$$
+ (b/\sqrt{2})(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2) + c|0\rangle_1|0\rangle_2). \quad (29)
$$

Hence, $[c, -b, a]^T = (\sigma_z \otimes \sigma_z)(\sigma_x \otimes \sigma_x)[a, b, c]^T$ implying that the two states are related to each other by local unitary transformation supporting the claim for the periodicity of quantum correlations.

Even r: In the case of even r, using Eq. (26), one obtains

$$
U^{2s} = \begin{pmatrix} \frac{1-(-1)^{r/2}}{2} & 0 & \frac{1+(-1)^{r/2}}{2} \\ 0 & -(-1)^{r/2} & 0 \\ \frac{1+(-1)^{r/2}}{2} & 0 & \frac{1-(-1)^{r/2}}{2} \end{pmatrix}. \quad (30)
$$

There are two cases depending on the value of r. If r is odd multiple of two, then

$$
U^{2s} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (31)
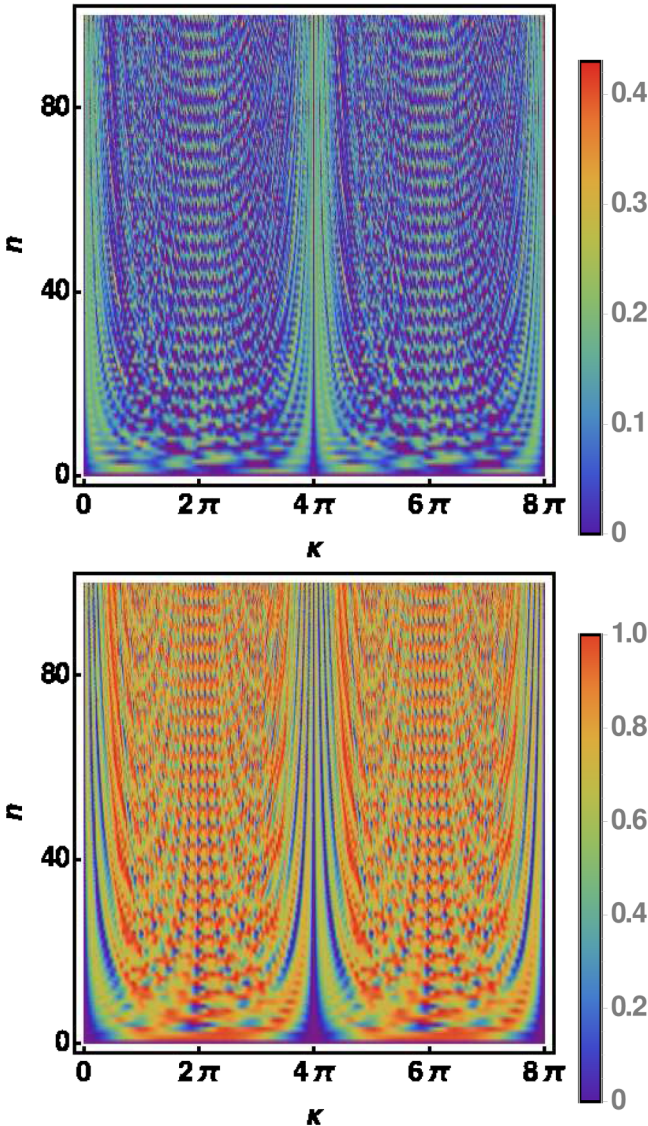$$

FIG. 7. (top) Concurrence (C_{12}) between any two qubits and (bottom) Meyer and Wallach Q measure for $j = 2$. It is color coded using the color map shown by the side.
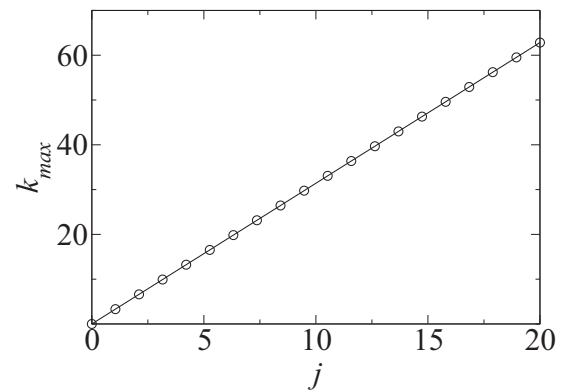

FIG. 8. Maximum value of chaos parameter $k_{\max} = j\pi$ such that phase-space effects on quantum correlations is unique as a function of number of qubits j.

which is an identity matrix implying the periodicity of quantum correlations. If r is even multiple of two, then

$$U^{2s} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{32}$$

Thus, $U^{2s}[a, b, c]^T = [c, -b, a]^T$. In the two-qubit basis $[c, -b, a]^T$ is equal to $c|1\rangle_1|1\rangle_2 - (b/\sqrt{2})(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2) + a|0\rangle_1|0\rangle_2$. Again using the formula for concurrence for two-qubit pure state [29] one obtains $2|b^2/2 - ac|$ for both the states, thus proving the claimed periodicity of quantum correlations. It can be shown that the same results hold true for $p = 3\pi/2$.

B. Case of $p = \pi$

For $p = \pi$ the Floquet operator reduces to

$$\begin{pmatrix} 0 & 0 & e^{-ik/2} \\ 0 & -1 & 0 \\ e^{-ik/2} & 0 & 0 \end{pmatrix}. \tag{33}$$

Its eigenvalues and eigenvectors are, respectively, given as $\{-e^{-ik/2}, e^{-ik/2}, -1\}$, $[-1/\sqrt{2}, 0, 1/\sqrt{2}]^T$, $[1/\sqrt{2}, 0, 1/\sqrt{2}]^T$, and $[0, 1, 0]^T$. Thus, using them the Floquet operator for nth time can be obtained and is given as follows:

$$U^n = \frac{1}{2} \begin{pmatrix} \alpha & 0 & \beta \\ 0 & (-1)^n & 0 \\ \beta & 0 & \alpha \end{pmatrix}, \tag{34}$$

where $\alpha = (-e^{-ik/2})^n + (e^{-ik/2})^n$ and $\beta = -(-e^{-ik/2})^n + (e^{-ik/2})^n$. Consider the case of chaos parameter $k = r\pi/s$. It will be proved that if r is odd then the time period of quantum correlations is $T = 2s$, otherwise, it is $T = s$.

Odd r: In this case using Eq. (34), one obtains

$$U^{2s} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{35}$$

It can be seen that U^{2s} is a diagonal matrix and it is shown in an identical case in Sec. IV that quantum correlations are invariant under its action. Apart from this periodicity of $2s$ additional temporal periodicity is also found. For initial separable state the quantum correlations at times $t = s + l$ and $t = s - l$ are same for $1 \leqslant l \leqslant s - 1$. This argument can be extended to $t > 2s$. Details of the derivation of this result are given in Appendix A.

Even r: Consider the case of even r which implies odd s. It will be now shown that the period is s. Using Eq. (34), one obtains

$$U^s = \begin{pmatrix} 0 & 0 & i^r \\ 0 & -1 & 0 \\ i^r & 0 & 0 \end{pmatrix}. \tag{36}$$

Thus, if r is odd multiple of 2 then $U^s[a, b, c]^T = [-c, -b, -a]$ otherwise $U^s[a, b, c]^T = [c, -b, a]$. It can be seen easily that the concurrence for both the state is $2|b^2/2 - ac|$ proving the claimed periodicity.

In this case, apart from this periodicity of s, additional temporal periodicity is found. For the initial separable state the quantum correlations at times $(s - 2l - 1)/2$ and $(s + 2l + 1)/2$ are same for $1 \leqslant l \leqslant (s - 3)/2$. Details of the derivation of this result are given in Appendix B. It can be shown that the same results holds true for $p = 0$ and 2π. It should be pointed here that no such time periodicity was observed for $j > 1$ (as also shown in Fig. 4, 5, 6, and 7) even if $t \gg 1$. It should also be pointed that these periodicities in k, and that of time for the case $j = 1$, of quantum correlations are of purely quantum origin and are independent of the underlying classical phase space.

VII. SUMMARY

Quantum kicked top is a fundamental model of Hamiltonian chaos and has been realized experimentally in various distinct test-beds, namely, the hyperfine states of cold atoms, coupled superconducting qubits and recently in a two-qubit system using Nuclear Magnetic Resonance techniques [50]. This model advantage that it can be represented in terms of qubits and lends itself naturally to theoretical studies on the connections between quantum correlation measures and classical dynamical properties. With increasing interest in the experimental results using quantum kicked top [4,5], this paper presents new results on the periodic behavior of quantum correlation measures (using j spins to represent the kicked top) as a function of either time or kick strength when certain conditions are satisfied. Due to the periodicity of quantum correlations, experimentally it is sufficient to explore the parameter space corresponding to the basic unit. This work provides an upper bound on the parameter values corresponding to this basic unit.

In particular, it is shown analytically as well as demonstrated numerically that, for a given initial quantum state, the quantum correlations are periodic in kick strength k with a period given by $\kappa = 2j\pi$. A special case of this result was reported in Ref. [7]. Since this is valid for large j, periodicity in k is seen in the semiclassical limit as well. This has also been verified through numerical simulations for bipartite measures of entanglement like the von Neumann entropy, quantum discord and concurrence. Similar numerical results have also been obtained for the multipartite entanglement measures such as 3-tangle and Meyer and Wallah Q measure. The phase space of the kicked top for any given value of k is isomorphic to that at $-k$. This observation, when combined with the periodicity of $\kappa = 2j\pi$ shows that the unique signatures of phase space are obtained only in the range $[0, j\pi]$. This can guide experimental implementations of the kicked top on the appropriate choice of parameters, given the value of j.

Temporal periodicity of quantum correlations are analytically shown to arise for $j = 1$ (two qubit case) if $k = r\pi/s$, where r and s are integers if the angular frequency p can take any of the values from the set $\{0, \pi/2, \pi, 3\pi/2, 2\pi\}$. In the case of $p = \pi/2$, the period is shown to be $T = 4s$ for odd r otherwise it is $T = 2s$, whereas for $p = \pi$ the period is shown to be $T = 2s$ for odd r otherwise it is $T = s$. In the case of $p = \pi$ (same results hold true for $p = 2\pi$) additional temporal periodicity are proved. If the initial state is separable then for odd r it is shown that quantum correlations are same

at $t = s + l$ and $t = s - l$ such that $1 \leqslant l \leqslant s - 1$. Whereas the same is true for even values of r at times $(s - 2l - 1)/2$ and $(s + 2l + 1)/2$ such that $1 \leqslant l \leqslant (s - 3)/2$. These results can be extended for times longer than the respective time periods T.

The case of $j = 1$ has one more experimental implication. Kicked top experiments are limited by the coherence time τ_{coh}, which is typically not large. The entire experiment including the readout should be completed by this timescale. If $k = r\pi/s$ and p is chosen from the set $\{0, \pi/2, \pi, 3\pi/2, 2\pi\}$, then the period T of quantum correlations as a function of time is known from the results obtained in this work. Thus, the relevant timescale for the experiments is $\min(\tau_{\mathrm{coh}}, T)$. This implies that in some cases T can be made smaller than τ_{coh} effectively improving the reliability of the experimental results.

APPENDIX A: DERIVATION OF ADDITIONAL TEMPORAL PERIODICITY FOR $p = \pi$ AND ODD r

In this Appendix additional temporal periodicity for $p = \pi$ and for odd r in the value of $k = r\pi/s$ will be proved. It will be proved that if the initial state $[a, b, c]^T$ is separable then the quantum correlations at time $t = s + l$ and $t = s - l$ are same for $1 \leqslant l \leqslant s - 1$. We will restrict ourselves to time interval $[0, 2s]$ and the argument can be extended to $t > 2s$. Consider the case of odd l. Then, $s \pm l$ will be odd. Thus, using Eq. (34) one obtains

$$U^{s \pm l} = \begin{pmatrix} 0 & 0 & e^{-ir(s \pm l)\pi/2s} \\ 0 & -1 & 0 \\ e^{-ir(s \pm l)\pi/2s} & 0 & 0 \end{pmatrix}. \quad (A1)$$

This implies

$$U^{s \pm l}[a, b, c]^T = [c \, e^{-ir(s \pm l)\pi/2s}, -b, a \, e^{-ir(s \pm l)\pi/2s}]. \quad (A2)$$

This can be written in the two-qubit basis as follows:

$$c \, e^{-ir(s \pm l)\pi/2s} |1\rangle_1 |1\rangle_2 - (b/\sqrt{2})(|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2) + a \, e^{-ir(s \pm l)\pi/2s} |0\rangle_1 |0\rangle_2. \quad (A3)$$

Concurrences for 2-qubit pure states in Eq. (A3) are $2|b^2/2 - a \, c \, e^{-i \, r \, (s \pm l)\pi/s}|$. Since the initial state $[a, b, c]^T$ is separable the concurrence formula gives $ac = b^2/2$, the concurrence becomes

$$2|ac| \, |1 - e^{-ir(s \pm l)\pi/s}| = 2|ac|\sqrt{2[1 - \cos(r\pi \pm rl\pi/s)]}. \quad (A4)$$

The cosines of both these angles are same since they are reflection of each other about x axis. Similarly, it can be shown for even l that the quantum correlations at times $t = s + l$ and $t = s - l$ are same.

APPENDIX B: DERIVATION OF ADDITIONAL TEMPORAL PERIODICITY FOR $p = \pi$ AND EVEN r

In this Appendix additional temporal periodicity for $p = \pi$ and for even r in the value of $k = r\pi/s$ will be proved. It will be proved that if the initial state $[a, b, c]^T$ is separable then the quantum correlations at times $(s - 2l - 1)/2$ and $(s + 2l + 1)/2$ are same for $1 \leqslant l \leqslant (s - 3)/2$. Consider the case of even $(s - 2l - 1)/2$ which implies $(s + 2l + 1)/2$ is odd since the difference between them is $2l + 1$. Thus, using Eq. (34), one obtains

$$U^{(s-2l-1)/2} = \begin{pmatrix} e^{-ir(s-2l-1)\pi/(4s)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-ir(s-2l-1)\pi/(4s)} \end{pmatrix}, \quad (B1)$$

whereas

$$U^{(s+2l+1)/2} = \begin{pmatrix} 0 & 0 & e^{-ir(s+2l+1)\pi/(4s)} \\ 0 & -1 & 0 \\ e^{-ir(s+2l+1)\pi/(4s)} & 0 & 0 \end{pmatrix}. \quad (B2)$$

This gives

$$U^{(s-2l-1)/2}[a, b, c]^T$$
$$= [a \, e^{-ir(s-2l-1)\pi/(4s)}, b, c \, e^{-ir(s-2l-1)\pi/(4s)}]^T, \quad (B3)$$

while

$$U^{(s+2l+1)/2}[a, b, c]^T$$
$$= [c \, e^{-ir(s+2l+1)\pi/(4s)}, -b, a \, e^{-ir(s+2l+1)\pi/(4s)}]^T. \quad (B4)$$

The concurrence for these states are then

$$2|b^2/2 - a \, c \, e^{-ir(s-2l-1)\pi/(2s)}| \quad \text{and}$$
$$2|b^2/2 - a \, c \, e^{-ir(s+2l+1)\pi/(2s)}|,$$

respectively. Since the initial state $[a, b, c]^T$ is separable implies $ac = b^2/2$. Then the concurrences becomes

$$2|ac| \, |1 - e^{-ir(s-2l-1)\pi/(2s)}| \quad \text{and}$$
$$2|ac| \, |1 - e^{-ir(s+2l+1)\pi/(2s)}|,$$

respectively, which can be written as

$$2\sqrt{2}|ac| \, |1 - \cos[r(s - 2l - 1)\pi/(2s)]| \quad \text{and}$$
$$2\sqrt{2}|ac| \, |1 - \cos[r(s + 2l + 1)\pi/(2s)]|,$$

respectively. It can be seen that for r even $\cos[r(s - 2l - 1)\pi/(2s)]$ and $\cos[r(s + 2l + 1)\pi/(2s)]$ are equal since the angles are reflection of each other about x axis. Similarly, this result can be proved for odd $(s - 2l - 1)/2$.

[1] F. Haake, J. Mod. Opt. **47**, 2883 (2000).

[2] S. Chaudhury, A. Smith, B. E. Anderson, S. Ghose, and P. S. Jessen, Nature **461**, 768 (2009).

[3] U. T. Bhosale and M. S. Santhanam, Phys. Rev. E **95**, 012216 (2017).

[4] M. Kumari and S. Ghose, Phys. Rev. E **97**, 052209 (2018).

[5] V. Madhok, S. Dogra, and A. Lakshminarayan, Opt. Commun. **420**, 189 (2018).

[6] S. Dogra, V. Madhok, and A. Lakshminarayan, arXiv:1808.07741 (2018).

[7] J. B. Ruebeck, J. Lin, and A. K. Pattanayak, Phys. Rev. E **95**, 062222 (2017).

[8] A. Piga, M. Lewenstein, and J. Q. Quach, arXiv:1804.10543 (2018).

[9] M. Lombardi and A. Matzkin, Phys. Rev. E **83**, 016207 (2011).

[10] S. Ghose, R. Stock, P. Jessen, R. Lal, and A. Silberfarb, Phys. Rev. A **78**, 042318 (2008).

[11] P. A. Miller and S. Sarkar, Phys. Rev. E **60**, 1542 (1999).

[12] A. Lakshminarayan, Phys. Rev. E **64**, 036207 (2001).

[13] J. N. Bandyopadhyay and A. Lakshminarayan, Phys. Rev. Lett. **89**, 060402 (2002).

[14] J. N. Bandyopadhyay and A. Lakshminarayan, Phys. Rev. E **69**, 016201 (2004).

[15] G. Stamatiou and D. P. K. Ghikas, Phys. Lett. A **368**, 206 (2007).

[16] L. J. Fiderer and D. Braun, Nat. Commun. **9**, 1351 (2018).

[17] V. Madhok, V. Gupta, D.-A. Trottier, and S. Ghose, Phys. Rev. E **91**, 032906 (2015).

[18] H. Fujisaki, T. Miyadera, and A. Tanaka, Phys. Rev. E **67**, 066201 (2003).

[19] C. Neill, P. Roushan, M. Fang, Y. Chen, M. Kolodrubetz, Z. Chen, A. Megrant, R. Barends, B. Campbell, B. Chiaro *et al.*, Nat. Phys. **12**, 1037 (2016).

[20] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A **53**, 2046 (1996).

[21] I. Bengtsson and K. Zyczkowski, *Geometry of Quantum States: An Introduction to Quantum Entanglement* (Cambridge University Press, Cambridge, 2006).

[22] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).

[23] L. Henderson and V. Vedral, J. Phys. A: Math. Gen. **34**, 6899 (2001).

[24] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[25] A. Datta, Ph.D. thesis, The University of New Mexico, 2008, arXiv:0807.4490; arXiv:1003.5256 (2010).

[26] W. H. Zurek, Ann. Phys. (Berlin) **9**, 855 (2000).

[27] V. Madhok and A. Datta, Phys. Rev. A **83**, 032323 (2011).

[28] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

[29] W. K. Wootters, Quantum Inf. Comput. **1**, 27 (2001).

[30] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, Rev. Mod. Phys. **80**, 517 (2008).

[31] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).

[32] S. Hill and W. K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997).

[33] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).

[34] U. T. Bhosale and A. Lakshminarayan, Phys. Rev. A **94**, 022344 (2016).

[35] D. A. Meyer and N. R. Wallach, J. Math. Phys. **43**, 4273 (2002).

[36] A. Lakshminarayan and V. Subrahmanyam, Phys. Rev. A **71**, 062334 (2005).

[37] J. Karthik, A. Sharma, and A. Lakshminarayan, Phys. Rev. A **75**, 022304 (2007).

[38] W. G. Brown, L. F. Santos, D. J. Starling, and L. Viola, Phys. Rev. E **77**, 021106 (2008).

[39] N. Lambert, C. Emary, and T. Brandes, Phys. Rev. A **71**, 053804 (2005).

[40] G. K. Brennen, Quantum Inf. Comput. **3**, 619 (2003).

[41] A. J. Scott, Phys. Rev. A **69**, 052330 (2004).

[42] D. J. Griffiths, *Introduction to Quantum Mechanics* (Cambridge, India, 2016).

[43] H. Ming-Liang and X. Xiao-Qiang, Chin. Phys. B **17**, 3559 (2008).

[44] F. Haake, M. Kus, and R. Scharf, Z. Phys. B **65**, 381 (1987).

[45] F. T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).

[46] R. J. Glauber and F. Haake, Phys. Rev. A **13**, 357 (1976).

[47] R. R. Puri, *Mathematical Methods of Quantum Optics* (Springer, Berlin, 2001).

[48] F. Haake, *Quantum Signatures of Chaos*, 3rd ed. (Springer, Berlin, 2010).

[49] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).

[50] V. R. Krithika, V. S. Anjusha, U. T. Bhosale, and T. S. Mahesh, arXiv:1810.11963 (2018).